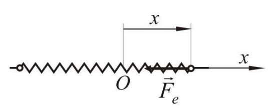
1 ПРАВОЛИНИЈСКЕ ОСЦИЛАЦИЈЕ МАТЕРИЈАЛНЕ ТАЧКЕ

1.1 УВОДНА РАЗМАТРАЊА

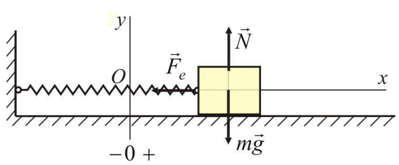
- *слободне*
- *принудне*
- *пригушене*
- *непригушене*



Слика 1

$$F_e = c\,\Delta l\,,\ \Delta l = |x|,\ F_{ex} = -cx$$

1.2 СЛОБОДНЕ НЕПРИГУШЕНЕ ОСЦИЛАЦИЈЕ МАТЕРИЈАЛНЕ ТАЧКЕ



Слика 2

$$m\vec{a} = \vec{F}_e + \vec{N} + m\vec{g}$$
$$m\ddot{x} = F_{ex} \qquad F_{ex} = -cx$$
$$m\ddot{y} = N - mg = 0 \qquad N = mg$$

$$m\ddot{x} = -cx \qquad \ddot{x} + \frac{c}{m}x = 0$$

$$\frac{c}{m} = \omega^2$$

$$\ddot{x} + \omega^2 x = 0$$

$$x = Ae^{\lambda t} \qquad \dot{x} = \lambda Ae^{\lambda t} \qquad \ddot{x} = \lambda^2 Ae^{\lambda t}$$

$$\left(\lambda^2 + \omega^2\right)Ae^{\lambda t} = 0 \qquad \lambda^2 + \omega^2 = 0$$

$$\lambda_{1,2} = \pm\, i\omega \qquad e^{\pm i\varphi} = \cos\varphi \pm i\sin\varphi$$

$$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \qquad x = \left(A_1 + A_2\right)\cos\omega t + i\left(A_1 - A_2\right)\sin\omega t$$

$$C_1 = A_1 + A_2 \qquad C_2 = i\left(A_1 - A_2\right)$$

$$x = C_1\cos\omega t + C_2\sin\omega t$$

$$\dot{x} = -C_1\omega\sin\omega t + C_2\omega\cos\omega t$$

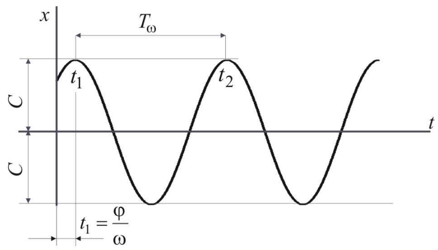
$$t = t_0 = 0 \qquad x(0) = x_0 \qquad \dot{x}(0) = \dot{x}_0$$

$$x_0 = C_1 \qquad \dot{x}_0 = C_2\omega$$

$$x = x_0 \cos\omega t + \frac{\dot{x}_0}{\omega}\sin\omega t$$

$$C_1 = C\cos\varphi \qquad C_2 = C\sin\varphi$$

$$x = C\left(\cos\omega t\cos\varphi + \sin\omega t\sin\varphi\right) \qquad x = C\cos\left(\omega t - \varphi\right)$$



Слика 3

$$tg\varphi = \frac{C_2}{C_1} \qquad tg\varphi = \frac{\dot{x}_0}{\omega x_0}$$

$$C = \sqrt{C_1^2 + C_2^2} \qquad C = \sqrt{x_0^2 + \frac{\dot{x}_0^2}{\omega^2}}$$

$$x = C\cos\left(\omega t - \varphi\right)$$

$$\cos\left(\omega t_1 - \varphi\right) = \cos\left(\omega t_2 - \varphi\right)$$

$$\omega t_1 - \varphi + 2\pi = \omega t_2 - \varphi$$

$$\omega\left(t_2 - t_1\right) = 2\pi \qquad \omega T_\omega = 2\pi \qquad T_\omega = \frac{2\pi}{\omega} \qquad T_\omega = 2\pi\sqrt{\frac{m}{c}}$$

$$f = \frac{1}{T_\omega} \qquad f = \frac{1}{2\pi\sqrt{\frac{m}{c}}} \qquad f = \frac{1}{\frac{2\pi}{\omega}} \qquad \omega = 2\pi f$$

