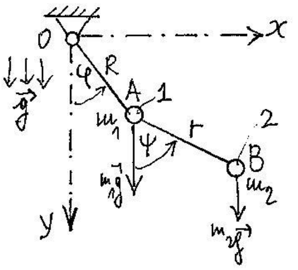
NA PRIMERU SA "DVOGUBIM" MATEMATIČKIM KLATNOM POKAŽIMO KAKO SE "AUTOMATSKI" MOGU IZRAČUNAVATI "INERCIONI" KOEFICIJENTI I GENERALISANE SILE T.J. FORMIRATI LAGRANŽEVE DIF. JED. II VRSTE.

ZA VEŽBU SVAKI STUDENT TREBA DA FORMIRA OVE DIFERENCIJALNE JEDNAČINE DVOGUBOG KLATNA NA "KLASIČAN" NAČIN (KOJI SMO SAVLADALI U OVOM KURSU) ($T=\frac{1}{2}m_1V_A^2+\frac{1}{2}m_2V_B^2$; $\delta A=Q_\varphi\delta\varphi+Q_\psi\delta\psi$)



SISTEM IMA 2 STEPENA SLOBODE KRETANJA, NEKA SU $q_1=\varphi$; $q_2=\psi$ (APSOLUTNI UGLOVI)

$$T=\frac{1}{2}\sum_{\alpha,\beta}a_{\alpha\beta}\dot q^\alpha\dot q^\beta$$

$$a_{\alpha\beta}=a_{\beta\alpha}(q^1,q^2)$$

POŠTO JE VEZA (VEZE) STACIONARNA SVI VEKTORI $\frac{\partial\vec r_k}{\partial t}=0 \Rightarrow B_i=0$ T.J.

$$\boxed{T=T_1}+\cancel{T_2}^{\,0}+\cancel{T_3}^{\,0}\qquad T_1=\frac{1}{2}\sum_{\alpha,\beta=1}^{s}\sum^{s}a_{\alpha\beta}\dot q^\alpha\dot q^\beta$$

$$a_{\alpha\beta}=\sum_{i=1}^{n}m_i\frac{\partial\vec r_i}{\partial q^\alpha}\cdot\frac{\partial\vec r_i}{\partial q^\beta}$$

[U NAŠEM ZADATKU ($i=1,2$; $\alpha,\beta=1,2$)]: $T=\frac{1}{2}a_{11}\dot\varphi^2+a_{12}\dot\varphi\dot\psi+\frac{1}{2}a_{22}\dot\psi^2$

$$a_{11}=m_1\frac{\partial\vec r_1}{\partial\varphi}\cdot\frac{\partial\vec r_1}{\partial\varphi}+m_2\frac{\partial\vec r_2}{\partial\varphi}\cdot\frac{\partial\vec r_2}{\partial\varphi}$$

$$a_{11}=m_1R^2+m_2R^2=(m_1+m_2)R^2$$

$$a_{12}=m_1\frac{\partial\vec r_1}{\partial\varphi}\cdot\frac{\partial\vec r_1}{\partial\psi}+m_2\frac{\partial\vec r_2}{\partial\varphi}\cdot\frac{\partial\vec r_2}{\partial\psi}$$

$$a_{12}=m_1(0)+m_2Rr(\cos\varphi\cos\psi+\sin\varphi\sin\psi)$$

$$a_{22}=m_1\frac{\partial\vec r_1}{\partial\psi}\cdot\frac{\partial\vec r_1}{\partial\psi}+m_2\frac{\partial\vec r_2}{\partial\psi}\cdot\frac{\partial\vec r_2}{\partial\psi}$$

$$a_{22}=m_1(0)+m_2r^2$$

$$\vec r_1\equiv\vec r_A=R\sin\varphi\,\vec i+R\cos\varphi\,\vec j$$

$$\vec r_2\equiv\vec r_B=(R\sin\varphi+r\sin\psi)\vec i+(R\cos\varphi+r\cos\psi)\vec j$$

$$\frac{\partial\vec r_1}{\partial\varphi}=R\cos\varphi\,\vec i-R\sin\varphi\,\vec j$$

$$\frac{\partial\vec r_1}{\partial\psi}=0\qquad\frac{\partial\vec r_2}{\partial\varphi}=R\cos\varphi\,\vec i-R\sin\varphi\,\vec j\qquad\frac{\partial\vec r_2}{\partial\psi}=r\cos\psi\,\vec i-r\sin\psi\,\vec j$$

$$\cos\alpha\cos\psi+\sin\varphi\sin\psi=\cos(\psi-\varphi)$$

$$T=\frac{1}{2}\left[(m_1+m_2)R^2\right]\dot\varphi^2+m_2\left[Rr\cos(\psi-\varphi)\right]\dot\varphi\dot\psi+\frac{1}{2}m_2\left[r^2\right]\dot\psi^2$$

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GENERALISANA SILA

"OSNOVNI VEKTOR" KRIVOLINIJSKE KOORDINATE: $\vec g_\alpha=\frac{\partial\vec r}{\partial q^\alpha}$

$$\underbrace{\vec F\cdot\vec g_\alpha\equiv Q_\alpha}\qquad Q_\alpha=\sum_{i=1}^{2}\vec F_i\cdot\frac{\partial\vec r_i}{\partial q^\alpha}$$

PROJEKCIJA AKTIVNE SILE $\vec F$ NA "α" KOORDINATNU LINIJU DUŽ LUKA ds_α PO TOJ LINIJI – "RAD TE PROJEKCIJE".

q^α (KRIVOLINIJSKA KOORDINATA)

$m_1\vec g=m_1g\vec j$ $\quad m_2\vec g=m_2g\vec j$ $\quad$ (DVE AKTIVNE SILE; VEZE SU IDEALNE)

$$Q_\varphi=m_1\vec g\cdot\frac{\partial\vec r_1}{\partial\varphi}+m_2\vec g\cdot\frac{\partial\vec r_2}{\partial\varphi}\qquad Q_\varphi=m_1g(-R\sin\varphi)+m_2g(-R\sin\varphi)\qquad Q_\varphi=-(m_1+m_2)gR\sin\varphi$$

$$Q_\psi=m_1\vec g\cdot\frac{\partial\vec r_1}{\partial\psi}+m_2\vec g\cdot\frac{\partial\vec r_2}{\partial\psi}\qquad Q_\psi=m_1g(0)+m_2g(-r\sin\psi)\qquad Q_\psi=-m_2gr\sin\psi$$

*

$$\begin{cases}\dfrac{d}{dt}\dfrac{\partial T}{\partial\dot\varphi}-\dfrac{\partial T}{\partial\varphi}=Q_\varphi\\[2mm]\dfrac{d}{dt}\dfrac{\partial T}{\partial\dot\psi}-\dfrac{\partial T}{\partial\psi}=Q_\psi\end{cases}$$

FORMIRATI OVE DIFERENCIJALNE JEDNAČINE KRETANJA

* (U LINEARNOM SLUČAJU ĆETE DOBITI) ($\sin\varphi\approx\varphi$ $\sin\psi\approx\psi$; $\cos\varphi\approx1$ $\cos\psi\approx1$)

$$\begin{cases}(m_1+m_2)R\ddot\varphi+m_2r\ddot\psi+(m_1+m_2)g\varphi=0\\ R\ddot\varphi+r\ddot\psi+g\psi=0\end{cases}$$