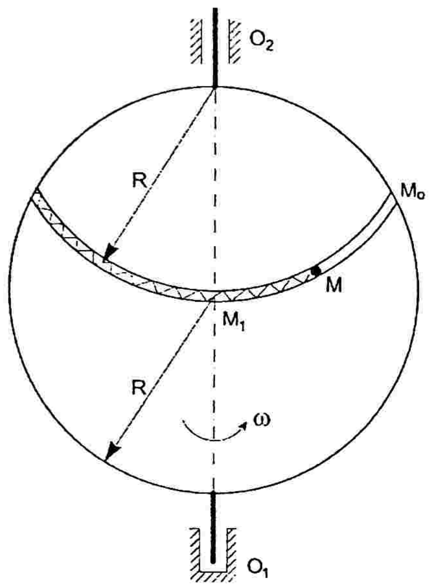
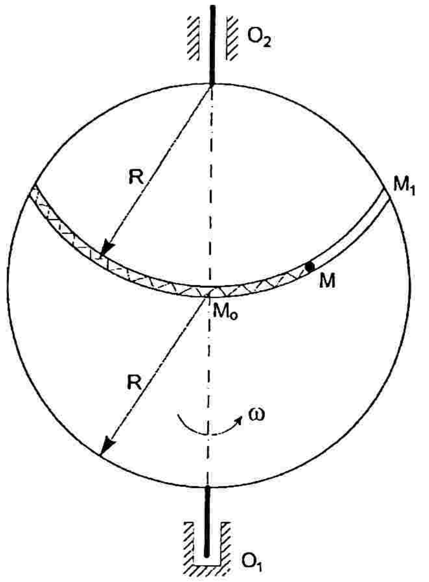
MEHANIKA 3 **1.**GRUPA OKTOBAR 2021

1. Homogeni disk mase m i poluprečnika R obrće se oko svoje vertikalne ose O_1O_2. U disk je urezan kružni žljeb poluprečnika R po kome može da klizi bez trenja tačka M mase m. Tačka je vezana oprugom nepoznate krutosti c čiji je drugi kraj vezan za obod diska. Dužina nenapregnute opruge iznosi $\frac{R\pi}{3}$. U početnom trenutku, tačka se nalazila u položaju M_0 bez početne relativne brzine, dok se disk obrtao ugaonom brzinom $\omega_0 = \sqrt{\frac{g}{R}}$. Odrediti ugaonu brzinu diska u trenutku kada se tačka nađe na njegovoj osi obrtanja (položaj M_1). Odrediti krutost opruge c iz uslova da se tačka zaustavi u položaju M_1.

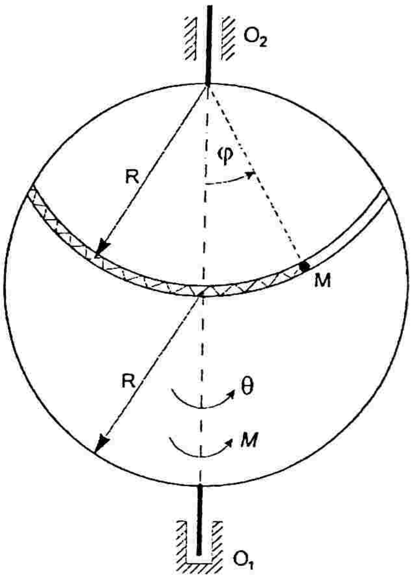
2. Disk poluprečnika R obrće se oko svoje vertikalne ose O_1O_2 konstantnom ugaonom brzinom ω. U disk je urezan kružni glatki žljeb poluprečnika R po kome može da se kreće tačka M mase m. Tačka je vezana oprugom krutosti c čiji je drugi kraj vezan za obod diska. Dužina nenapregnute opruge iznosi $\frac{2R\pi}{3}$. U početnom trenutku, tačka se nalazila na osi obrtanja diska (položaj M_0) bez početne relativne brzine. Ako je $\omega = \sqrt{\frac{g}{R}}$ i $c = \frac{9}{4\pi^2}\frac{mg}{R}$, odrediti relativnu brzinu tačke u položaju M_1. Odrediti ukupnu reakciju veze koja deluje na tačku u položaju M_1.



Zadatak 1.



Zadatak 2.



Zadatak 3.

