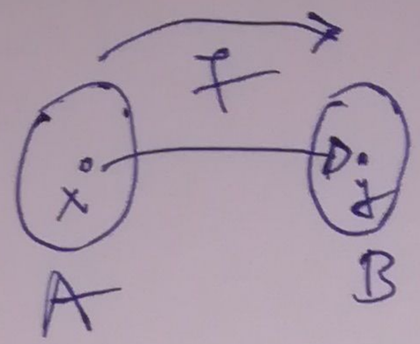
Функција

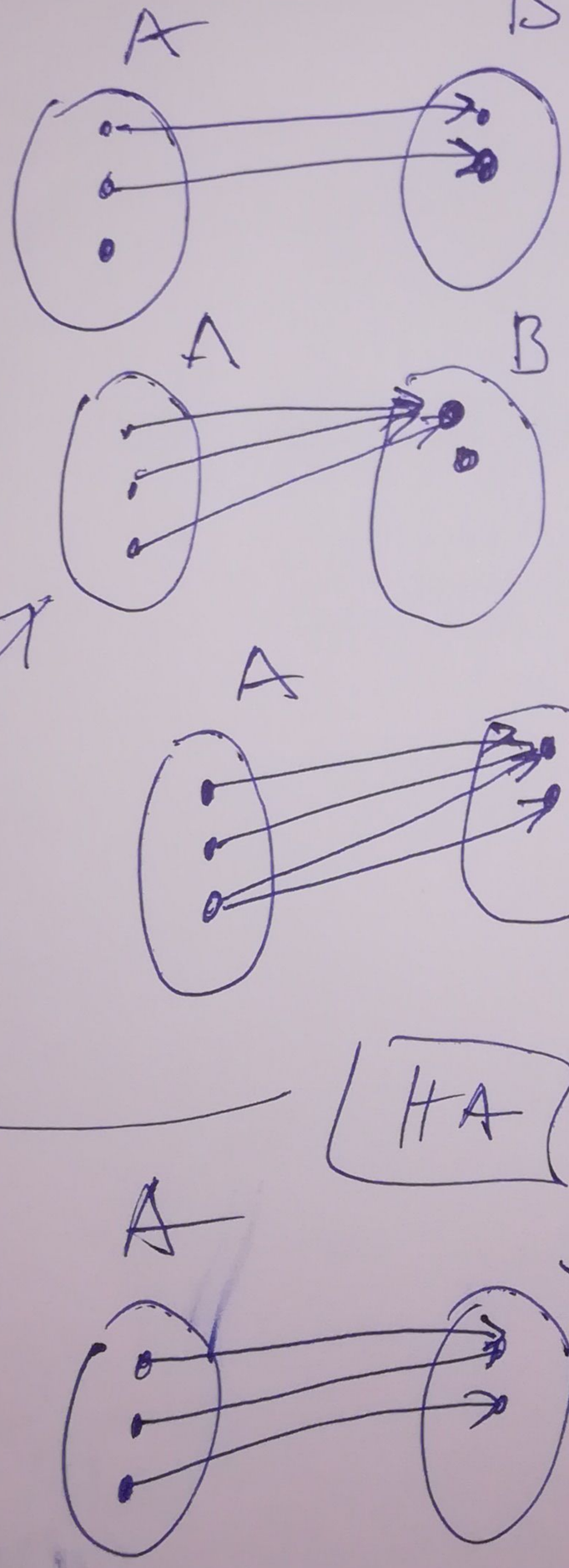
A

$f : A \to B$ $\quad (A, B, f)$



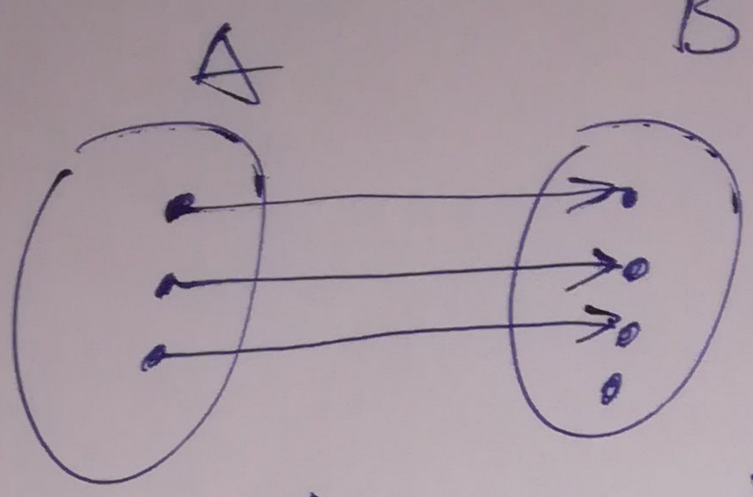
$y = f(x)$

Шта је функција?

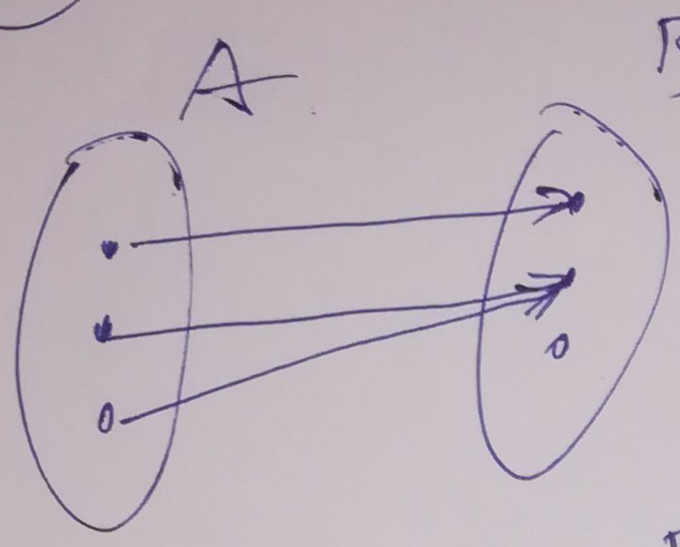


1-1

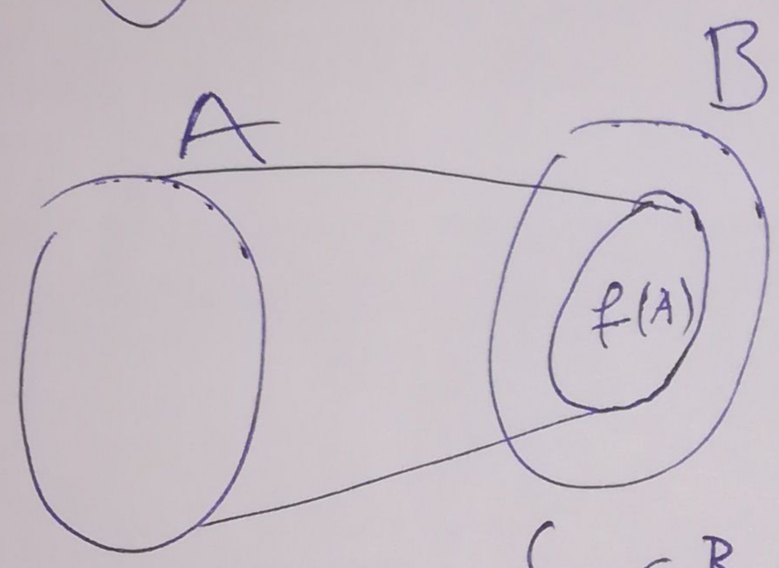
$x_1 \neq x_2 \longrightarrow f(x_1) \neq f(x_2)$



ресет 1-1



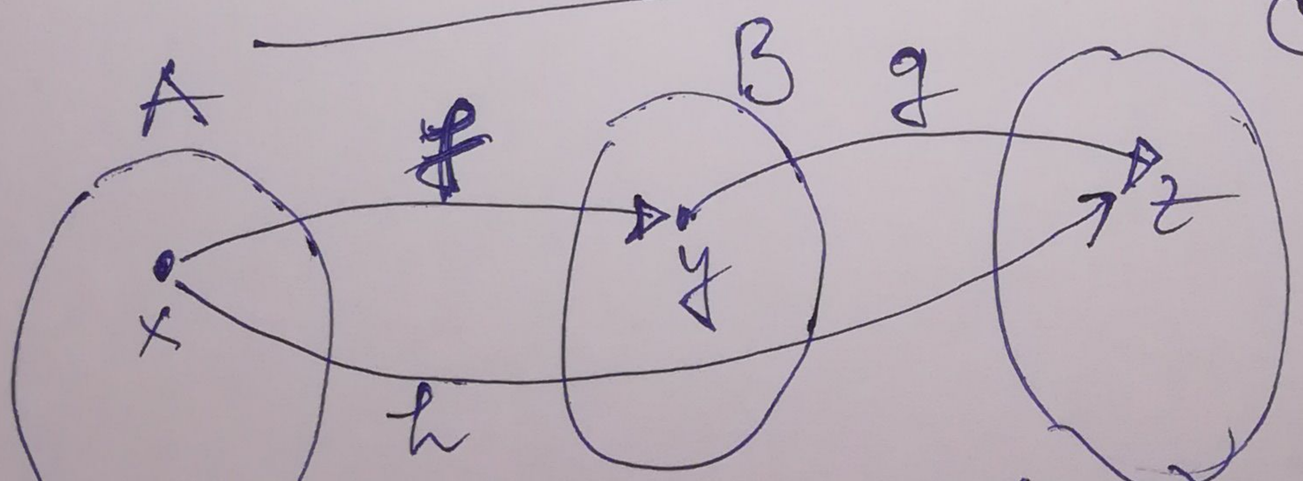
инјекс 1-1



$f(A)$ слика скупа A

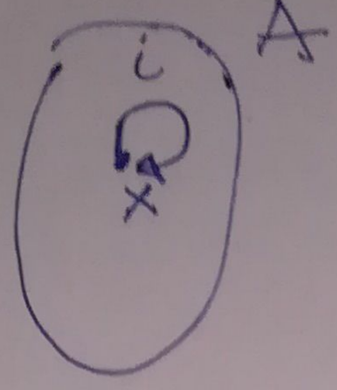
$$f(A) = \{y \in B \mid (\exists x \in A)\ y = f(x)\}$$

