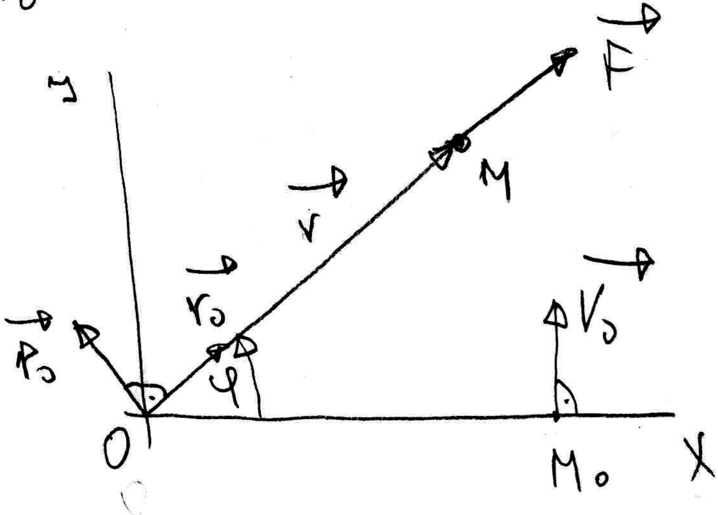
5.3 m

$\vec{F} = k^2 m \vec{r}$

$t_0 = 0$, r_0, $V_0 = k r_0$

$\alpha = \pi/2$

$r(\varphi) - V(\varphi) - ?$



$$C = \frac{1}{2} r_0 V_0 \sin\alpha = \frac{1}{2} r_0 k r_0 \cdot 1 = \frac{1}{2} r_0^2 k$$

$$\boxed{2C = r_0^2 k}$$

$$2C = r^2 \dot{\varphi} = r_0^2 \dot{\varphi}_0 = r_0^2 k \Rightarrow \boxed{\dot{\varphi}_0 = k}$$

$$F_r(r) = + k^2 m r$$

$$T - T_0 = A(\vec{F}_r)$$

$$\frac{m}{2}(V^2 - V_0^2) = \int_{M_0}^{M} F_r \, dr$$

$$V^2 = V_0^2 + \frac{2}{m} k^2 m \int_{r_0}^{r} r \, dr = V_0^2 + k^2 (r^2 - r_0^2)$$

$$V^2 = 4C^2 \left[\left(\left(\frac{1}{r}\right)' \right)^2 + \left(\frac{1}{r}\right)^2 \right] = V_0^2 + k^2 (r^2 - r_0^2)$$

$$\left(\left(\frac{1}{r}\right)' \right)^2 = \frac{1}{r_0^4 k^2} \left[V_0^2 - k^2 r_0^2 + k^2 r^2 - r_0^4 k^2 \left(\frac{1}{r}\right)^2 \right]$$

$$\left(\frac{1}{r}\right)' = \pm\frac{1}{r_0^2 k}\sqrt{V_0^2 - k^2 r_0^2 + k^2 r^2 - r_0^4 k^2 \frac{1}{r^2}}$$

$\frac{1}{r} = z$ – СМЕНА

$$z' = \frac{\pm 1}{r_0^2 k}\sqrt{V_0^2 - k^2 V_0^2 + \left(\frac{k}{z}\right)^2 - V_0^4 k^2 z^2} = \frac{dz}{d\varphi}$$

$$\frac{z\,dz}{\sqrt{1 - r_0^4 z^4}} = \pm\frac{d\varphi}{V_0^2} \quad \Big| \cdot \int \qquad (*)$$

$$\int_{z_0}^{z} \frac{z\,dz}{\sqrt{1 - V_0^4 z^4}} = \left| \begin{array}{l} (V_0 z)^2 = w \\ 2V_0^2 z\,dz = dw \\ z\,dz = \frac{dw}{2V_0^2} \end{array} \right| = \frac{1}{2V_0^2}\int_{w_0}^{w} \frac{dw}{\sqrt{1 - w^2}}$$

