Vežba 2

FUNKCIJA OBLIKA BARUTNOG ZRNA

Smatra se da se sagorevanje baruta obavlja po geometrijskom zakonu sagorevanja. To podrazumeva 3 osnovne predpostavke:

- masa baruta je hemijski i fizički jednorodna (po strukturi i gustini); barutna zrna su jednakih dimenzija.
- površine svih barutnih zrna se pripaljuju istovremeno.
- sagorevanje baruta obavlja se po paralelnim slojevima sa jednakom linearnom brzinom u svim pravcima normalnom na površinu sagorevanja baruta.

Veza izmedju geometrije baruta i obrazovanja gasova

Označimo:

e_o	-	polovina ukupne dedljine baruta ($2e_o$ - "svod" barutnog zrna)
e	-	debljina baruta koja je sagorela u datom trenutku u jednom pravcu
$z=e/e_o$	-	relativna debljina sagorelog baruta
$\psi=\omega_s/\omega_o=W_s/W_o$	-	relativni (maseni - sa ω, odnosno zapreminski - sa W) udeo sagorelog baruta
$\sigma=S/S_o$	-	relativna površina baruta

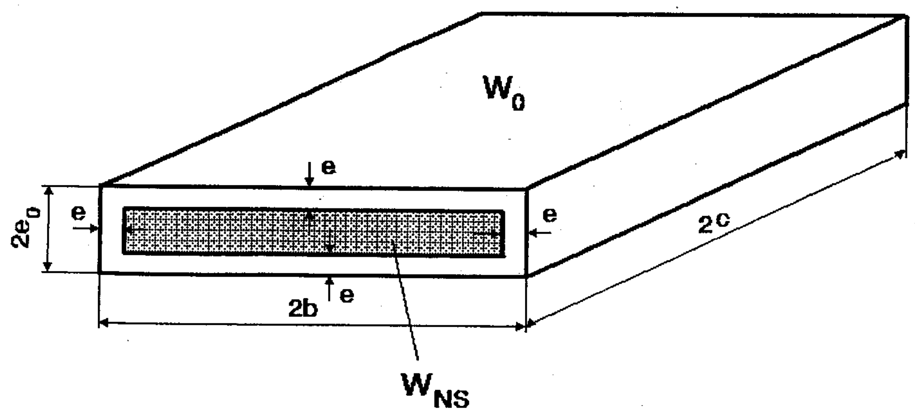
(indeks "o" označava vrednosti u početnom trenutku sagorevanja)

Treba odrediti vrednosti $\psi=f_1(z)$ i $\sigma=f_2(z)$.

Zavisnost $\psi=f_1(z)$ je oblika:

$$\psi=\kappa z\left(1+\lambda z+\mu z^2\right)$$

κ, λ, μ - karakteristike oblika baruta - konstante koje zavise od oblika zrna.
Izvešćemo zavisnost $\psi=f_1(z)$ za barutno zrno u obliku trake, čija je šema sagorevanja data na slici.



$$\psi = \frac{W_s}{W_o} = 1 - \frac{W_{ns}}{W_o} \qquad W_0 = 2e_0 \cdot 2b \cdot 2c$$

$$W_{ns} = 2(e_0 - e) \cdot 2(b-e) \cdot 2(c-e)$$

Uvodimo oznake: $2e_o/2b = \alpha$ $\quad 2e_o/2c = \beta$

$$\frac{W_{ns}}{W_o} = \left(\frac{e_o - e}{e_o}\right) \cdot \left(\frac{b-e}{b}\right) \cdot \left(\frac{c-e}{c}\right) = \left(1 - \frac{e}{e_o}\right) \cdot \left(1 - \frac{e}{b}\right) \cdot \left(1 - \frac{e}{c}\right) =$$

$$\left(1 - \frac{e}{e_o}\right) \cdot \left(1 - \frac{e_o}{b} \cdot \frac{e}{e_o}\right) \cdot \left(1 - \frac{e_o}{c} \cdot \frac{e}{e_o}\right) = (1-z) \cdot (1 - \alpha z) \cdot (1 - \beta z) =$$

$$= 1 - (1 + \alpha + \beta) z + (\alpha + \beta + \alpha\beta) z^2 - \alpha\beta z^3$$

$$\psi = 1 - \frac{W_{ns}}{W_o} \quad \Rightarrow \quad \psi = (1 + \alpha + \beta) z - (\alpha + \beta + \alpha\beta) z^2 + \alpha\beta z^3$$

opšti izraz: $\boxed{\psi = \kappa z \left(1 + \lambda z + \mu z^2\right)}$

$$\boxed{\kappa = 1 + \alpha + \beta} \qquad \boxed{\lambda = -\frac{\alpha + \beta + \alpha\beta}{1 + \alpha + \beta}} \qquad \boxed{\mu = \frac{\alpha\beta}{1 + \alpha + \beta}}$$