1.2.1 Слободне осцилације терета обешеног о опругу

$$\Delta l = \left| f_{st} + x \right|$$

$$F_{ex} = -c\left(x + f_{st}\right)$$

$$m\ddot{x} = F_{ex} + mg$$

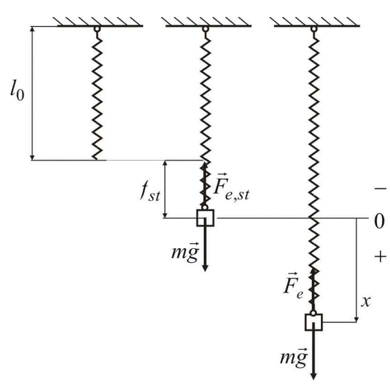
$$m\ddot{x} = -cx + mg - cf_{st}$$

$$\dot{x} = \ddot{x} = 0, \quad x = 0$$

$$0 = mg - cf_{st}$$

$$m\ddot{x} + cx = 0 \qquad \ddot{x} + \frac{c}{m}x = 0$$

$$T_\omega = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{f_{st}}{g}}$$

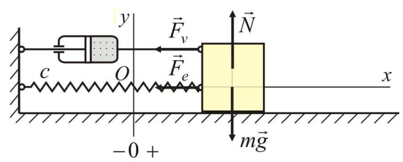


Слика 4

1.3 СЛОБОДНЕ ПРИГУШЕНЕ ОСЦИЛАЦИЈЕ МАТЕРИЈАЛНЕ ТАЧКЕ

1.3.1 Слободне пригушене осцилације материјалне тачке под дејством силе отпора пропорционалне првом степену брзине тачке

$$\vec{F}_v = -b\vec{v} \qquad b > 0$$



Слика 5

$$m\vec{a} = \vec{F}_v + \vec{F}_e + \vec{N} + m\vec{g}$$

$$m\ddot{x} = F_{v,x} + F_{e,x}$$

$$F_{e,x} = -cx \qquad F_{v,x} = -b\dot{x}$$

$$m\ddot{y} = N - mg = 0 \qquad N = mg$$

$$m\ddot{x} = -b\dot{x} - cx \qquad \ddot{x} + \frac{b}{m}\dot{x} + \frac{c}{m}x = 0 \qquad \frac{b}{m} = 2\delta \qquad \frac{c}{m} = \omega^2$$

$$\ddot{x} + 2\delta\dot{x} + \omega^2 x = 0$$

$$x = Ae^{\lambda t} \qquad \dot{x} = \lambda Ae^{\lambda t} \qquad \ddot{x} = \lambda^2 Ae^{\lambda t}$$

$$\left(\lambda^2 + 2\delta\lambda + \omega^2\right)Ae^{\lambda t} = 0 \qquad \lambda^2 + 2\delta\lambda + \omega^2 = 0$$

$$\lambda_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega^2}$$

1. **Случај малог отпора (привидно периодично кретање):** $\delta < \omega$

$$\lambda_{1,2} = -\delta \pm i\,p \qquad p = \sqrt{\omega^2 - \delta^2}$$

$$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \qquad x = e^{-\delta t}\left(A_1 e^{ipt} + A_2 e^{-ipt}\right)$$

$$x = e^{-\delta t}\left(C_1 \cos pt + C_2 \sin pt\right)$$

$$\dot{x} = -\delta e^{-\delta t}\left(C_1 \cos pt + C_2 \sin pt\right) + e^{-\delta t}\left(-pC_1 \sin pt + pC_2 \cos pt\right)$$