$$= \frac{1}{2V_0^2} \arcsin w \Big|_{w_0}^{w} = \frac{1}{2V_0^2} \arcsin (V_0 z)^2 \Big|_{z_0}^{z} =$$

$$= \frac{1}{2V_0^2} \arcsin\left(\frac{r_0}{r}\right)^2 \Big|_{r_0}^{r} = \frac{1}{2V_0^2}\left[\arcsin\left(\frac{V_0}{r}\right)^2 - \arcsin\left(\frac{V_0}{V_0}\right)^2\right]$$

$$= \frac{1}{2V_0^2}\left[\arcsin\left(\frac{V_0}{V}\right)^2 - \frac{\pi}{2}\right]$$

$(*)$

$$\frac{1}{2V_0^2}\left[\arcsin\left(\frac{V_0}{r}\right)^2 - \frac{\pi}{2}\right] = \pm\frac{\varphi}{V_0^2} \quad \Big| \cdot 2V_0^2$$

$$\arcsin\left(\frac{V_0}{V}\right)^2 - \frac{\pi}{2} = \pm 2\varphi$$

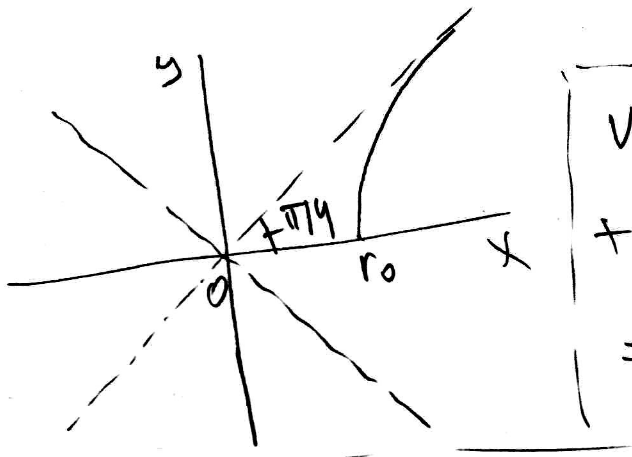
$$\arcsin\left(\frac{r_0}{r}\right)^2 = \pm 2\varphi + \frac{\pi}{2}$$

$$\left(\frac{r_0}{r}\right)^2 = \sin\left(\pm 2\varphi + \frac{\pi}{2}\right) = \cos(\pm 2\varphi)$$

понеже $r \geq 0,\ 0 \leq 2\varphi < \frac{\pi}{2}$

$0 \leq \varphi < \frac{\pi}{4}$

$$r = \frac{\pm r_0}{\sqrt{\cos(\pm 2\varphi)}} \qquad r = \frac{r_0}{\sqrt{\cos(2\varphi)}}$$



$$v^2 = v_0^2 + k^2(r^2 - r_0^2) = k^2 r_0^2 + k^2\left(\frac{r_0^2}{\cos 2\varphi} - r_0^2\right) = k^2 r_0^2/\cos 2\varphi = k^2 \frac{r_0^2}{\cos 2\varphi} = k^2 r^2 \quad \boxed{v = kr}$$

