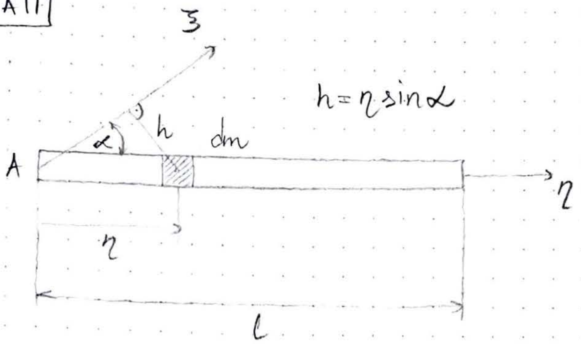
ШТАП



$h = \eta \sin\alpha$

$\xi \Rightarrow$ произвољна оса под углом α

ШТАП $\Rightarrow$ линијски елемент

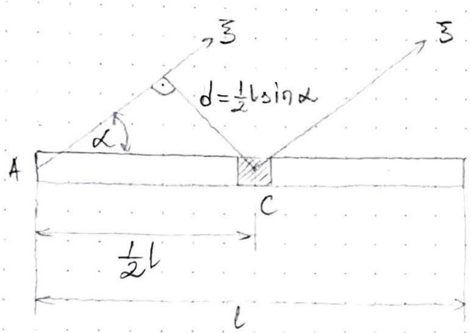
линијска густина $\rho = \frac{m}{l} = const.$

$$\rho = \frac{dm}{d\eta} \Rightarrow dm = \rho\, d\eta = \frac{m}{l} d\eta$$

$$J_{A\xi} = \int_m h^2 dm = \int_0^l \eta^2 \sin^2\alpha \cdot \frac{m}{l} d\eta$$

$$J_{A\xi} = \frac{m}{l}\sin^2\alpha \int_0^l \eta^2 d\eta = \frac{m}{\cancel{l}}\sin^2\alpha \cdot \frac{l^{\cancel{3}\,2}}{3}$$

$$\boxed{\begin{array}{l} J_{A\xi} = \frac{1}{3} m l^2 \sin^2\alpha \\ J_{A\eta} = 0 \quad (h=0) \end{array}}$$



$J_{A\xi} = J_{C\xi'} + m d^2 \Rightarrow$ ШТАЈНЕРОВА ТЕОРЕМА

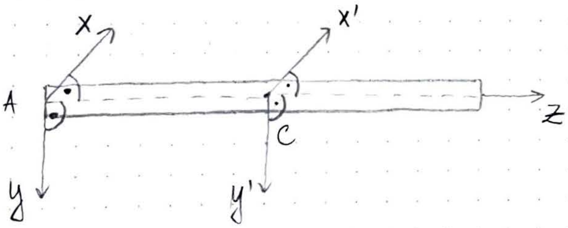
d – нормално растојање од ξ

$$J_{C\xi'} = J_{A\xi} - m d^2 , \quad d = \frac{1}{2} l \sin\alpha$$

$$= \frac{1}{3} m l^2 \sin^2\alpha - \frac{1}{4} m l^2 \sin^2\alpha$$

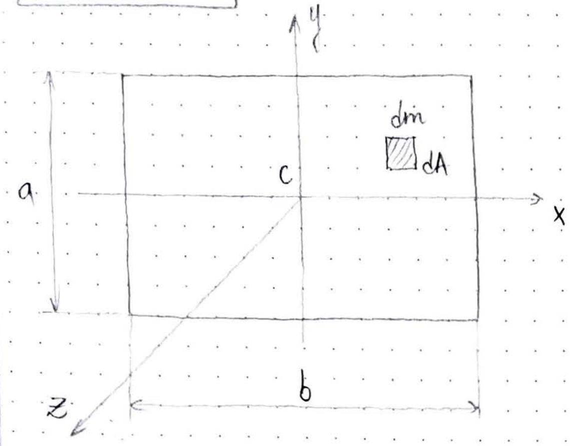
$$\boxed{\begin{array}{l} J_{C\xi'} = \frac{1}{12} m l^2 \sin^2\alpha \\ J_{C\eta'} = 0 \end{array}}$$

$\alpha = 90° = \frac{\pi}{2}$



$$\boxed{\begin{array}{ll} J_{Ax} = J_{Ay} = \frac{1}{3} m l^2 & [kgm^2] \\ J_{Cx} = J_{Cy} = \frac{1}{12} m l^2 & [kgm^2] \end{array}}$$

ПРАВОУГАОНИК



$\rho = \frac{m}{A} \Rightarrow$ површинска густина , $A = ab$, $\rho = \frac{m}{ab}$