Koeficijenti κ, λ, μ nazivaju se koeficijenti oblika barutnog zrna
Na kraju sagorevanja barutnog zrna $z_k=1$ i $\psi_k=1$, pa je:

$$1 = \kappa(1 + \lambda + \mu)$$

Ova jednakost služi za proveru proračuna koeficijenata oblika baruta. Na osnovu sličnog izvodjenja moguće je odrediti koeficijente oblika i za druge oblike barutnog zrna.

OBLIK I DIMENZIJE		α	β	κ	λ	μ
Traka	dužina L=2c, širina 2b debljina $2e_o$	$\frac{2e_0}{2b}$	$\frac{2e_0}{2c}$	$1+\alpha+\beta$	$-\frac{\alpha+\beta+\alpha\beta}{1+\alpha+\beta}$	$\frac{\alpha\beta}{1+\alpha+\beta}$
Kvadratna pločica	2b = 2c $\alpha = \beta$	$\frac{2e_0}{2b}$	$\frac{2e_0}{2c}$	$1+2\beta$	$-\frac{2\beta+\beta^2}{1+2\beta}$	$\frac{\beta^2}{1+2\beta}$
Cevčica	$2b = \infty$, L = 2c e_o = (D-d)/4	0	$\frac{2e_0}{2c}$	$1+\beta$	$-\frac{\beta}{1+\beta}$	0
Štapić	$2e_o$ = 2b	1	$\frac{2e_0}{2c}$	$2+\beta$	$-\frac{1+2\beta}{2+\beta}$	$\frac{\beta}{2+\beta}$
Kocka	$2e_o$ = 2b = 2c	1	1	3	-1	$\frac{1}{3}$

Odredimo sada funkciju $\sigma = S/S_o = f_2(z)$.

Diferencirajmo $\psi=f_1(z)$ po z:

$$\frac{d\psi}{dz} = \kappa\left(1+2\lambda z+3\mu z^2\right)$$

$$\frac{d\psi}{dz} = \frac{d\psi}{dt}\frac{dt}{de}\frac{de}{dz} \qquad z = \frac{e}{e_0} \quad \Rightarrow \quad \frac{de}{dz} = e_0$$

$$\frac{d\psi}{dt} = \frac{S\,de}{W_0\,dt} = \frac{S_0}{W_0}\frac{S}{S_0}\frac{de}{dt} \qquad \Rightarrow$$

$$\frac{d\psi}{dz} = \frac{S_0}{W_0}\frac{S}{S_0}\frac{de}{dt}\frac{dt}{de}e_0 = \frac{S_0}{W_0}\sigma e_0$$

$$\frac{S_o}{W_o}\sigma e_0 = \kappa\left(1+2\lambda z+3\mu z^2\right)$$

U početnom trenutku sagorevanja: $t = 0$, $z = 0$ i $\sigma = S/S_0 = 1$ $\Rightarrow$

$$\frac{S_o}{W_o}e_0 = \kappa \qquad \boxed{\sigma = 1+2\lambda z+3\mu z^2}$$

Radi uprošćenja izraza pri analitičkom rešavanju zadataka unutrašnje balistike, često se koristi dvočlana zavisnost:

$$\psi = \kappa_1 z\left(1+\lambda_1 z\right)$$

Za odredjivanje κ_1 i λ_1 u zavisnosti od κ, λ i μ postavljaju se dva dopunska uslova.