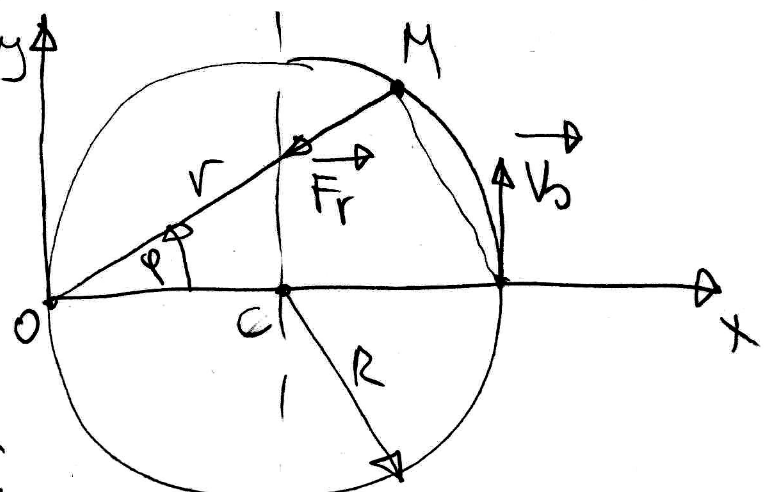
5.4 m, $F_r(r) < 0$

$$\vec{F}_r(r) = -\frac{mk^2}{r^5}\vec{r}_0$$

$\vec{v}_0 \perp \vec{r}_0$, $\alpha = 90^\circ$

$R = r_0/2$

$v_0 - ?$



$$T - T_0 = A(\vec{F}_r)$$

$$\frac{m}{2}(v^2 - v_0^2) = -mk^2 \int_{r_0}^{r} \frac{dr}{r^5}$$

$$v^2 - v_0^2 = 2k^2 \int_{\varphi_0}^{\varphi} \frac{r_0 \sin\varphi\, d\varphi}{r_0^5 \cos^5\varphi} = \left|\begin{array}{c} \cos\varphi = w \\ -\sin\varphi\, d\varphi \\ = dw \end{array}\right|$$

$$\cos\varphi = \frac{r}{2R} = \frac{r}{r_0}$$

$$r = r_0 \cos\varphi$$

$$dr = -r_0 \sin\varphi\, d\varphi$$

$$v^2 = v_0^2 + \frac{2k^2}{v_0^4}\frac{1}{4}\left[\left(\frac{r_0}{r}\right)^4 - 1\right] \Rightarrow$$

$$v_0^2 = v^2 + \frac{k^2}{2r_0^4}\left[1 - \left(\frac{r_0}{r}\right)^4\right]$$

$$v^2 = 4c^2\left[\left(\frac{1}{r}\right)'^2 + \left(\frac{1}{r}\right)^2\right]$$

$$2c = r_0 v_0 \sin\alpha = 2R \cdot v_0 \cdot 1 = r_0 v_0 \quad)^2$$

$$4c^2 = r_0^2 v_0^2$$

$$r = r_0\cos\varphi$$

$$\frac{1}{r} = \frac{1}{r_0\cos\varphi} \Rightarrow \left(\frac{1}{r}\right)' = \frac{d}{d\varphi}\left(\frac{1}{r_0\cos\varphi}\right) = \frac{\sin\varphi}{r_0\cos^2\varphi}$$

$$v_0^2 + \frac{k^2}{2r_0^4}\left[\frac{r_0^4}{r_0^4\cos^4\varphi} - 1\right] = r_0^2 v_0^2\left(\frac{\sin^2\varphi + \cos^2\varphi}{r_0^2\cos^4\varphi}\right)$$

⋮

$$\boxed{v_0^2 = \frac{k^2}{2r_0^4}}$$

5.5

$m, \; F_r(r) < 0$

$$F_r(r) = -\frac{m p V_0^2}{4 r^2}, \quad p = const$$

$$t_0 = 0 \quad r_0 = p/2, \quad \alpha = 90^\circ$$

$V(r)$ — ?

$$2C = r_0 V_0 \sin\alpha = \frac{p}{2} \cdot V_0 \cdot 1 = \frac{p V_0}{2}$$

$$4C^2 = \frac{p^2 V_0^2}{4}$$

$$T - T_0 = A(\vec{F_r})$$

$$\frac{m}{2}(V^2 - V_0^2) = \int_{r_0}^{r} F_r(r)\,dr$$

$$V^2 = V_0^2 + \frac{2}{m}\int_{r_0}^{r} F_r(r)\,dr = V_0^2 - \frac{2}{m}\frac{m p V_0^2}{4}\int_{r_0}^{r}\frac{dr}{r^2}$$

$$V^2 = V_0^2 + \frac{p V_0^2}{2}\left(\frac{1}{r} - \frac{2}{p}\right) = V_0^2\left[1 + \frac{1}{2}\left(\frac{p - 2r}{r}\right)\right]$$

$$V^2 = 4C^2\left[\left(\frac{1}{r}\right)'^2 + \left(\frac{1}{r}\right)^2\right]$$

$$V_0^2\left[1 + \frac{1}{2}\left(\frac{p - 2r}{r}\right)\right] = \frac{1}{4}p^2 V_0^2\left[\left(\frac{1}{r}\right)'^2 + \frac{1}{r}\right]$$