$dm = \rho \cdot dA$, $dA = dx \cdot dy$

$dm = \frac{m}{ab} dx\, dy$

$$J_{cx} = \int_m (y^2 + \cancel{z^2}) dm = \int_m y^2 dm = \frac{m}{ab} \int_{-\frac{1}{2}b}^{\frac{1}{2}b} \int_{-\frac{1}{2}a}^{\frac{1}{2}a} y^2 dy\, dx =$$

$$= \frac{m}{ab} \int_{-\frac{1}{2}b}^{\frac{1}{2}b} \left(\frac{y^3}{3} \Big|_{-\frac{1}{2}a}^{\frac{1}{2}a} \right) dx = \frac{m}{3ab} \left(\frac{1}{8}a^3 + \frac{1}{8}a^3 \right) \cdot x \Big|_{-\frac{1}{2}b}^{\frac{1}{2}b} =$$

$$= \frac{m}{3ab} \cdot \frac{2}{48} a^{3^2} \cdot b$$

$$\boxed{\begin{aligned} J_{cx} &= \frac{1}{12} m a^2 \\ J_{cy} &= \frac{1}{12} m b^2 \end{aligned}}$$

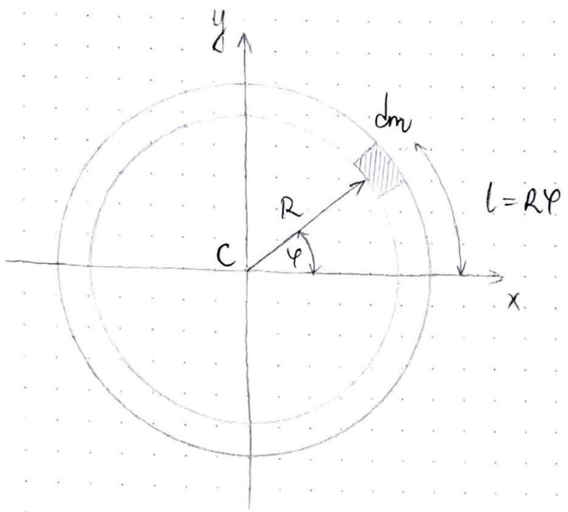
$$J_{cz} = \int_m (x^2 + y^2) dm = \frac{m}{ab} \int_{-\frac{1}{2}b}^{\frac{1}{2}b} \int_{-\frac{1}{2}a}^{\frac{1}{2}a} x^2 dx\, dy + \frac{m}{ab} \int_{-\frac{1}{2}b}^{\frac{1}{2}b} \int_{-\frac{1}{2}a}^{\frac{1}{2}a} y^2 dx\, dy =$$

$$= \frac{m}{3ab} \frac{1}{4} b^{3^2} \cdot a + \frac{m}{3ab} \frac{1}{4} a^{3^2} \cdot b$$

$$\boxed{J_{cz} = \frac{1}{12} m (a^2 + b^2)}$$

$* \; a = b \Rightarrow \underline{\underline{J_{cz} = \frac{1}{6} m a^2}}$

ТАНАК ПРСТЕН



$$\rho = \frac{dm}{dl} \Rightarrow$$ линијска густина

$$l = R\varphi \Rightarrow dl = R\,d\varphi$$

$$\rho = \frac{dm}{R\,d\varphi} = \frac{m}{2R\pi} \Rightarrow dm = \frac{m}{2\pi}\,d\varphi$$

$$x = R\cos\varphi$$

$$y = R\sin\varphi$$

$$J_{cx} = \int_m (y^2 + \cancel{z^2}^{\,0})\,dm = \frac{m}{2\pi}\int_0^{2\pi} R^2 \sin^2\varphi\,d\varphi$$

$$\sin^2\varphi = \frac{1-\cos\varphi}{2}$$

$$J_{cx} = \frac{mR^2}{4\pi}\int_0^{2\pi} d\varphi - \frac{mR^2}{4\pi}\int_0^{2\pi}\cos\varphi\,d\varphi$$

$$J_{cx} = \frac{mR^2}{\cancel{4\pi}\,2}\cdot\cancel{2\pi} - \frac{mR^2}{4\pi}(\cancel{\sin 2\pi}^{\,0} - 0)$$

