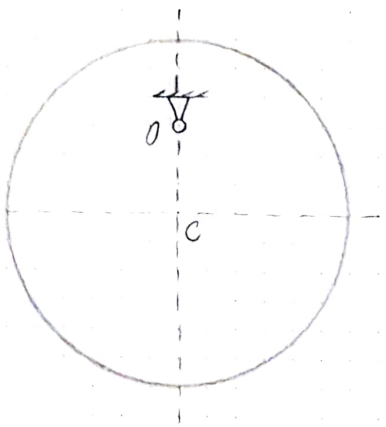
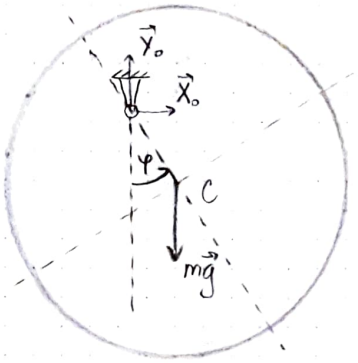
8.22. r $\frac{r}{2}$ Og C Одредити период малих осцилација диска.





(1) $\frac{d\mathcal{L}_{Oz}}{dt} = \sum M_{Oz}(\vec{F}_i^S)$

$\curvearrowleft + M_{Oz}(m\vec{g}) = -mg\frac{r}{2}\sin\varphi = -\frac{1}{2}mgr\sin\varphi$ (2)

$\curvearrowleft + M_{Oz}(\vec{X}_o) = M_{Oz}(\vec{Y}_o) = 0$

(3) $\mathcal{L}_{Oz} = J_{Oz}\dot{\varphi}$

(4) $J_{Oz} = J_{Cz'} + m\left(\frac{r}{2}\right)^2 = \frac{1}{2}mr^2 + \frac{1}{4}mr^2 = \frac{3}{4}mr^2$

(4) $\rightarrow$ (3) $\Rightarrow$ $\mathcal{L}_{Oz} = \frac{3}{4}mr^2\dot{\varphi}$

(5) $\frac{d\mathcal{L}_{Oz}}{dt} = \frac{3}{4}mr^2\ddot{\varphi}$

(2), (5) $\rightarrow$ (1) $\Rightarrow$ $\frac{3}{4}mr^2\ddot{\varphi} = -\frac{1}{2}mgr\sin\varphi \;/:\frac{3}{4}mr^2$

$\ddot{\varphi} + \frac{2}{3}\frac{g}{r}\sin\varphi = 0$

$\sin\varphi \approx \varphi \Rightarrow$ МАЛЕ ОСЦИЛАЦИЈЕ

$\ddot{\varphi} + \underbrace{\frac{2}{3}\frac{g}{r}}_{\omega^2}\varphi = 0$

$\omega = \sqrt{\frac{2}{3}\frac{g}{r}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3r}{2g}} = \pi\sqrt{\frac{4\cdot 3r}{2g}}$

$\underline{T = \pi\sqrt{\frac{6r}{g}}}$