$\vdots$

$$\left(\frac{1}{r}\right)'^2 = \frac{2}{pr} - \frac{1}{r^2}, \quad z = \frac{1}{r}, \quad z_0 = \frac{1}{r_0} = \frac{2}{p}$$

$$z'^2 = \frac{2}{p}z - z^2$$

$$z' = \pm\sqrt{\frac{2}{p} - z^2} = \frac{dz}{d\varphi}$$

$$\pm d\varphi = \frac{dz}{\sqrt{\frac{2}{p} - z^2}} \quad \Big| \cdot \int$$

(*)

$$\pm\int_0^{\varphi} d\varphi = \int_{z_0}^{z} \frac{dz}{\sqrt{\frac{2}{p} - z^2}}$$

$$\int_{z_0}^{z} \frac{dz}{\sqrt{\frac{2}{p} - z^2}} = \left| \begin{array}{l} u = zp - 1 \Rightarrow \frac{1-u^2}{p^2} = \frac{2}{p}z - z^2 \\ du = p\,dz \Rightarrow dz = \frac{du}{p} \end{array} \right| =$$

$$= \int_{u_0}^{u} \frac{\frac{1}{p}\,du}{\frac{1}{p}\sqrt{1-u^2}} = \arcsin u \Big|_{u_0}^{u} = \arcsin(zp-1)\Big|_{z_0}^{z} =$$

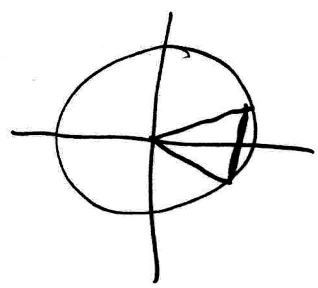
$$= \arcsin(zp-1) - \arcsin 1 = \arcsin(zp-1) - \frac{\pi}{2}$$

(*)

$$\pm\varphi = \arcsin(zp-1) - \frac{\pi}{2}$$

$$zp - 1 = \sin\left(\pm\varphi + \frac{\pi}{2}\right) = \cos(\pm\varphi)$$

$$\cos(\varphi) = \cos(-\varphi)$$



⋮

$$r = \frac{p}{1+\cos(\pm\varphi)}$$

$$\boxed{r = \frac{p}{1+\cos\varphi}}$$

$$\frac{1}{r} = \frac{1+\cos\varphi}{p}$$

$$\frac{d}{d\varphi}\left(\frac{1}{r}\right) = -\frac{1}{p}\sin\varphi$$

$$v^2 = \frac{1}{4}p^2 v_0^2\left(\frac{\sin^2\varphi}{p^2} + \frac{1+2\cos\varphi+\cos^2\varphi}{p^2}\right) =$$

$$= \dots = \frac{p}{2r} v_0^2$$

5.8 m

$v = kr$, $k = \text{const.}$

$t_0 = 0$ $r_0 = b$ $\alpha = 45^\circ$

$F_r(r) - r - ?$

$v_0 = k r_0 = kb$

$2C = r_0 v_0 \sin\alpha = b \cdot kb \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} kb^2$