Слагање (композиција) функција



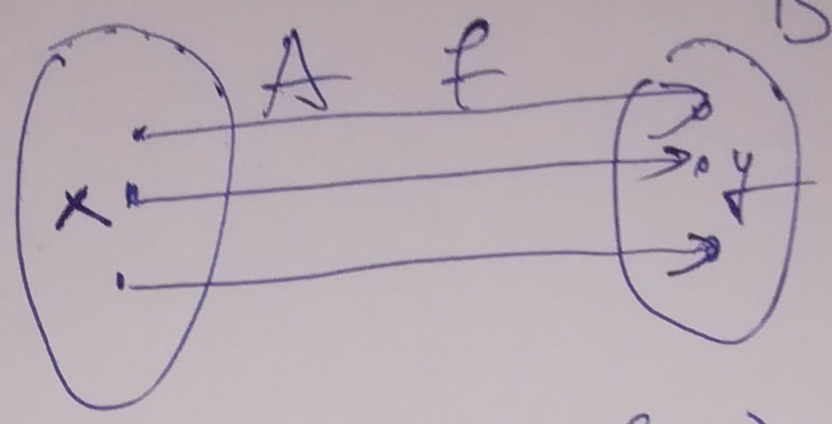
$$h(x) = g(y) = g(f(x)) = (g \circ f)(x)$$

$$h = g \circ f$$



идентичко пресликавање
$i(x) = x$

Нека је f 1-1 и на, тј. бијекција
на A и B



$f: A \to B \quad (y = f(x))$

можемо дефинисати инверзно пресликавање $f^{-1}: B \to A$, за

$f^{-1}(y) = x$, тј.

$f^{-1}(f(x)) = i(x)$

$(f^{-1} \circ f)(x) = i(x)$

$\boxed{f^{-1} \circ f = i}$

и

$i(z) = y = f(x) = f(f^{-1}(y)) = (f \circ f^{-1})(z)$

$\boxed{f \circ f^{-1} = i}$

Пример

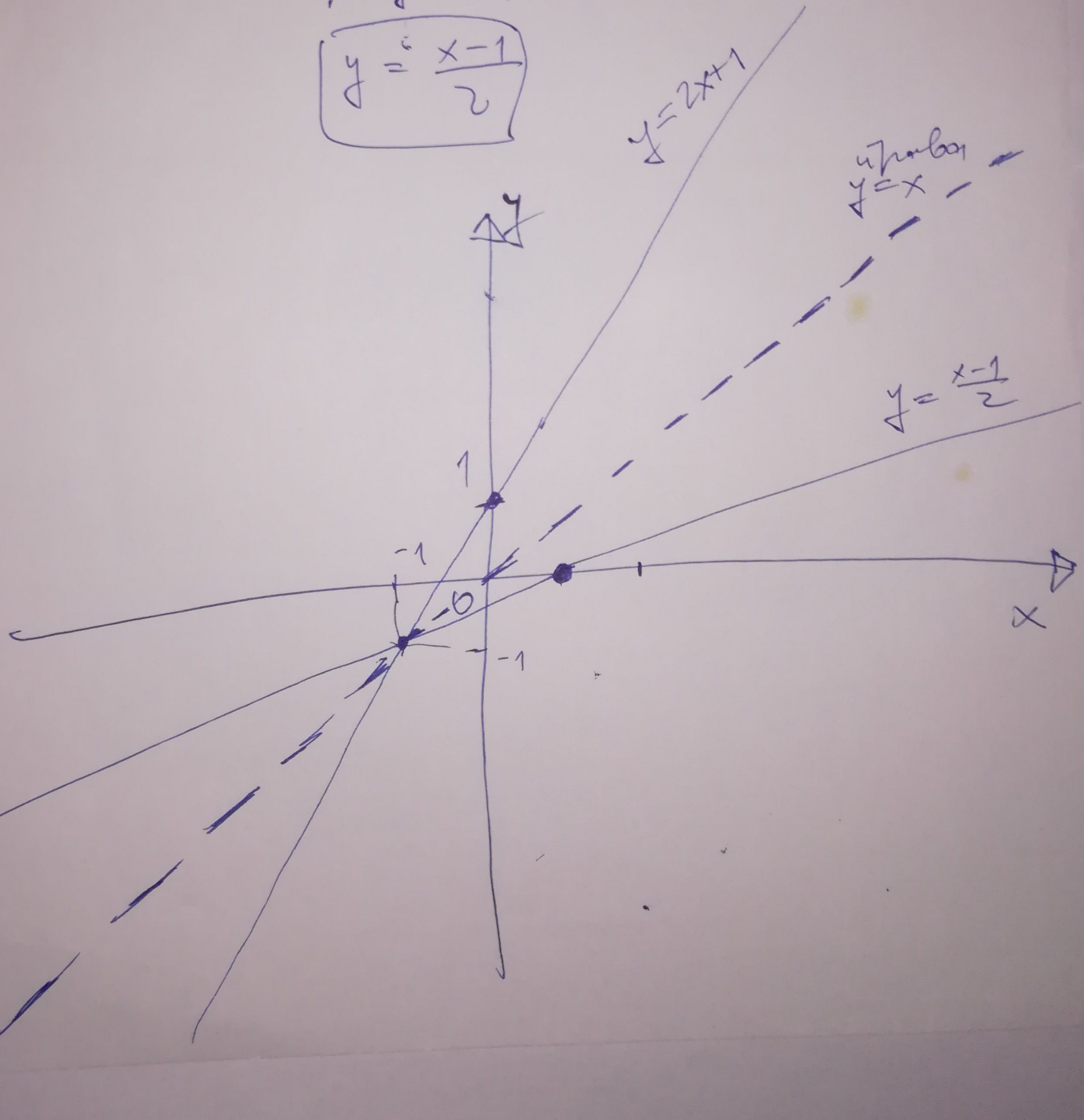
Наћи инверзну функције функције $y = 2x + 1$.

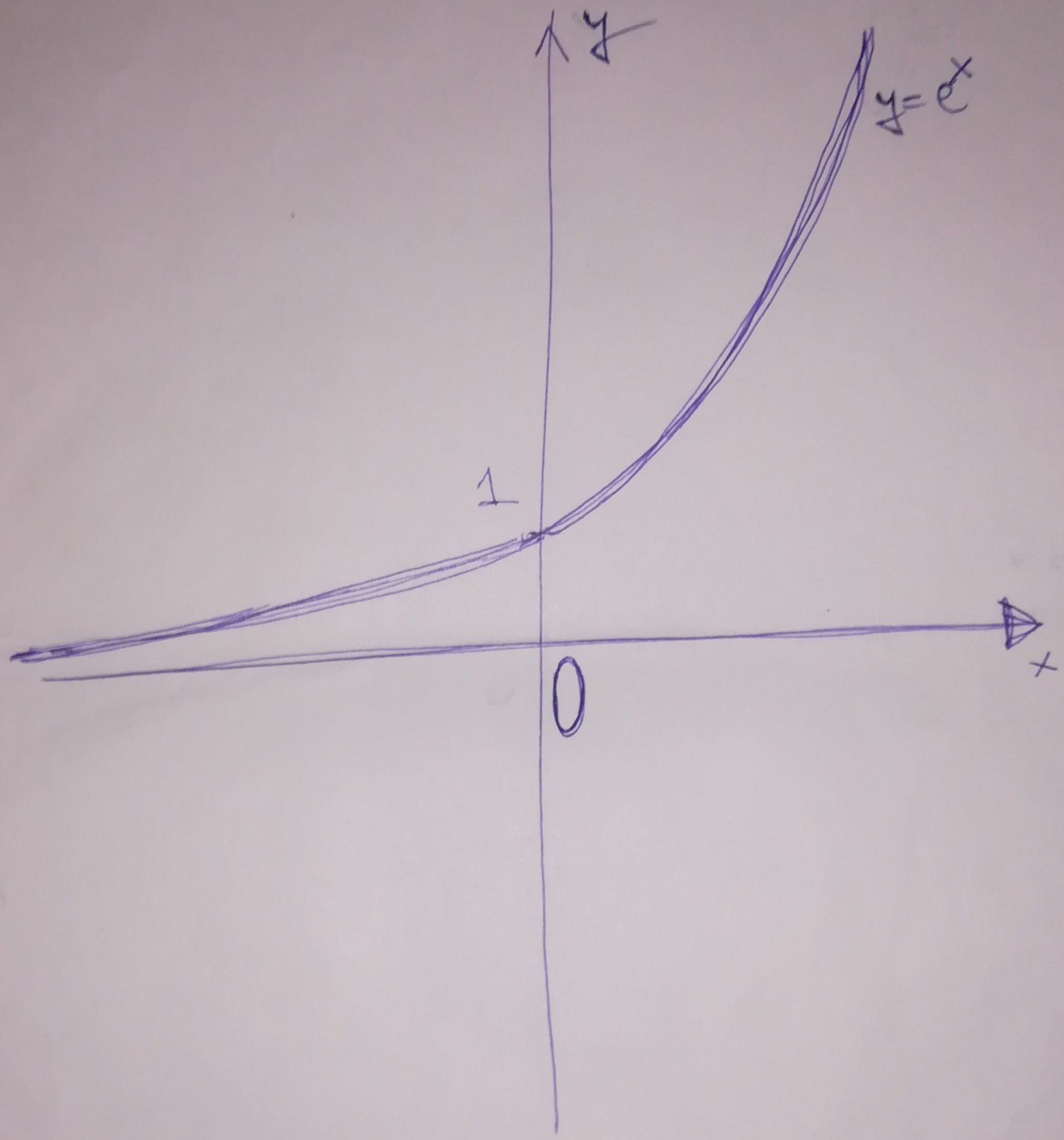
Решимо по x:

