Алгебарски изрази

A

$$a^2 - b^2 = (a-b)(a+b)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Неке једнакости

$$a^n \cdot b^n = (a \cdot b)^n$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$(a^n)^k = a^{n \cdot k}$$

$$a^n \cdot a^m = a^{n+m}$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{-1} = \frac{1}{a} \quad (a > 0)$$

$$a^0 = 1$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\sqrt{x^2} = |x| \quad (\text{важно!})$$

$$\frac{\frac{x}{y}}{\frac{z}{t}} = \frac{x \cdot t}{y \cdot z}$$

$$\frac{x}{y} : \frac{z}{t} = \frac{x}{y} \cdot \frac{t}{z} = \frac{x \cdot t}{y \cdot z}$$

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[m \cdot n]{x}$$

$\boxed{\text{Пр.}}$ Средити израз, тј. израчунати $\boxed{\text{Б}}$

$$A=\left(\frac{2^{-4}-3^{-4}}{2^{-1}-3^{-1}}\left(2^{-1}+3^{-1}\right)^{-1}-3^{-1}\cdot 81^{-\left(2^{-2}\right)}\right)^{-1/2}.$$

$$A=\left(\frac{\cancel{\left(2^{-1}-3^{-1}\right)}\cancel{\left(2^{-1}+3^{-1}\right)}\left(2^{-2}+3^{-2}\right)}{\cancel{\left(2^{-1}-3^{-1}\right)}\cancel{\left(2^{-1}+3^{-1}\right)}}-3^{-1}\cdot 81^{-\frac{1}{4}}\right)^{-\frac{1}{2}}=$$

$$=\left(2^{-2}+3^{-2}-\frac{1}{3}\cdot\frac{1}{\sqrt[4]{81}}\right)^{-1/2}=\left(\frac{1}{4}+\cancel{\frac{1}{9}}-\cancel{\frac{1}{9}}\right)^{-\frac{1}{2}}=$$

$$=\left(\frac{1}{4}\right)^{-1/2}=\frac{1}{4^{-1/2}}=\frac{1}{\frac{1}{\sqrt{4}}}=2.\ \square$$

$\boxed{\text{Пр.}}$

$$\frac{\left(0{,}5:1{,}25+\frac{7}{5}:1\frac{4}{7}-\frac{3}{11}\right)\cdot 3}{\left(1{,}5+\frac{1}{4}\right):18\frac{1}{3}}=\frac{\left(\frac{1}{2}\cdot\frac{4}{5}+\frac{7}{5}\cdot\frac{7}{11}-\frac{3}{11}\right)\cdot\cancel{3}}{\left(\frac{3}{2}+\frac{1}{4}\right)\cdot\frac{\cancel{3}}{55}}=$$

$$=\frac{\frac{2}{5}+\frac{49}{55}-\frac{3}{11}}{\frac{7}{4\cdot 55}}=\frac{\frac{22+49-15}{\cancel{55}}}{\frac{7}{4\cdot\cancel{55}}}=32$$

$\boxed{\text{Пр.}}$

$$\left(\left(3\sqrt{2}-4\right)^{-1}-\left(3\sqrt{2}+4\right)^{-1}\right):\left(\frac{1}{5+2\sqrt{6}}+5-2\sqrt{6}\right)=$$

$$=\left(\frac{1}{3\sqrt{2}-4}-\frac{1}{3\sqrt{2}+4}\right):\left(5-2\sqrt{6}+5-2\sqrt{6}\right)$$

$$=\frac{\cancel{3\sqrt{2}}+4-\cancel{3\sqrt{2}}+4}{2}\cdot\frac{1}{2\left(5-2\sqrt{6}\right)}=2\cdot\frac{5+2\sqrt{6}}{1}=10+4\sqrt{6}$$

B

$$\frac{(a-b)^2+3ab}{a^3-b^3}\cdot\frac{a^2-b^2-a+b}{a^2b+ab^2-ab}=$$

$$=\frac{a^2-2ab+b^2+3ab}{(a-b)(a^2+ab+b^2)}\cdot\frac{(a-b)(a+b)-(a-b)}{ab(a+b)-ab}$$

$$=\frac{\cancel{a^2+ab+b^2}}{\cancel{(a-b)}\cancel{(a^2+ab+b^2)}}\cdot\frac{\cancel{(a-b)}\cancel{(a+b-1)}}{ab\cancel{(a+b-1)}}=\frac{1}{ab}$$

$\boxed{\text{Пр.}}$ $(a>0)$

$$\left(\frac{\sqrt{a}}{1+a}\right)^{-1}+\sqrt{\left(1-\frac{1}{a}\right)(a-1)}=$$

ово је дефинисано за $a>0$

$$=\frac{1+a}{\sqrt{a}}+\sqrt{\frac{(a-1)^2}{a}}=$$

$$=\frac{1+a}{\sqrt{a}}+\frac{|a-1|}{\sqrt{a}}=\frac{1+a+|a-1|}{\sqrt{a}}=\begin{cases}2\sqrt{a}, & a>1\\ 2, & a=1\\ \frac{2}{\sqrt{a}}, & a<1\end{cases}$$

Пр. $(x>0)$

$$\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}}=\sqrt{x\sqrt{x\sqrt[4]{x^3}}}=$$

$$=\sqrt{x\sqrt[8]{x^7}}=\sqrt[16]{x^{15}}=x^{15/16}$$

или $$\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}}=\left(x\left(x\left(x\cdot x^{1/2}\right)^{1/2}\right)^{1/2}\right)^{1/2}=\left(x\left(x\cdot x^{3/4}\right)^{1/2}\right)^{1/2}=\left(x\cdot x^{7/8}\right)^{1/2}=x^{\frac{15}{16}}$$

$\boxed{\text{Пр.}}$ Г

$$\frac{a^2-b^2}{a-b}-\frac{a^3-b^3}{a^2-b^2}=$$

$$=\frac{\cancel{(a-b)}(a+b)}{\cancel{a-b}}-\frac{\cancel{(a-b)}(a^2+ab+b^2)}{\cancel{(a-b)}(a+b)}=$$

$$=a+b-\frac{a^2+ab+b^2}{a+b}=$$

$$=\frac{(a+b)^2-a^2-ab-b^2}{a+b}=\frac{\cancel{a^2}+2ab\cancel{+b^2}\cancel{-a^2}-ab\cancel{-b^2}}{a+b}$$

$$=\frac{ab}{a+b}.\ \square$$