$4c^2 = \frac{1}{2} b^4 k^2$

$T = \frac{1}{2} m v^2 = \frac{1}{2} m k^2 r^2$

$dT = m k^2 r\,dr$

$dT = F_r(r)\,dr \Rightarrow F_r(r) = \frac{dT}{dr} = m k^2 r$

$$F_r(r) = -\frac{4c^2 m}{r^2}\left[\left(\frac{1}{r}\right)'' + \left(\frac{1}{r}\right)\right]$$

$$\left(\frac{1}{r}\right)'' + \frac{1}{r} = -\frac{2r^3}{b^4}, \quad z = \frac{1}{r}$$

$$z'' = -z - \frac{2}{b^4 z^3} = \frac{dz'}{d\varphi}\frac{dz}{dz} = \frac{z'\,dz'}{dz} \quad \Big|\cdot dz$$

$$z'\,dz' = -z\,dz - \frac{2}{b^4 z^3}\,dz \quad \Big|\int$$

$$\frac{1}{2}z'^2\Big|_{z_0'}^{z'} = -\frac{1}{2}z^2\Big|_{z_0}^{z} + \frac{1}{b^4}\frac{1}{z^2}\Big|_{z_0}^{z} \quad (*) \quad \Big|\cdot 2$$

$$z_0 = \frac{1}{r_0} = \frac{1}{b}, \quad z' = \left(\frac{1}{r}\right)' = \frac{d}{d\varphi}\left(\frac{1}{r}\right) = -\frac{dr}{d\varphi}\frac{1}{r^2} =$$

$$= -\frac{1}{r^2}\frac{\frac{dr}{dt}}{\frac{d\varphi}{dt}} = -\frac{1}{r^2}\frac{\dot r}{\dot\varphi}$$

$$z_0' = \left(\frac{1}{r}\right)'_0 = -\frac{1}{r_0^2}\frac{\dot r_0}{\dot\varphi_0} = -\frac{1}{r_0}\frac{\dot r_0}{V_0\sin\alpha} = -\frac{1}{r_0}\frac{V_0\cos\alpha}{V_0\sin\alpha}$$

$$= -\frac{\operatorname{ctg}\alpha}{r_0} = -\frac{1}{b}$$

$$2c = r_0 V_0 \sin\alpha = r_0 \cdot r_0\dot\varphi_0 = r_0^2\dot\varphi_0, \quad V_0\sin\alpha = r_0\dot\varphi_0$$

$$(*) \quad z'^2 - z_0'^2 = -z^2 + z_0^2 + \frac{2}{b^4}\left(\frac{1}{z^2} - \frac{1}{z_0^2}\right)$$

$$z'^2 = \frac{2 - b^4 z^4}{b^4 z^2}$$

$$z' = \frac{dz}{d\varphi} = -\frac{\sqrt{2 - b^4 z^4}}{b^2 z} \qquad \left(z_0' = -\frac{1}{b} < 0 \right)$$

$$d\varphi = -\frac{b^2 z \, dz}{\sqrt{2 - b^4 z^4}} \qquad \Big| \cdot \int \quad (*)$$

$$-\int_{z_0}^{z} \frac{b^2 z \, dz}{\sqrt{2 - b^4 z^4}} = \frac{1}{2} \int_{z_0}^{z} -\frac{\sqrt{2}\, b^2 z \, dz}{\sqrt{1 - \frac{1}{2} b^4 z^4}} = \left| \begin{array}{l} \frac{b^2 z^2}{\sqrt{2}} = w \\ \sqrt{2}\, b^2 z \, dz = dw \end{array} \right|$$

$$= \frac{1}{2} \int_{w_0}^{w} -\frac{dw}{\sqrt{1 - w^2}} = \frac{1}{2} \arccos w \Big|_{w_0}^{w} =$$

$$= \frac{1}{2} \arccos\left(\frac{b^2 z^2}{\sqrt{2}} \right) \Big|_{z_0}^{z} = \frac{1}{2} \left[\arccos\left(\frac{b^2 z^2}{\sqrt{2}} \right) - \frac{\pi}{4} \right]$$