$$x = \frac{y-1}{2}$$

Замена $x \leftrightarrow y$ даје инверзну ϕ^{-1}:

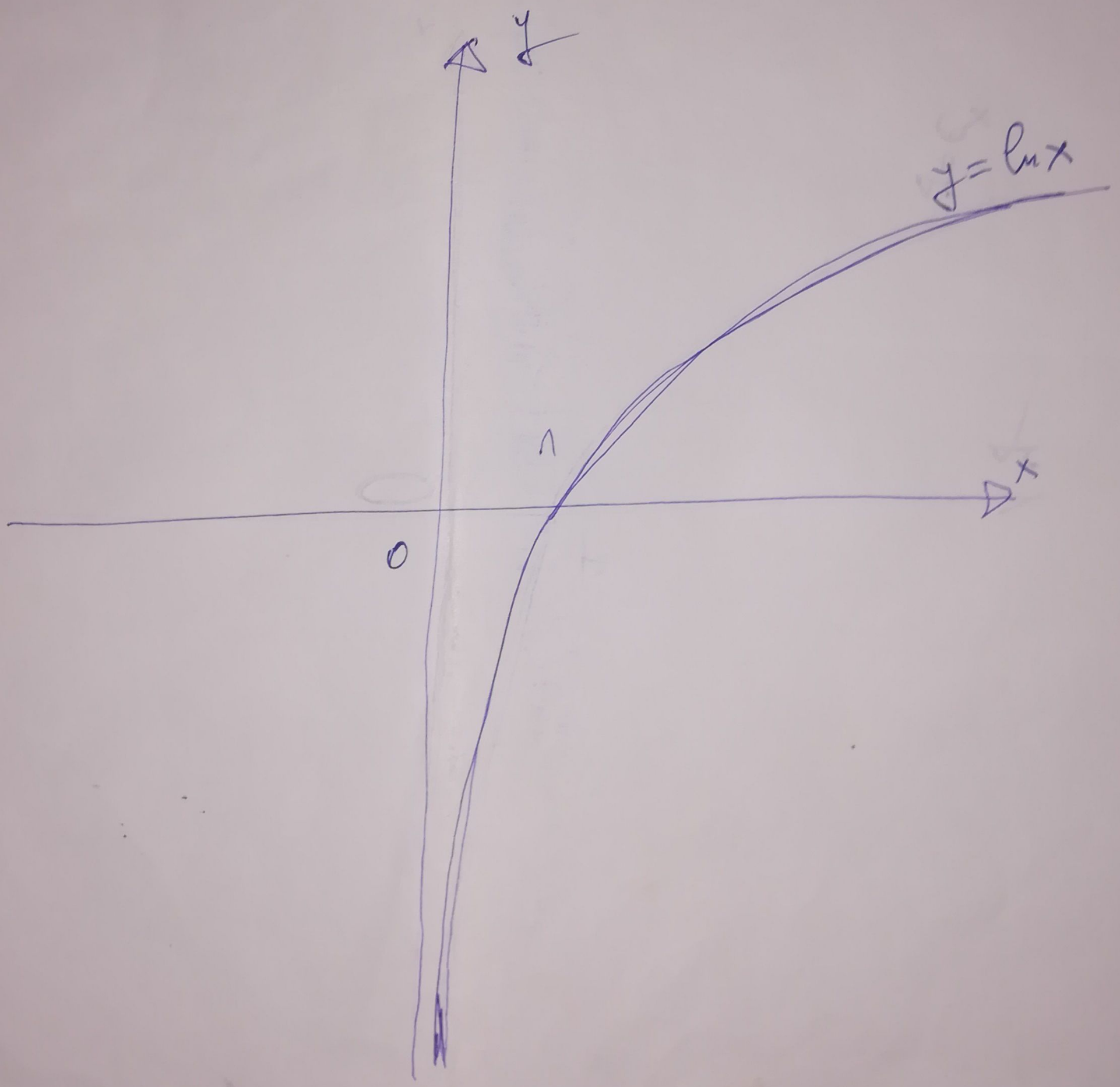
$$\boxed{y = \frac{x-1}{2}}$$





$$\ln e^x = x$$

$$e^{\ln|x|} = |x|, \; x \neq 0$$


y
y = ln x
1
x
0

пр. Ако је $f(x)=\frac{2x+1}{x-2}$ и $g(x)=\frac{1-x}{1+x}$, одредити $f(f(x))$ и $g(f(x))$.

—o—

$$f(f(x))=f\left(\frac{2x+1}{x-2}\right)=\frac{2\cdot\frac{2x+1}{x-2}+1}{\frac{2x+1}{x-2}-2}=$$

$$=\frac{\frac{2(2x+1)+x-2}{x-2}}{\frac{2x+1-2x+4}{x-2}}=\frac{4x+2+x-2}{5}=$$

$$=\frac{5x}{5}=x$$

$$g(f(x))=g\left(\frac{2x+1}{x-2}\right)=\frac{1-\frac{2x+1}{x-2}}{1+\frac{2x+1}{x-2}}=$$

$$=\frac{\frac{x-2-2x-1}{x-2}}{\frac{x-2+2x+1}{x-2}}=\frac{-x-3}{3x-1}=\frac{x+3}{1-3x}$$

пр. Ако је $f(3x+2)=5x-1$, одредити $f(x)$.

—o—

$3x+2=t$

$x=\frac{t-2}{3}$

$$f(t)=5\cdot\frac{t-2}{3}-1=\frac{5t-10-3}{3}$$

$$f(x)=\frac{5x-13}{3}$$

Пр Определи област на дефиниция

а) $f(x) = \sqrt{\left(\frac{x-1}{x+1}\right)^3}$

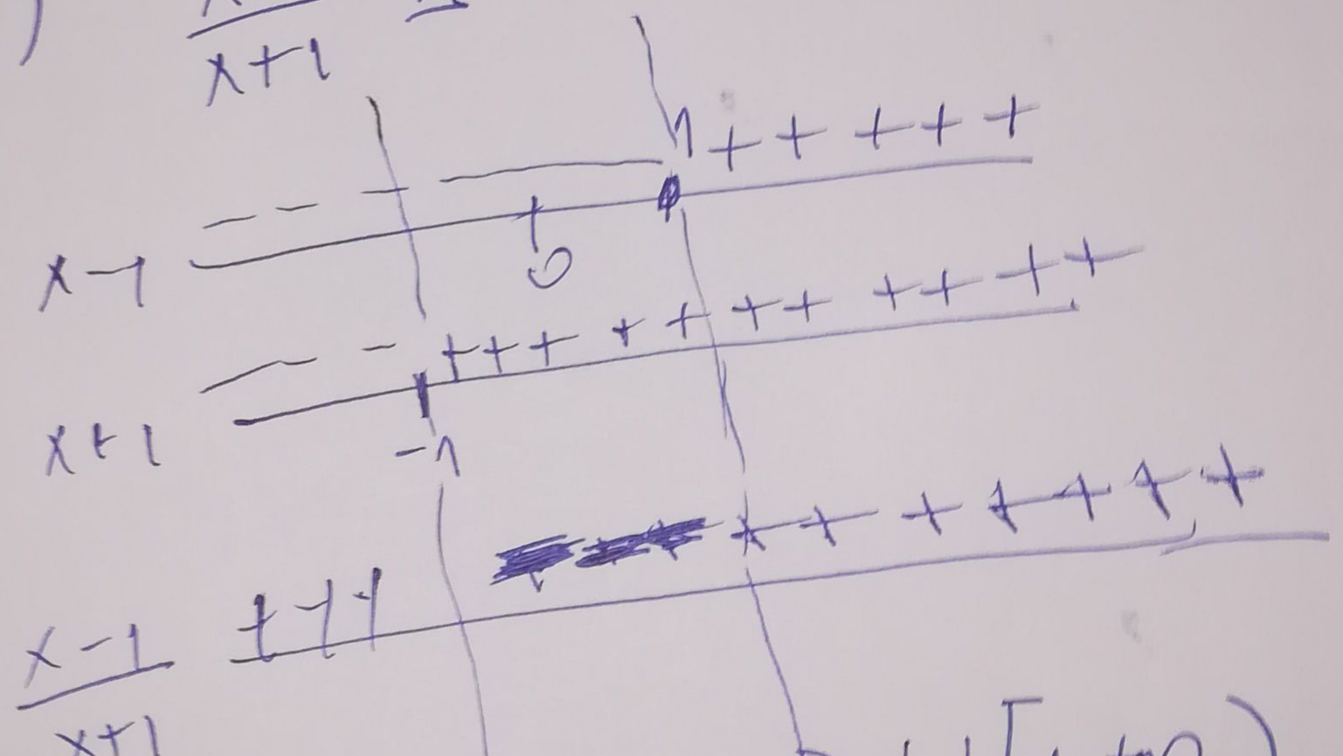
б) $f(x) = \sqrt{\frac{x^2}{x^2-9}} + \sqrt{144 - x^2}$

в) $f(x) = \ln \frac{\sqrt{x^2-1}}{x^2}$

г) $f(x) = \sqrt{1-x^2}\ \operatorname{arctg} x$.

