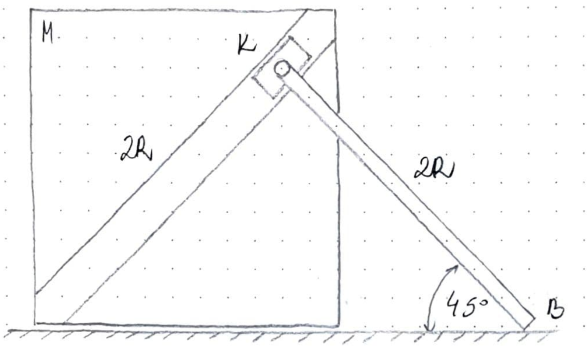
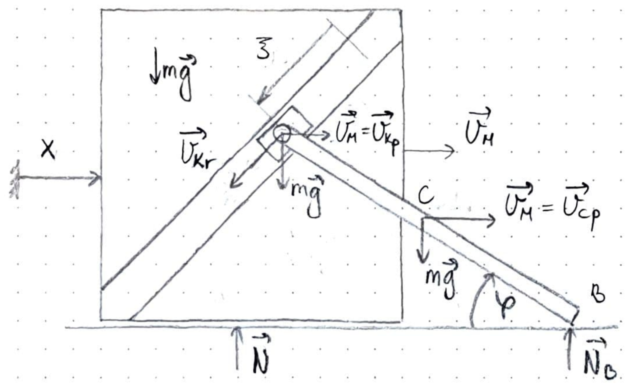
8.58. Тело M масе m може да клизи без трења по хоризонталној равни. У тело је, под углом од 45° у односу на хоризонталу урезан жлеб по коме може да клизи без трења клизач K масе m. За клизач је зглобно везан хомогени штап масе m и дужине 2R чији крај B може да клизи без трења по хоризонталној равни. У почетном тренутку, када је штап са хоризонталом градио угао од 45°, систем је мировао. Одредити брзину тела M у тренутку када штап дође у хоризонтални положај.





(1) $E_{k_1} - E_{k_0} = A^K_{0-1}(m\vec g) + A^C_{0-1}(m\vec g) + \cancel{A^M_{0-1}(m\vec g)}_0 + \cancel{A_{0-1}(\vec N)}_0 + \cancel{A_{0-1}(\vec N_B)}_0$

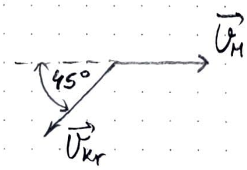
(2) $E_k = E_k^K + E_k^M + E_k^{\check{S}}$

ТРАНС. $E_k^M = \frac{1}{2} m v_M^2$ (3), $v_M = \dot x$

ТАЧКА $E_k^K = \frac{1}{2} m v_{K_{APS}}^2$, $\vec v_{K_{APS}} = \vec v_{Kp} + \vec v_{Kr}$

$\vec v_{K_{APS}} = \vec v_M + \vec v_{Kr} \; /\cdot\vec i/\cdot\vec j$

*ИСТО КАО ЗА ТРАНСЛАЦИЈУ

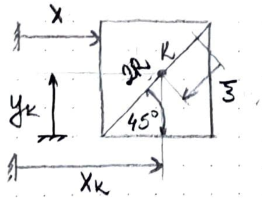


$v_{Kr} = \dot\xi$

$$\left.\begin{aligned} v_{K_{APS}x} &= v_M - v_{Kr}\frac{\sqrt2}{2} \\ v_{K_{APS}y} &= -v_{Kr}\frac{\sqrt2}{2} \end{aligned}\right\} v_{K_{APS}}^2 = v_{K_{APS}x}^2 + v_{K_{APS}y}^2 = v_M^2 - \sqrt2 v_M v_{Kr} + v_{Kr}^2$$

(4) $E_k^K = \frac{1}{2} m\left(v_M^2 - \sqrt2 v_M v_{Kr} + v_{Kr}^2\right) = \frac{1}{2} m\left(\dot x^2 - \sqrt2 \dot x \dot\xi + \dot\xi^2\right)$

II НАЧИН => "ЛАКШИ"



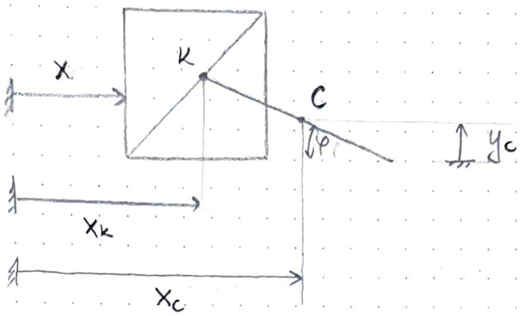
$x_K = x + (2R - \xi)\frac{\sqrt2}{2} \Rightarrow \dot x_K = \dot x - \frac{\sqrt2}{2}\dot\xi = v_M - \frac{\sqrt2}{2} v_{Kr}$

$y_K = (2R - \xi)\frac{\sqrt2}{2} \Rightarrow \dot y_K = -\frac{\sqrt2}{2}\dot\xi = -\frac{\sqrt2}{2} v_{Kr}$