3. Homogeni disk mase m i poluprečnika R može da se obrće oko svoje vertikalne ose O_1O_2. Ugao obrtanja diska određen je uglom θ. U disk je urezan kružni žljeb poluprečnika R po kome može da se kreće bez trenja tačka M mase m. Položaj tačke unutar žljeba određen je uglom φ, kao što je prikazano na slici. Tačka je vezana oprugom krutosti c čiji je drugi kraj vezan za obod diska. Dužina nenapregnute opruge iznosi $\frac{R\pi}{3}$. Na disk deluje spreg sila momenta intenziteta M u naznačenom smeru.

a) Za date generalisane koordinate (θ, φ) odrediti diferencijalne jednačine kretanja mehaničkog sistema.

b) Odrediti $M(\varphi)$, tj. zavisnost momenta od ugla φ tako da se disk obrće konstantnom ugaonom brzinom ω. U početnom trenutku važi $\varphi(0) = 0, \dot{\varphi}(0) = \dot{\varphi}_0$.

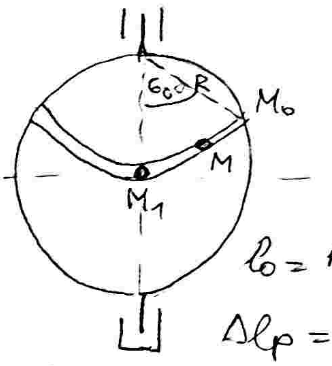
MEX. 3 (OKT. 121)

$L_{Oz} = L_{Oz,0}$

$J_D \omega_1 = J_D \omega_0 + R\frac{\sqrt{3}}{2} m \left(R\frac{\sqrt{3}}{2}\omega_0\right)$

$\cancel{\frac{1}{4}} \cancel{m} R^2 \omega_1 = \frac{1}{4} \cancel{m} R^2 \omega_0 + \frac{3}{4} \cancel{m} R^2 \omega_0$

$\boxed{\omega_1 = 4\omega_0}$ $\boxed{\omega_1 = 4\sqrt{g/R}}$



$l_0 = R\pi/3$

$\Delta l_p = R\pi/3$

$\Delta l_k = 0$

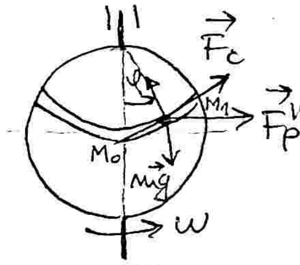
$E_{K1} - E_{K,0} = A(m\vec{g}) + A(\vec{F}_c)$

$\frac{1}{2} J_D \omega_1^2 - \frac{1}{2} J_D \omega_0^2 - \frac{1}{2} m V_{p0}^2 = mg\Delta h + \frac{c}{2}(\Delta l_p^2 - \cancel{\Delta l_k^2}^{\,0})$

$\frac{1}{2}\frac{1}{4} mR^2\omega_1^2 - \frac{1}{2}\frac{1}{4} mR^2\omega_0^2 - \frac{m}{2}\left(\frac{\sqrt{3}}{2}R\omega_0\right)^2 = mg\left(R - \frac{R}{2}\right) + \frac{c}{2}\left(\frac{R\pi}{3}\right)^2$

... $\boxed{c = 18\frac{mg}{R\pi^2}}$

2



$\omega = \sqrt{g/R}$

$l_0 = 2\frac{R\pi}{3}$

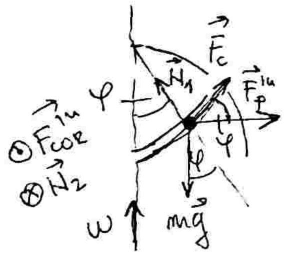
$E_{Kr,1} - \cancel{E_{Kr,0}}^{\,0} = A(mg) + A(\vec{F}_c) + A(\vec{F}_p^{in})$

$\frac{m}{2}V_{r1}^2 = -mg\frac{R}{2} + \frac{c}{2}\left(\frac{R\pi}{3}\right)^2 + \int_0^{60^\circ} m\omega^2 R\sin\varphi R\,d\varphi\cdot\cos\varphi$

$\frac{m}{2}V_{r1}^2 = -\frac{mgR}{2} + \frac{1}{2}\frac{\cancel{9}}{4\cancel{\pi^2}}\cdot\frac{mg}{\cancel{R}}\frac{R^{\cancel{2}}\cancel{\pi^2}}{\cancel{9}} + m\omega^2R^2\int_0^{60^\circ}\sin\varphi\cos\varphi\,d\varphi$