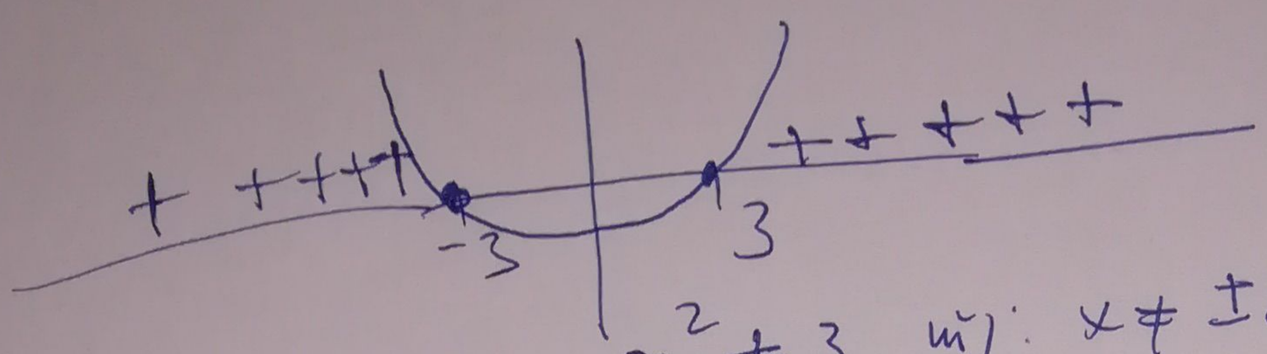
Решение.

а) $\frac{x-1}{x+1} \geq 0$ и $x \neq -1$



$D(f) = (-\infty, -1) \cup [1, +\infty)$

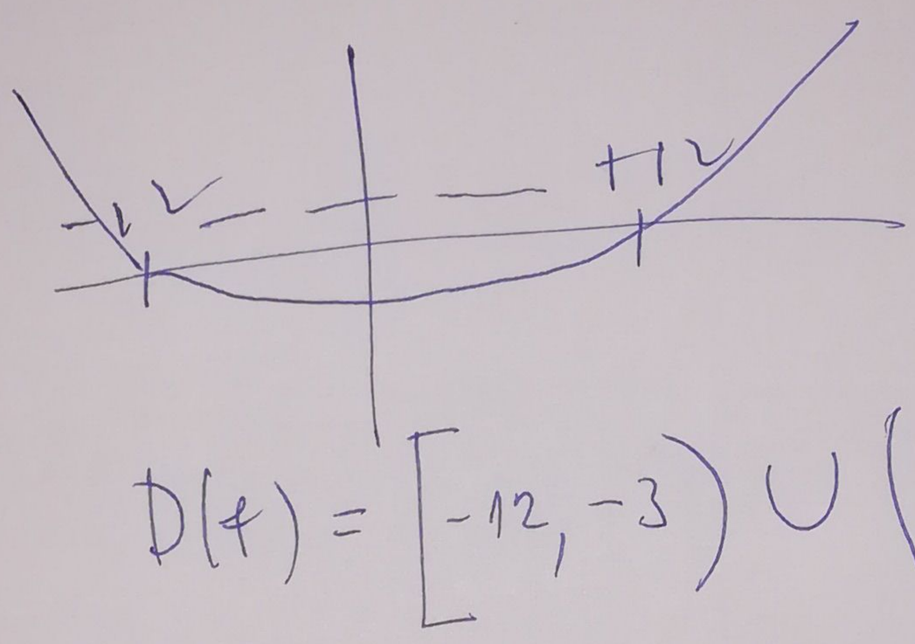
6) $x^2 - 9 \geq 0$



$x^2 \neq 3$, adică: $x \neq \pm 3$

$144 - x^2 \geq 0$

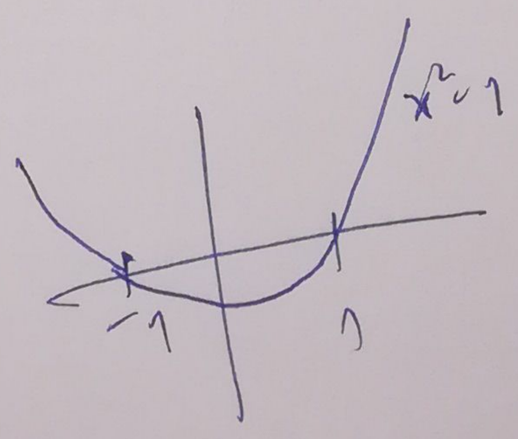
$x^2 - 144 \leq 0$



$D(f) = [-12, -3) \cup (3, 12]$.

c) $\frac{\sqrt{x^2-1}}{x^2} > 0$

$x \neq 0$

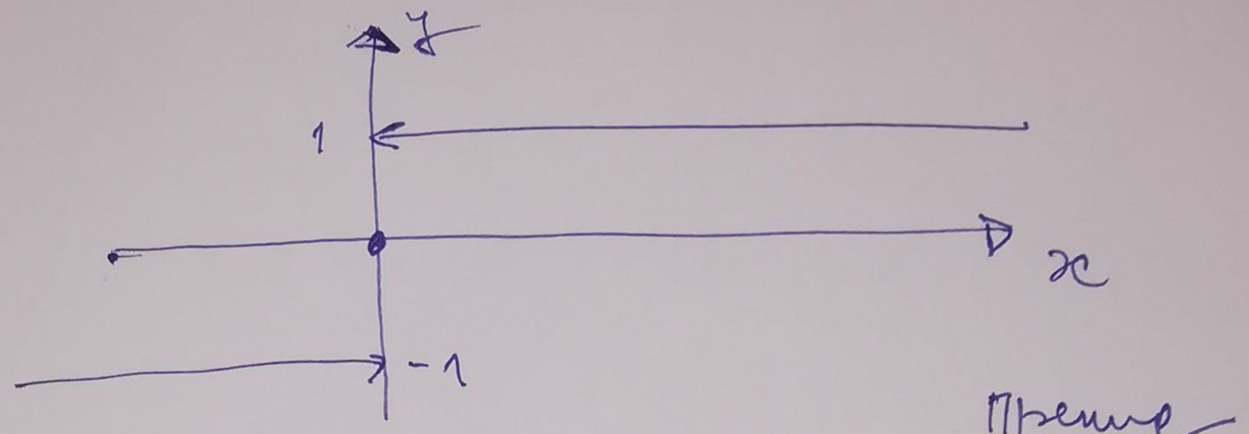


$D(f) = (-\infty, -1) \cup (1, +\infty)$

d) $D(f) = [-1, 1]$.