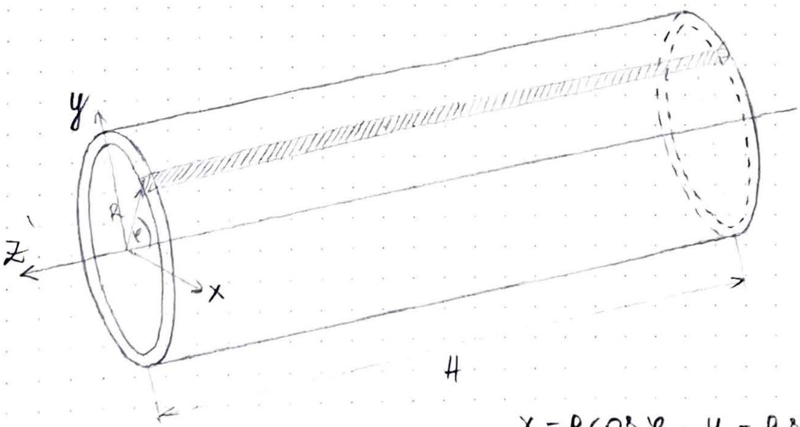
$$\boxed{\begin{aligned} J_{cx} &= \frac{1}{2}mR^2 \\ J_{cy} &= \frac{1}{2}mR^2 \end{aligned}}$$

$$J_{cz} = \int_m (x^2+y^2)\,dm = \int_m (R^2\cos^2\varphi + R^2\sin^2\varphi)\,dm$$

$$= \int_m R^2\,dm = R^2\int_m dm$$

$$\boxed{J_{cz} = mR^2}$$

ЦИЛИНДАР



$$dA = H\cdot R\,d\varphi$$

$$A = H\cdot 2R\pi$$

$$\rho = \frac{m}{A} = \frac{dm}{dA} \Rightarrow$$ површинска густина

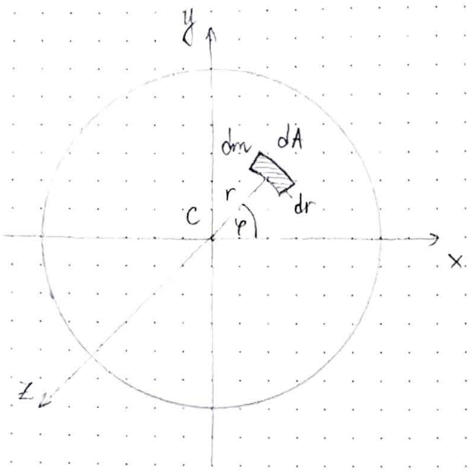
$$dm = \frac{m}{A}\,dA = \frac{m}{2R\pi H}\cdot HR\,d\varphi = \frac{m}{2\pi}\,d\varphi$$

$$x = R\cos\varphi, \quad y = R\sin\varphi$$

$$J_{cz} = \int_m (x^2+y^2)\,dm = \int_0^{2\pi}(R^2\cos^2\varphi + R^2\sin^2\varphi)\,\frac{m}{2\pi}\,d\varphi = \frac{mR^2}{2\pi}\int_0^{2\pi} d\varphi$$

$$\boxed{J_{cz} = mR^2}$$

ТАНАК КРУЖНИ ДИСК



$$\rho = \frac{dm}{dA} = \frac{m}{A}, \quad A = R^2\pi, \quad dA = dr \cdot r\,d\varphi$$

$$dm = \frac{m}{R^2\pi} \cdot r\,dr\,d\varphi$$

$$x = r\cos\varphi, \quad y = r\sin\varphi$$

$$J_{cx} = \int_m (y^2 + \overset{0}{z^2})\,dm = \int_0^R \int_0^{2\pi} r^2 \sin^2\varphi \cdot \frac{m}{R^2\pi}\, r\,dr\,d\varphi$$

$$= \frac{m}{R^2\pi} \int_0^{2\pi} \left(\int_0^R r^3\,dr \right) \sin^2\varphi\,d\varphi, \quad \sin^2\varphi = \frac{1-\cos\varphi}{2}$$

$$= \frac{m}{\cancel{R^2}\pi} \frac{R^{\cancel{4}\,2}}{4} \frac{1}{2} \left(\int_0^{2\pi} d\varphi - \int_0^{2\pi} \cos\varphi\,d\varphi \right)$$

$$= \frac{mR^2}{4\cancel{8\pi}} \cdot \cancel{2\pi}$$

$$\boxed{\begin{aligned} J_{cx} &= \frac{1}{4} mR^2 \\ J_{cy} &= \frac{1}{4} mR^2 \end{aligned}}$$

$$J_{cz} = \int_m (x^2 + y^2)\,dm = \int_m (r^2\cos^2\varphi + r^2\sin^2\varphi)\,dm$$

$$= \int_m r^2\,dm = \frac{m}{R^2\pi} \int_0^R \int_0^{2\pi} r^2 \cdot r\,dr\,d\varphi$$

$$= \frac{m}{\cancel{R^2\pi}} \cdot \frac{R^{\cancel{4}\,2}}{\cancel{2}4} \cdot \cancel{2\pi}$$

$$\boxed{J_{cz} = \frac{1}{2} mR^2}$$