$\frac{m}{2}V_{r1}^2 = -\frac{mgR}{2} + \frac{mgR}{8} + m\omega^2R^2\frac{\sin^2\varphi}{2}\Big|_0^{60^\circ}, \quad \left(\omega^2 = \frac{g}{R}\right)$

..., $\frac{m}{2}V_{r1}^2 = 0$, $\boxed{V_{r1} = 0}$



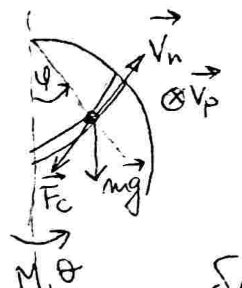
$m\frac{V_r^2}{R} = N_1 - mg\cos\varphi - F_p^{in}\sin\varphi$

$\varphi_1 = 60^\circ$, $N_1 = mg\frac{1}{2} + m\omega^2R\frac{3}{4} = \frac{5}{4}mg$

$N_2 = F_{cor}^{in} = 2m\omega \cancel{V_{r1}}^{\,0}\sin\beta = 0$, $\boxed{N = N_1 = \frac{5}{4}mg}$

3

$l_0 = \frac{R\pi}{3}$



$E_K = \frac{1}{2}J_D\dot{\theta}^2 + \frac{1}{2}m(V_r^2 + V_p^2)$

$E_K = \frac{1}{2}\frac{mR^2}{4}\dot{\theta}^2 + \frac{m}{2}\cdot(R^2\dot{\varphi}^2 + R^2\sin^2\varphi\cdot\dot{\theta}^2)$

$E_K = \frac{mR^2}{8}\dot{\theta}^2 + \frac{mR^2}{2}\dot{\varphi}^2 + \frac{mR^2}{2}\dot{\theta}^2\sin^2\varphi$

$\delta A(\vec{M}) = M\delta\theta$, $\delta A(\vec{F}_c) = \vec{F}_c\delta\vec{r}_M = -F_cR\delta\varphi = -cR^2\varphi\delta\varphi$

$\delta A(m\vec{g}) = m\vec{g}\delta\vec{r}_M = -mgR\sin\varphi\,\delta\varphi$

a)

$$\boxed{\begin{aligned} & mR^2\ddot{\theta}\left(\frac{1}{4}+\sin^2\varphi\right)+2mR^2\dot{\varphi}\dot{\theta}\sin\varphi\cos\varphi = M \quad \ldots(1)\\ & mR^2\ddot{\varphi} - mR^2\dot{\theta}^2\sin\varphi\cos\varphi = -mgR\sin\varphi - CR^2\varphi \quad \ldots(2)\end{aligned}}$$

б) $\dot{\theta}=\omega$, $\ddot{\theta}=0$

(1) $\rightarrow M = mR^2\omega\sin 2\varphi\cdot\dot{\varphi}$

(2) $\rightarrow$ $$\ddot{\varphi} = \omega^2\sin\varphi\cos\varphi - \frac{g}{R}\sin\varphi - \frac{c}{m}\varphi$$

$$\int_{\dot{\varphi}_0}^{\dot{\varphi}}\dot{\varphi}\,d\dot{\varphi} = \omega^2\int_{\varphi_0}^{\varphi}\sin\varphi\cos\varphi\,d\varphi - \frac{g}{R}\int_{\varphi_0}^{\varphi}\sin\varphi\,d\varphi - \frac{c}{m}\int_{\varphi_0}^{\varphi}\varphi\,d\varphi$$

$$\frac{\dot{\varphi}^2-\dot{\varphi}_0^2}{2} = \omega^2\frac{\sin^2\varphi}{2}\Big|_{\varphi_0}^{\varphi} - \frac{g}{R}(-\cos\varphi)\Big|_{\varphi_0}^{\varphi} - \frac{c}{m}\frac{\varphi^2}{2}\Big|_{\varphi_0}^{\varphi}$$

$$\dot{\varphi}^2 = \dot{\varphi}_0^2 + \omega^2(\sin^2\varphi - \overset{0}{\cancel{\sin^2\varphi_0}}) + \frac{2g}{R}(\cos\varphi - \overset{1}{\cancel{\cos\varphi_0}}) - \frac{c}{m}(\varphi^2 - \overset{0}{\cancel{\varphi_0^2}}) \ldots($$

(3) $\rightarrow \boxed{M = mR^2\omega\sin 2\varphi\cdot\dot{\varphi}(\varphi)}$