Vrednosti ψ za $z = 1$ i $z = 1/2$ proračunate po dvočlanoj i tročlanoj formuli moraju da budu iste.

za $z = 1$ $\qquad \kappa\left(1+\lambda+\mu\right) = \kappa_1\left(1+\lambda_1\right) = 1$

za $z = 1/2$ $\qquad \frac{\kappa}{2}\left(1+\frac{\lambda}{2}+\frac{\mu}{4}\right) = \frac{\kappa_1}{2}\left(1+\frac{\lambda_1}{2}\right)$

Rešavanjem ovog sistema dobijamo:

$$\kappa_1 = \kappa - \frac{\kappa\mu}{2} = \kappa\left(1-\frac{\mu}{2}\right) = 1+\alpha+\beta-\frac{\alpha\beta}{2} \qquad \boxed{\lambda_1 = \frac{1}{\kappa_1}-1}$$

$$\boxed{\sigma = 1+2\lambda_1 z}$$

ZADATAK 1

Odrediti $\psi(z)$ i $\sigma(z)$ po dvočlanoj i tročlanoj formuli za traku dimenzija: dužina 95 mm, širina 1.5 mm i debljina 0.36 mm (NGB - 161, ide u 120 mm LTF M56 OP).

Rešenje:

$$2e_0 = 0.36 \text{ mm} \qquad \alpha = \frac{2e_0}{2b} = \frac{0.36}{1.5} = 0.24$$
$$2b = 1.5 \text{ mm}$$
$$2c = 95 \text{ mm} \qquad \beta = \frac{2e_0}{2c} = \frac{0.36}{95} = 0.00379$$

$$\kappa = 1+\alpha+\beta = 1+0.24+0.00379 = 1.24379$$

$$\lambda = -\frac{\alpha+\beta+\alpha\beta}{1+\alpha+\beta} = -\frac{0.24+0.00379+0.24\cdot 0.00379}{1.24379} = -0.1967$$

$$\mu = \frac{\alpha\beta}{1+\alpha+\beta} = \frac{0.24\cdot 0.00379}{1.24379} = 0.0007313$$

$$\psi_{3c} = \kappa z\left(1+\lambda z+\mu z^2\right) = 1.24379\cdot z\cdot\left(1-0.1967\cdot z+0.0007313\cdot z^2\right)$$

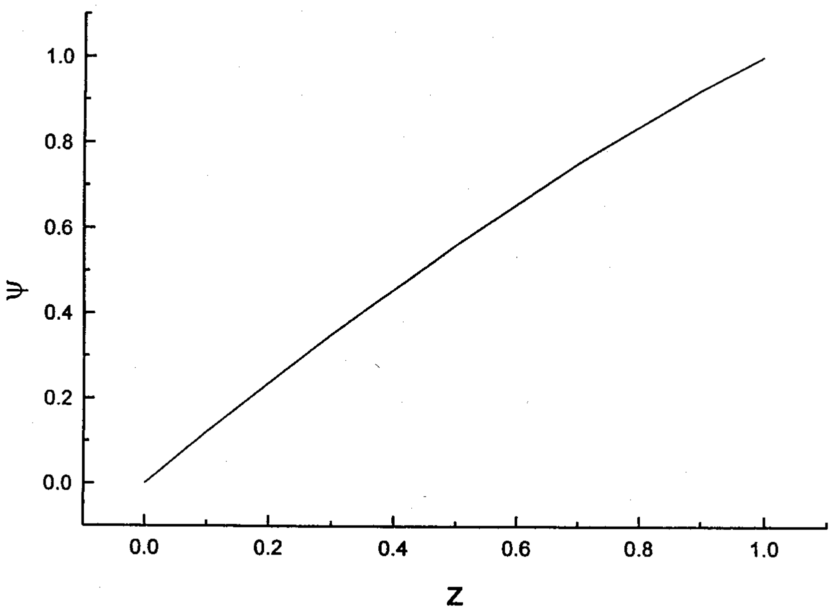
$$\sigma_{3c} = 1+2\lambda z+3\mu z^2 = 1-0.3934\cdot z+0.002194\cdot z^2$$