$$t = t_0 = 0 \qquad x(0) = x_0 \qquad \dot{x}(0) = \dot{x}_0$$

$$x_0 = C_1 \qquad \dot{x}_0 = -\delta C_1 + pC_2$$

$$C_2 = \frac{\dot{x}_0 + \delta C_1}{p} \qquad C_2 = \frac{\dot{x}_0 + \delta x_0}{p}$$

$$x = e^{-\delta t}\left(x_0 \cos pt + \frac{\dot{x}_0 + \delta x_0}{p}\sin pt\right)$$

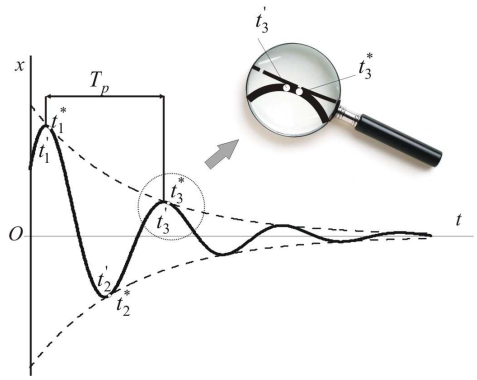
$$C_1 = R\cos\alpha \qquad C_2 = R\sin\alpha$$

$$x = Re^{-\delta t}\left(\cos\alpha \cos pt + \sin\alpha \sin pt\right)$$

$$x = Re^{-\delta t}\cos\left(pt-\alpha\right)$$

$$R=\sqrt{C_1^2+C_2^2} \qquad R=\sqrt{x_0^2+\left(\frac{\dot{x}_0+\delta x_0}{p}\right)^2}$$

$$tg\alpha=\frac{C_2}{C_1} \qquad tg\alpha=\frac{\dot{x}_0+\delta x_0}{px_0}$$



Слика 6

$$\cos\left(pt_1-\alpha\right)=\cos\left(pt_2-\alpha\right)$$

$$pt_1-\alpha+2\pi=pt_2-\alpha$$

$$p\left(t_2-t_1\right)=2\pi \quad T_p=\frac{2\pi}{p}$$

$$T_p=\frac{2\pi}{\sqrt{\omega^2-\delta^2}}$$

$$T_p>T_\omega \qquad T_p=\frac{2\pi}{\omega}\frac{1}{\sqrt{1-\left(\frac{\delta}{\omega}\right)^2}}$$

$$\psi=\frac{\delta}{\omega}$$

$$T_p=\frac{T_\omega}{\sqrt{1-\psi^2}}\approx T_\omega\left(1+\frac{1}{2}\psi^2+\ldots\right)$$

$$\cos\left(pt_n^*-\alpha\right)=1 \qquad x\left(t_n^*\right)=Re^{-\delta t_n^*}$$

1.3.1.1 Логаритамски декремент пригушених осцилација материјалне тачке

$$\dot{x}\left(t_1'\right)=0$$

$$0=-\delta e^{-\delta t_1'}\left(C_1\cos pt_1'+C_2\sin pt_1'\right)+e^{-\delta t_1'}\left(-pC_1\sin pt_1'+pC_2\cos pt_1'\right)$$

$$0=\cos pt_1'\left[-e^{-\delta t_1'}\left(\delta C_1+\delta C_2 tgpt_1'\right)+e^{-\delta t_1'}\left(-pC_1 tgpt_1'+pC_2\right)\right]$$

$$tg\left(pt_1'\right)=\frac{pC_2-\delta C_1}{\delta C_2+pC_1} \qquad pt_1'=arctg\left(\frac{pC_2-\delta C_1}{\delta C_2+pC_1}\right)$$