Осим домена код реалне функције ...

$$f(x) = \operatorname{sgn}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

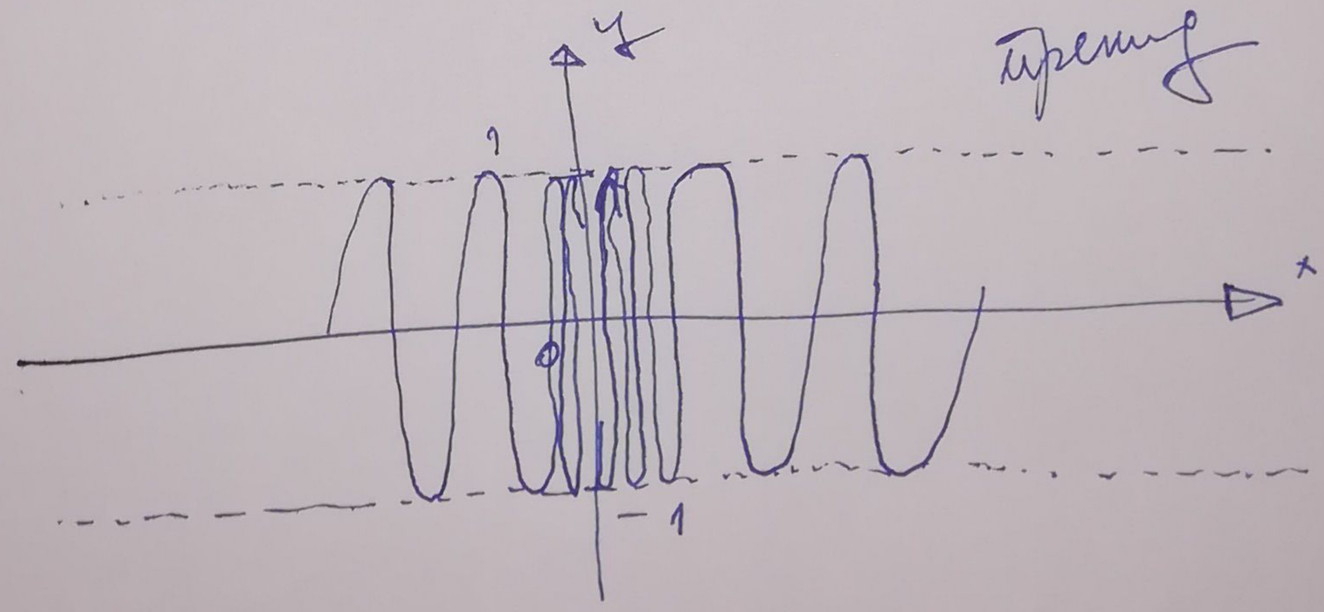


Прекид

$f(x) = \sin \frac{1}{x}$

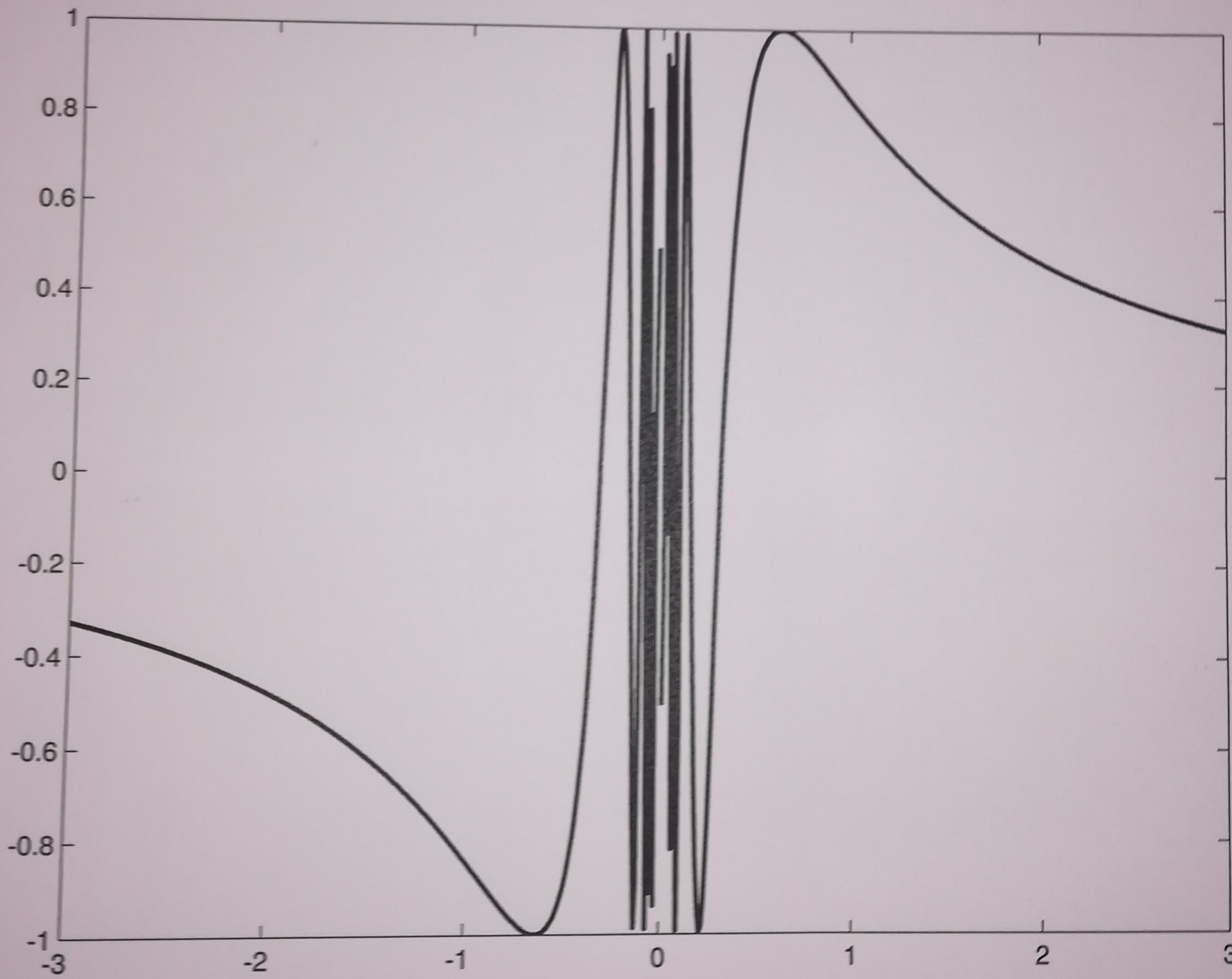
$\lim_{x \to 0} \sin \frac{1}{x}$?