$v_{K_{APS}}^2 = \dot x_K^2 + \dot y_K^2 = v_M^2 - \sqrt2 v_M v_{Kr} + v_{Kr}^2$

РАВНО $\quad E_k^{\check{s}} = \frac{1}{2} m v_c^2 + \frac{1}{2} J_{Cz} \dot{\varphi}^2 \quad , \quad J_{Cz} = \frac{1}{12} \cdot m \cdot (2R)^2 = \frac{1}{3} m R^2$

$$v_c^2 = \dot{x}_c^2 + \dot{y}_c^2$$



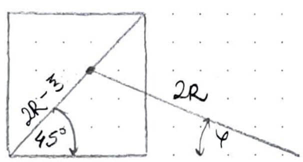
$$x_C = x_K + R\cos\varphi$$
$$y_C = R\sin\varphi$$
$$\dot{x}_C = \dot{x}_K - R\dot{\varphi}\sin\varphi = \dot{x} - \frac{\sqrt{2}}{2}\dot{\xi} - R\dot{\varphi}\sin\varphi$$
$$\dot{y}_C = R\dot{\varphi}\cos\varphi$$

$$(5)\quad E_k^{\check{s}} = \frac{1}{2} m (\dot{x}_c^2 + \dot{y}_c^2) + \frac{1}{6} m R^2 \dot{\varphi}^2$$

$$(3),(4),(5) \rightarrow (2) \Rightarrow E_k = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m (\dot{x}^2 - \sqrt{2}\dot{x}\dot{\xi} + \dot{\xi}^2) + \frac{1}{2} m (\dot{x}_c^2 + \dot{y}_c^2) + \frac{1}{6} m R^2 \dot{\varphi}^2 \quad (6)$$

$t_0 = 0 \Rightarrow \varphi = 45^\circ$, систем је мировао $\Rightarrow E_{k0} = 0 \quad (7)$

$t_1 \Rightarrow \varphi_1 = 0, \quad \dot{\varphi}_1, v_{Mr_1} \ldots ?$



$$(2R - \xi)\frac{\sqrt{2}}{2} = 2R\sin\varphi \,/\cdot \frac{d}{dt}$$

$$-\frac{\sqrt{2}}{2}\dot{\xi} = 2R\dot{\varphi}\cos\varphi \qquad \dot{\xi} = v_{Mr}$$

$$t_1 \Rightarrow \varphi_1 = 0 \quad \Rightarrow \quad -\frac{\sqrt{2}}{2}\dot{\xi} = 2R\dot{\varphi}_1 \overset{1}{\cos 0}$$

$$\dot{\varphi}_1 = -\frac{\sqrt{2}}{4}\frac{\dot{\xi}}{R} \quad (8)$$

ТЕОРЕМА О КРЕТАЊУ ЦЕНТРА МАСА

$$m\vec{a}_M + m\vec{a}_K + m\vec{a}_C = 3m\vec{g} + \vec{N} + \vec{N}_B \,/\cdot \vec{i}$$
$$m\ddot{x} + m\ddot{x}_K + m\ddot{x}_C = 0 \,/: m \,/ \int$$
$$\dot{x} + \dot{x}_K + \dot{x}_C = const. = \dot{x}_M(0) + \dot{x}_K(0) + \dot{x}_C(0) = 0$$
$$\dot{x} + \dot{x}_K + \dot{x}_C = 0$$
$$\dot{x} + \dot{x} - \frac{\sqrt{2}}{2}\dot{\xi} + \dot{x} - \frac{\sqrt{2}}{2}\dot{\xi} - R\dot{\varphi}\sin\varphi = 0 \Rightarrow 3\dot{x} - \sqrt{2}\dot{\xi} - R\dot{\varphi}\sin\varphi = 0$$

$$t_1 \Rightarrow 3\dot{x}_1 - \sqrt{2}\dot{\xi}_1 - R\dot{\varphi}_1 \overset{0}{\sin 0} = 0 \quad \Rightarrow \quad \dot{\xi}_1 = v_{Mr_1} = \frac{3\sqrt{2}}{2}\dot{x}_1 = \frac{3\sqrt{2}}{2} v_{M_1} \quad (9)$$

$$(8) \Rightarrow \dot{\varphi}_1 = -\frac{\sqrt{2}}{4} \cdot \frac{3\sqrt{2}}{2}\frac{v_{M1}}{R} = -\frac{3}{4}\frac{v_{M1}}{R}$$

$$E_{k_1} = \frac{1}{2} m v_{M_1}^2 + \frac{1}{2} m \left(v_{M_1}^2 - \sqrt{2} v_{M_1} \frac{3\sqrt{2}}{2} v_{M_1} + \frac{9}{2} v_{M_1}^2\right)$$
$$+ \frac{1}{2} m \left[\left(v_{M_1} - \frac{\sqrt{2}}{2}\cdot\frac{3\sqrt{2}}{2} v_{M_1} - R\dot{\varphi}_1 \overset{0}{\sin 0}\right)^2 + \left(-\frac{3}{4}\frac{v_{M1}}{R}\cdot R \overset{1}{\cos 0}\right)^2\right] + \frac{1}{6} m R^2 \frac{9}{16}\frac{v_{M_1}^2}{R^2}$$

$$\left.\begin{aligned} E_{k_1} &= \frac{9}{4} m v_{M_1}^2 \\ A_{0-1}^K(m\vec{g}) &= +mg\cdot 2R\frac{\sqrt{2}}{2} = \sqrt{2}mgR \\ A_{0-1}^C(m\vec{g}) &= +mg\cdot \underbrace{R\frac{\sqrt{2}}{2}}_{\text{ПРОМЕНА ВИСИНЕ ТАЧКЕ C}} = \frac{\sqrt{2}}{2}mgR \end{aligned}\right\} \rightarrow (1) \Rightarrow \frac{9}{4} m v_{M_1}^2 = \frac{3\sqrt{2}}{2} mgR$$

$$v_{M_1}^2 = \frac{2\sqrt{2}}{3} gR$$