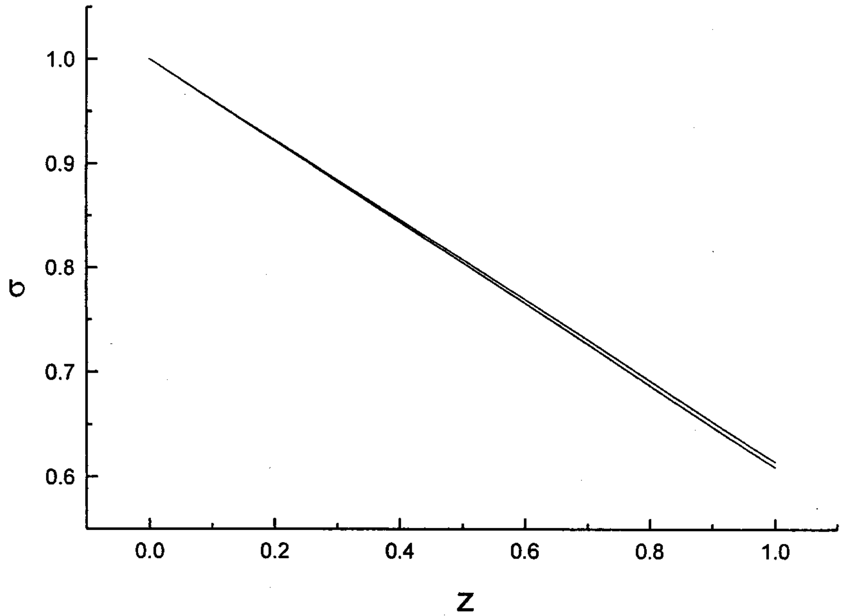
$$\kappa_1 = \kappa\left(1-\frac{\mu}{2}\right) = 1.24379\cdot\left(1-\frac{0.0007313}{2}\right) = 1.2392$$

$$\lambda_1 = \frac{1}{\kappa_1}-1 = \frac{1}{1.2392}-1 = -0.193 \qquad \psi_{2c} = \kappa_1 z\left(1+\lambda_1 z\right) = 1.2392\cdot z\cdot\left(1-0.193\cdot z\right)$$

$$\sigma_{2c} = 1+2\lambda_1 z = 1-0.386\cdot z$$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
ψ_{3c}	0	0.1219	0.2390	0.3511	0.4584	0.5608	0.6584	0.7511	0.8389	0.9219	1
ψ_{2c}	0	0.1215	0.2383	0.3502	0.4574	0.5598	0.6574	0.7502	0.8383	0.9216	1
σ_{3c}	1	0.9607	0.9214	0.8822	0.8430	0.8038	0.7647	0.7257	0.6867	0.6477	0.6088
σ_{2c}	1	0.9614	0.9228	0.8842	0.8456	0.8070	0.7684	0.7298	0.6912	0.6526	0.614





ZADATAK 2

Odrediti funkcije $\psi(z)$ i $\sigma(z)$ za jednokanalne cevčice sledećih dimenzija:

1. D = 1.25 mm, d = 0.30 mm, L = 6.5 mm (NC - 27 , haubica 122 mm D-30)
2. D = 5.8 mm, d = 2.2 mm,L = 256 mm

Rešenje:

1. $$2e_0 = \frac{D-d}{2} = \frac{1.25-0.30}{2} = 0.476 \qquad L = 2c$$

$$\beta = \frac{2e_0}{2c} = \frac{2e_0}{L} = \frac{0.475}{6.5} = 0.0731$$

$$\kappa = 1+\beta = 1+0.0731 = 1.0731 \qquad \lambda = -\frac{\beta}{1+\beta} = -\frac{0.0731}{1+0.0731} = -0.06812 \qquad \mu = 0$$

$$\psi_1 = \kappa z(1+\lambda z) = 1.0731 \cdot z \cdot (1-0.06812 \cdot z)$$

$$\sigma_1 = 1+2\lambda z = 1-0.13624 \cdot z$$

2. $$2e_0 = \frac{D-d}{2} = \frac{5.8-2.2}{2} = 1.8$$

$$\beta = \frac{2e_0}{L} = \frac{1.8}{256} = 0.00703$$

$$\kappa = 1+\beta = 1+0.00703 = 1.00703 \qquad \lambda = -\frac{\beta}{1+\beta} = -\frac{0.00703}{1+0.00703} = -0.00698 \qquad \mu = 0$$

$\psi_2 = \kappa z(1+\lambda z) = 1.00703 \cdot z \cdot (1 - 0.00698 \cdot z)$

$\sigma_2 = 1 - 0.01396 \cdot z$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
ψ_1	0	0.1066	0.2117	0.3154	0.4175	0.5183	0.6175	0.7154	0.8147	0.9066	1
σ_1	1	0.9864	0.9728	0.9591	0.9455	0.9319	0.9183	0.9046	0.8910	0.8774	0.8638
ψ_2	0	0.1006	0.2011	0.3015	0.4017	0.5018	0.6017	0.7015	0.8011	0.9006	1
σ_2	1	0.9986	0.9972	0.9958	0.9944	0.9930	0.9916	0.9902	0.9888	0.9874	0.9860