$(*)$

$$\varphi = \frac{1}{2} \left[\arccos\left(\frac{b^2 z^2}{\sqrt{2}} \right) - \frac{\pi}{4} \right] \quad \Big| \cdot 2$$

$$2\varphi + \frac{\pi}{4} = \arccos\left(\frac{b^2 z^2}{\sqrt{2}} \right)$$

$$\frac{b^2 z^2}{\sqrt{2}} = \cos\left(2\varphi + \frac{\pi}{4} \right) \Rightarrow r = \frac{b}{\sqrt[4]{2}\, \sqrt{\cos\left(2\varphi + \frac{\pi}{4} \right)}}$$

5.13 m

$r^2 = 4 + t^2$

$V_0 = 2\ m/s$, $\alpha = 90^\circ$

$F_r - r - ?$

$r^2 = 4 + t^2 \quad \Big| \frac{d}{dt}$ $\qquad r = \sqrt{4+t^2}$

$\cancel{2} r \dot{r} = \cancel{2} t$

$$\dot{r} = \frac{t}{r} = \frac{t}{\sqrt{4+t^2}} = \sqrt{1 - \frac{4}{r^2}}$$

$\dot{r}_0 = V_0 \cos\alpha = 0$

$V_0 \sin\alpha = r_0 \dot{\varphi}_0 = 2 \Rightarrow \boxed{\dot{\varphi}_0 = \frac{2}{2} = 1}$

$2C = r_0 V_0 \sin\alpha = r_0 \cdot r_0 \dot{\varphi}_0 = r_0^2 \dot{\varphi}_0 = 4$

$4C^2 = 16$

$$2C = r^2 \dot{\varphi} = r_0^2 \dot{\varphi}_0 \Rightarrow \dot{\varphi} = \frac{2C}{r^2} = \frac{d\varphi}{dt} \frac{dr}{dr}$$

$$\frac{2C}{r^2 \dot{r}} = \frac{d\varphi}{dr} \Rightarrow d\varphi = \frac{2C\, dr}{r^2 \sqrt{1 - \frac{4}{r^2}}} \quad \Big| \cdot \int$$

$$\varphi = \int_{r_0}^{r} \frac{4\, dr}{r^2 \sqrt{1 + \frac{4}{r^2}}} = \left| \begin{matrix} 2r^{-1} = w \\ -2 \frac{dr}{r^2} = dw \end{matrix} \right| = -\int_{w_0}^{w} \frac{2\, dw}{\sqrt{1 - w^2}}$$

$$= -2\arcsin w \Big|_{w_0}^{w} = 2\arcsin\left(\frac{2}{r}\right)\Big|_{r}^{r_0} =$$

$$= 2\left[\arcsin\left(\frac{2}{r_0}\right) - \arcsin\left(\frac{2}{r}\right)\right]$$

$$\varphi = 2\left[\frac{\pi}{2} - \arcsin\left(\frac{2}{r}\right)\right]$$

$$\frac{\varphi}{2} = \frac{\pi}{2} - \arcsin\left(\frac{2}{r}\right)$$

$$r = \frac{2}{\sin\left(\frac{\pi}{2} - \frac{\varphi}{2}\right)} = \frac{2}{\cos\frac{\varphi}{2}} \qquad 0 \le \frac{\varphi}{2} < \frac{\pi}{2} \qquad 0 \le \varphi < \pi$$

$$\frac{1}{r} = \frac{1}{2}\cos\frac{\varphi}{2}$$

$$\left(\frac{1}{r}\right)' = -\frac{1}{4}\sin\frac{\varphi}{2}$$

$$\left(\frac{1}{r}\right)'' = -\frac{1}{8}\cos\frac{\varphi}{2} = -\frac{1}{4}\frac{1}{r}$$

$$F_r = -4c^2\frac{m}{r^2}\left[\left(\frac{1}{r}\right)'' + \left(\frac{1}{r}\right)\right] =$$

$$= -16\frac{m}{r^2}\left[-\frac{1}{4}\frac{1}{r} + \frac{1}{r}\right] = -\cancel{16}\frac{m}{r^2}\frac{3}{\cancel{4}}\frac{1}{r}$$

$$F_r = -\frac{12m}{r^3}$$