Ессенцијалан прекид



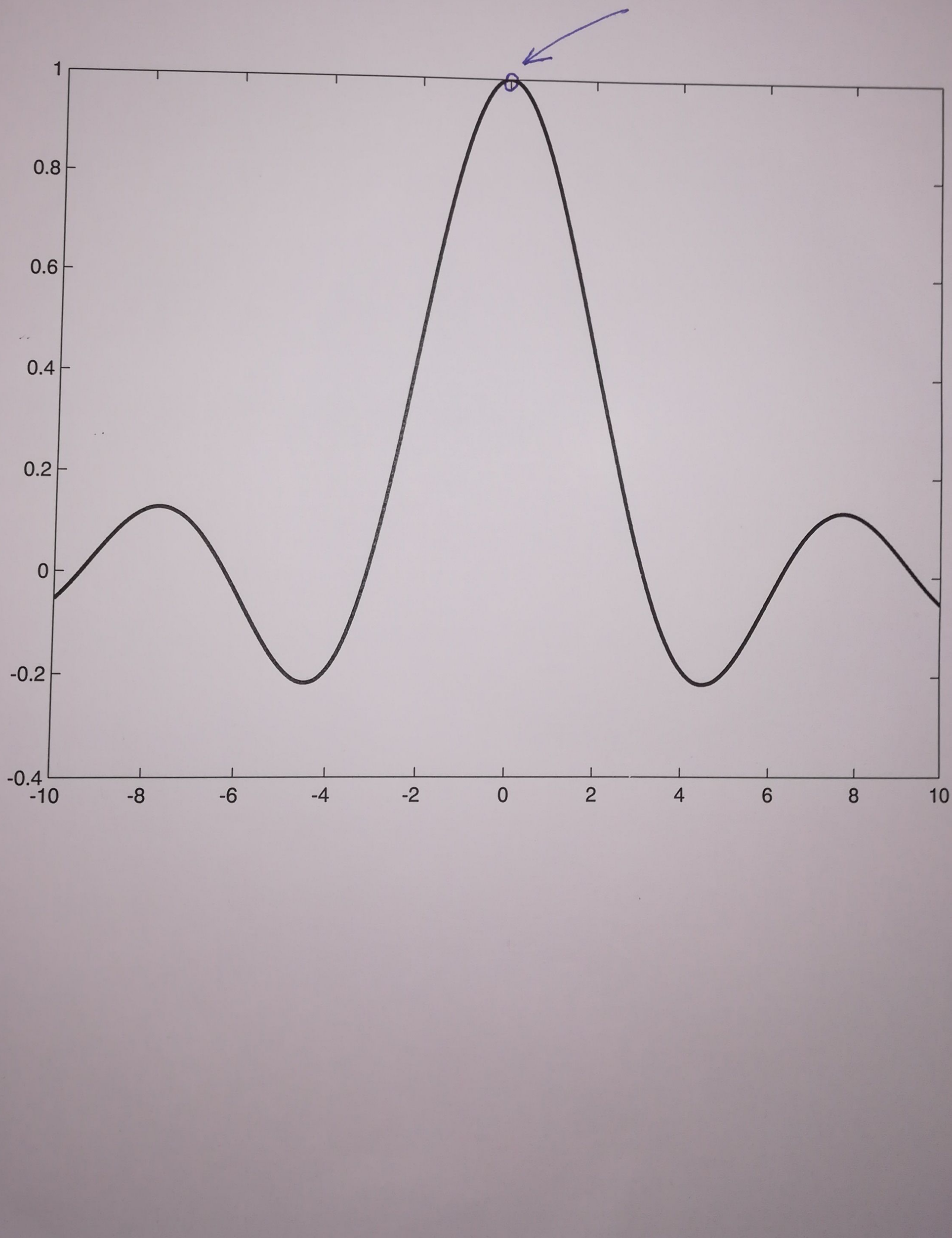
3.1

$f(x) = \sin \frac{1}{x}$



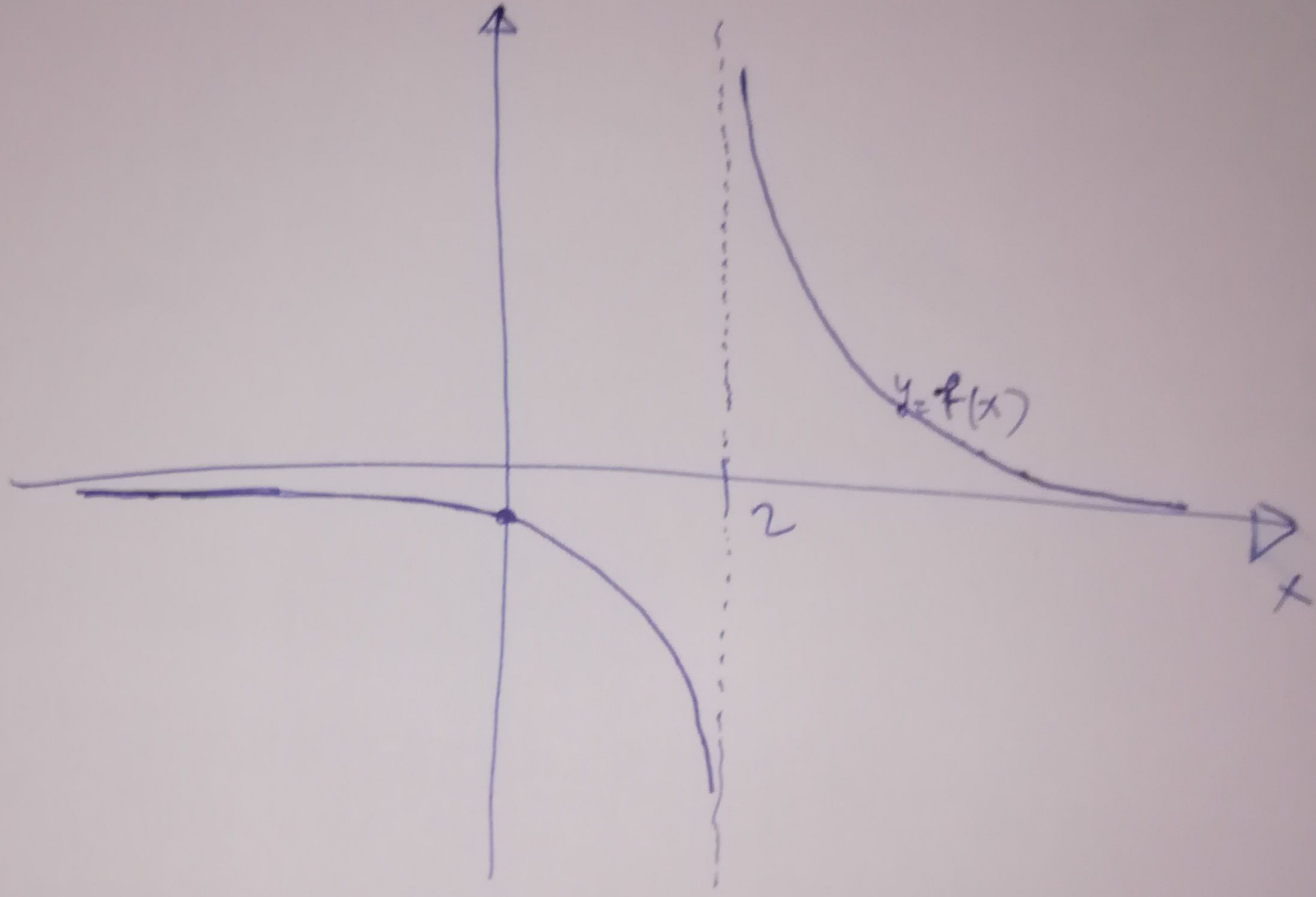
3.2

$$f(x) = \frac{\sin x}{x}$$



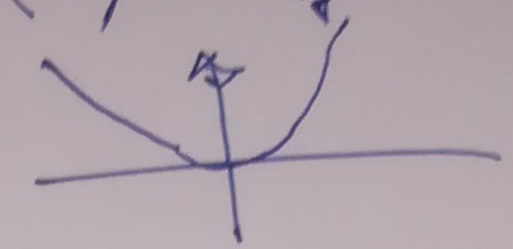
3.3

$$f(x) = \frac{1}{x-2}$$

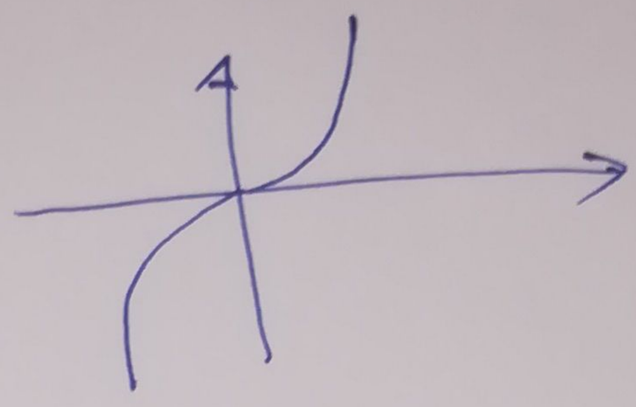


Неке функције чији графици | I.1
треба да знате

(*) $y = ax^2 + bx + c$ (параболу поседно)

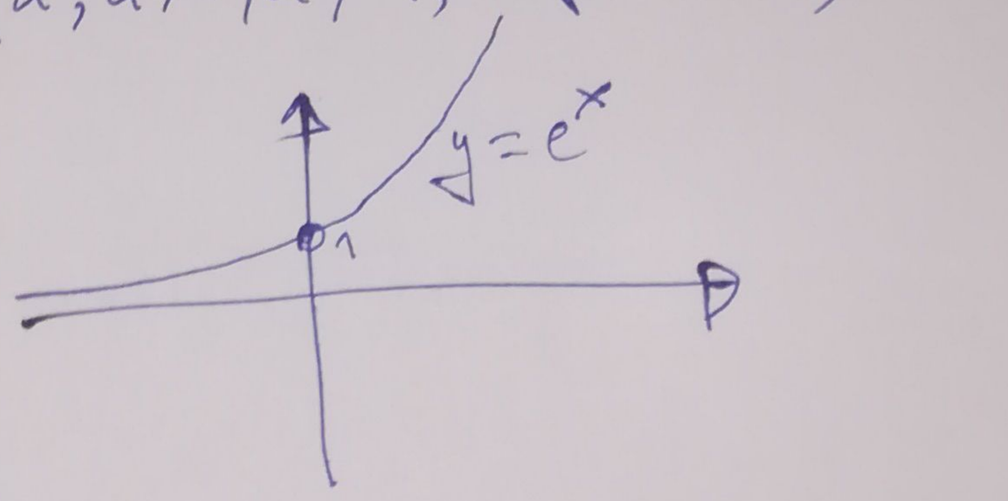


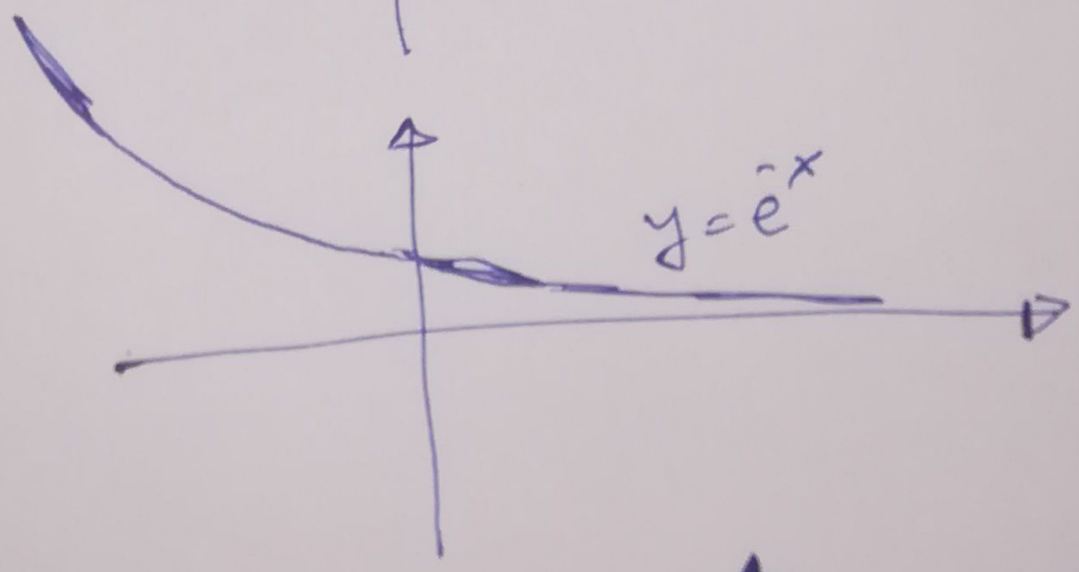
$y = x^2$

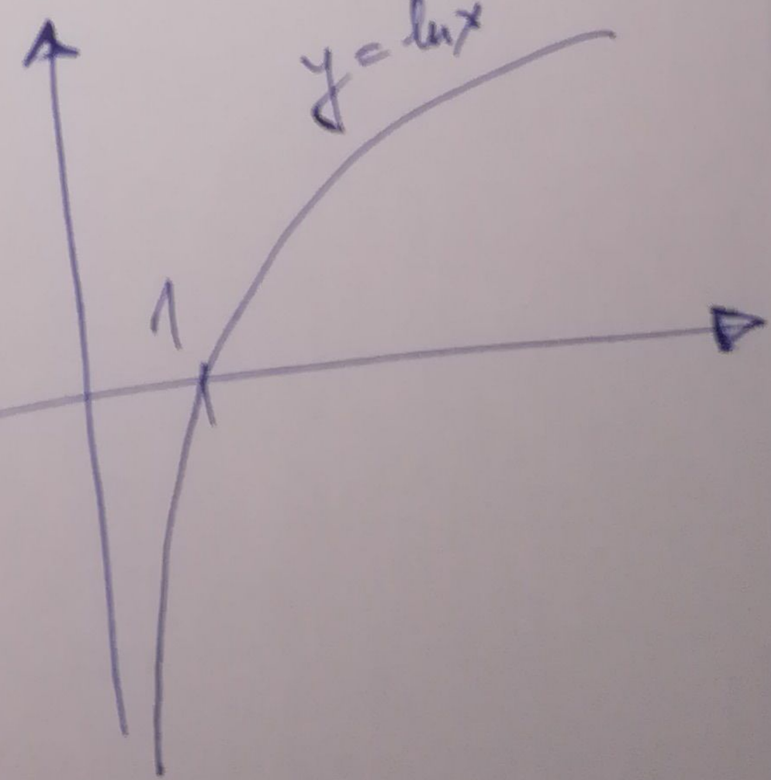


$y = x^3$

$y = e^x$ $(a^x; a>0, a\neq 1,$ обје $a>1)$

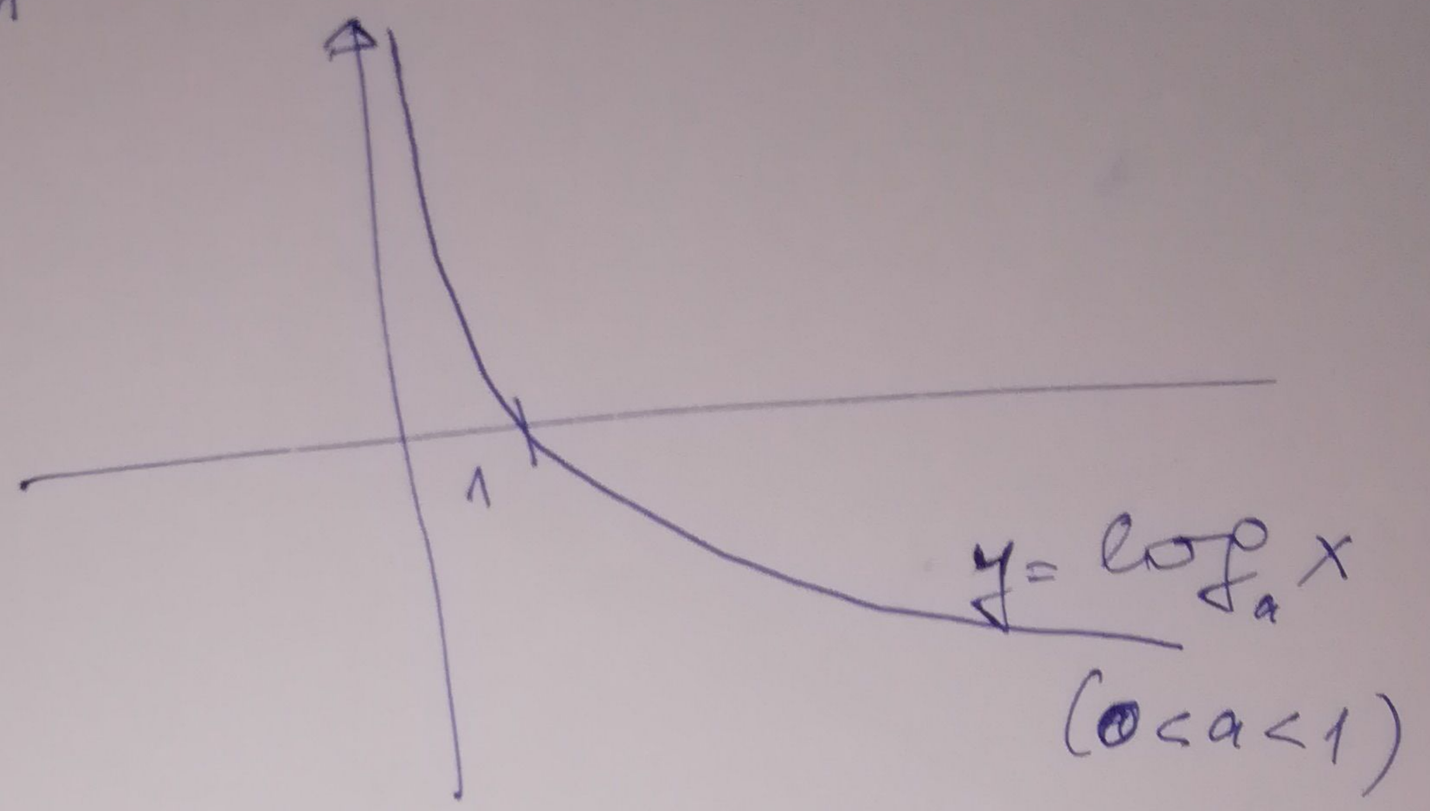




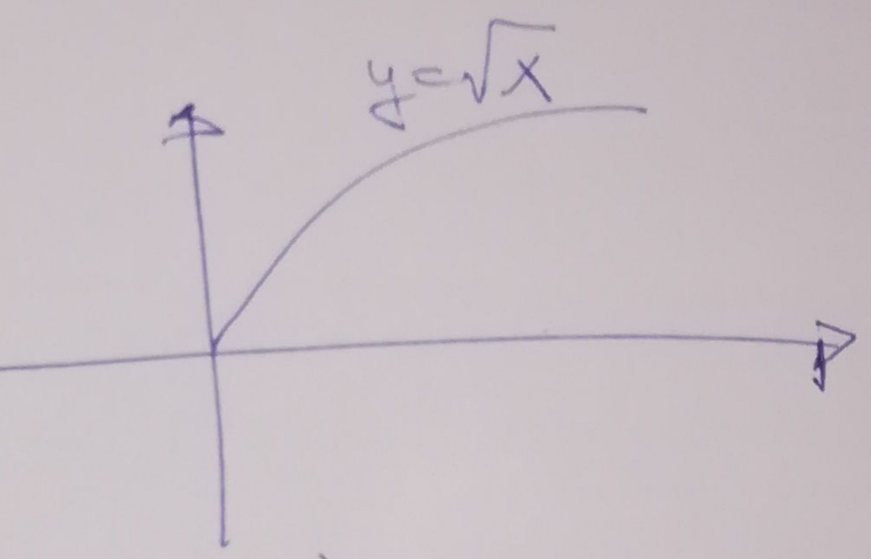


$y = \ln x = \log_e x$

$y = \log_{0.1} x$ $(= \log_a x,\ a < 1)$ 1.2



$y = \sqrt{x}$

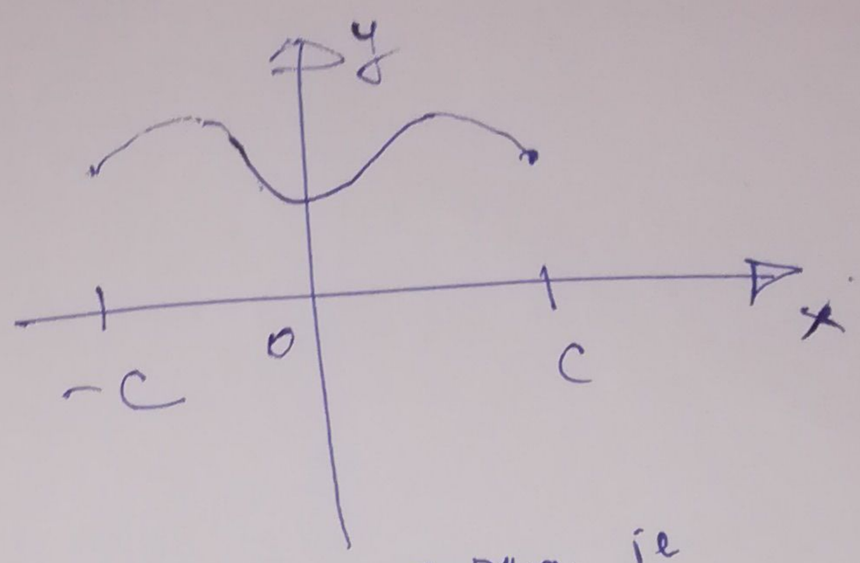


има их још

тригонометријске, хиперболометријске, ...

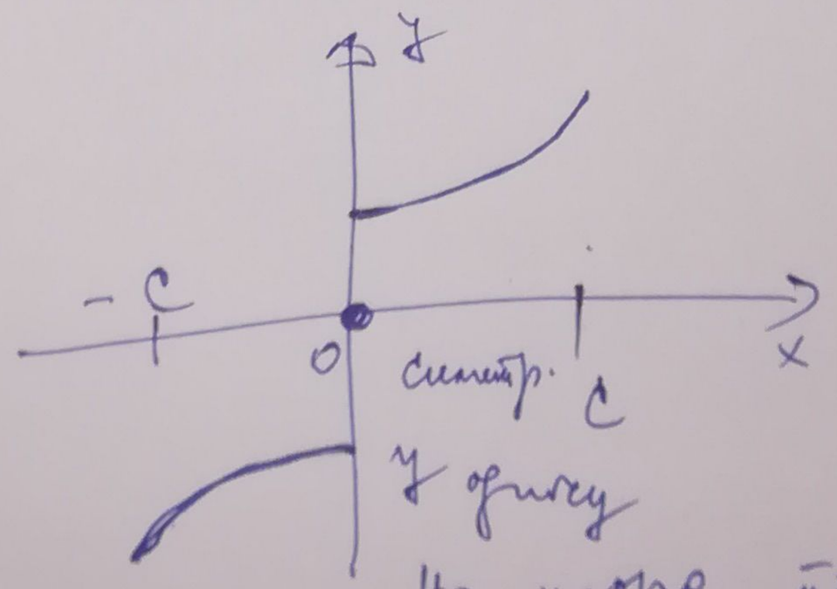
$y = f(x)$, за $x \in [-c, c]$ | J. 1

$f(-x) = f(x)$ – ПАРНА



Симетрична је у односу на y осу

$f(-x) = -f(x)$ – НЕПАРНА



Ако за неко $x \in D_f$ важи $f(x+w) = f(x)$ за неко w (> 0) кажемо да је f периодична са периодом w.

$w = w_0$ – најмањи период, основни пер.

sin, cos имају основни период 2π | I.2

а tg, ctg π

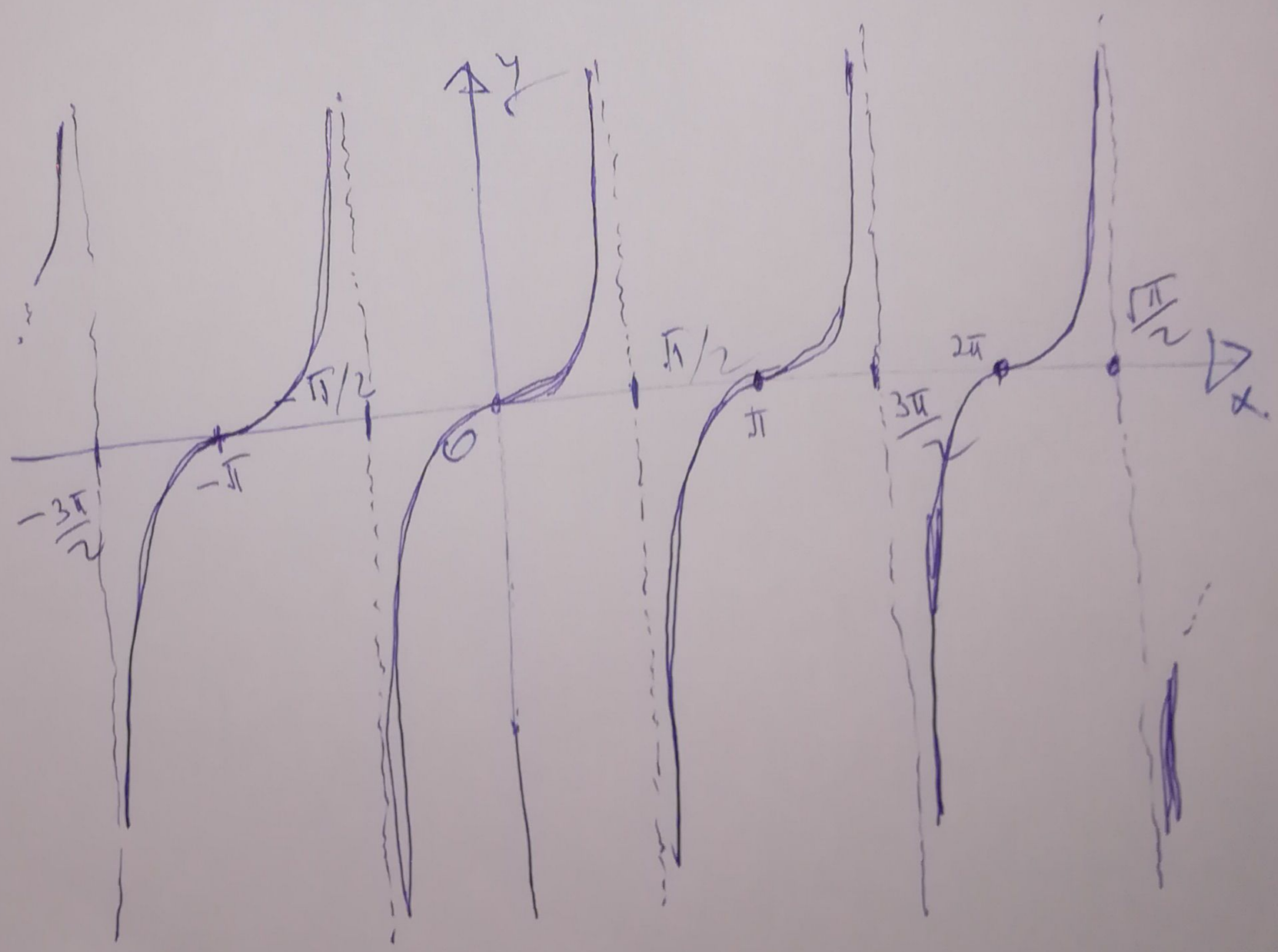
Пр. $f(x) = \ln(10 + \cos x)$

је периодична са периодом 2π

јер $f(x+2\pi) = \ln(10 + \cos(x+2\pi)) =$

$= \ln(10 + \cos x) = f(x)$

$f(x) = \operatorname{tg} x$



Шта је реалан низ? [k]

(a_n)

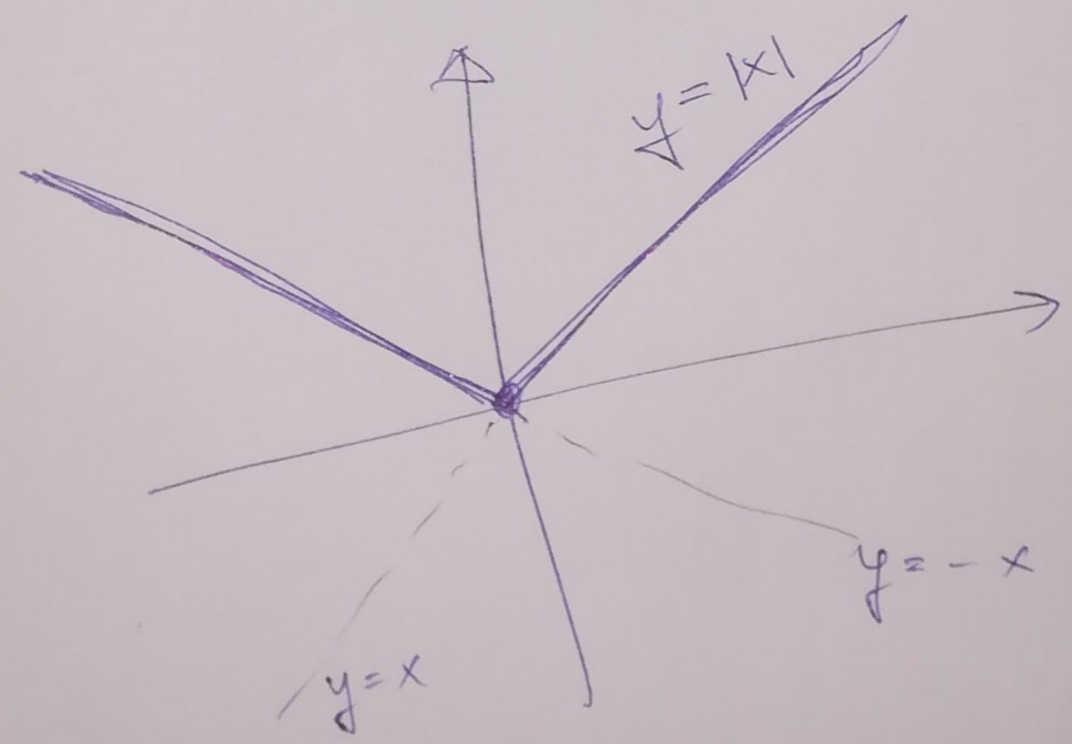
$a_1, a_2, \dots, a_n, a_{n+1}, \dots$

$a(1), a(2), \dots, a(n), a(n+1), \dots$

$$a: \begin{pmatrix} 1 & 2 & \dots & n & n+1 & \dots \\ a(1) & a(2) & \dots & a(n) & a(n+1) & \dots \end{pmatrix}$$

$$a: \mathbb{N} \longrightarrow \mathbb{R}$$

За наведен и $y = |x| = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$



$$f(x) = |g(x)| = \begin{cases} g(x), & \text{тамо где је } g \text{ позитивно} \\ 0, & \text{ч. } g = 0 \\ -g(x), & \text{ш. где је } g < 0 \end{cases}$$