$$x = Re^{-\delta t}\cos\left(pt-\alpha\right)$$

$$t_2' = t_1' + \frac{T_p}{2}$$

$$t_3' = t_2' + \frac{T_p}{2} = t_1' + 2\,\frac{T_p}{2}$$

$$\vdots$$

$$t_n' = t_{n-1}' + \frac{T_p}{2} = t_1' + (n-1)\,\frac{T_p}{2}$$

$$\left|\frac{x_n}{x_{n+1}}\right| = \left|\frac{Re^{-\delta t_n'}\cos\left(pt_n' - \alpha\right)}{Re^{-\delta t_{n+1}'}\cos\left(pt_{n+1}' - \alpha\right)}\right| = \left|\frac{e^{-\delta t_n'}}{e^{-\delta\left(t_n' + \frac{T_p}{2}\right)}}\;\frac{\cos\left(pt_n' - \alpha\right)}{\cos\left(pt_n' + p\,\frac{T_p}{2} - \alpha\right)}\right|$$

$$\left|\frac{x_n}{x_{n+1}}\right| = \left|\frac{1}{e^{-\frac{\delta T_p}{2}}}\;\frac{\cos\left(pt_n' - \alpha\right)}{\cos\left(pt_n' - \alpha + \pi\right)}\right| = e^{\frac{\delta T_p}{2}} = D$$

$$\ln\left|x_n\right| - \ln\left|x_{n+1}\right| = \ln e^{\frac{\delta T_p}{2}} = \ln D = \frac{\delta T_p}{2} = \mathcal{D}$$

2. Гранични случај (апериодично кретање): $\delta = \omega$

$$\lambda_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega^2} = -\delta$$

$$x = C_1 e^{-\delta t} + C_2 t e^{-\delta t} = e^{-\delta t}\left(C_1 + C_2 t\right)$$

$$\dot{x} = -\delta e^{-\delta t}\left(C_1 + C_2 t\right) + e^{-\delta t} C_2$$

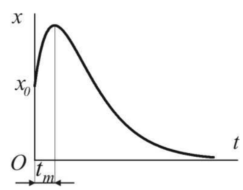
$$x_0 = C_1$$

$$\dot{x}_0 = -\delta C_1 + C_2 \qquad\qquad C_2 = \dot{x}_0 + \delta x_0$$

$$x = x_{\max} = x(t_m)$$

$$\dot{x}(t_m) = 0 \qquad \delta(C_1 + C_2 t_m) = C_2 \qquad t_m = \frac{C_2 - \delta C_1}{\delta C_2} = \frac{\dot{x}_0}{\delta(\dot{x}_0 + \delta x_0)}$$

$$x(t_1) = 0 \qquad C_1 + C_2 t_1 = 0 \qquad t_1 = -\frac{C_1}{C_2} = -\frac{x_0}{\delta x_0 + \dot{x}_0}$$

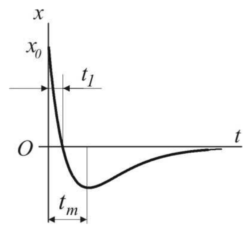


Слика 7

а)

$x_0 > 0 \quad \dot{x}_0 > 0$

$t_m > 0 \quad t_1 < 0$



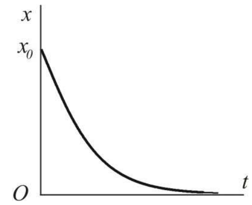
Слика 8

б)

$x_0 > 0 \quad \dot{x}_0 < 0$

$\dot{x}_0 + \delta x_0 < 0$

$t_m > 0 \quad t_1 > 0$



Слика 9

в)

$x_0 > 0 \quad \dot{x}_0 < 0$

$\dot{x}_0 + \delta x_0 > 0$

$t_m < 0 \quad t_1 < 0$

3. Велико пригушење (апериодично кретање): $\delta > \omega$

$$\lambda_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega^2} \qquad \lambda_{1,2} = -\delta \pm \nu \qquad \nu^2 = \delta^2 - \omega^2$$

$$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \qquad x = e^{-\delta t}\left(A_1 e^{\nu t} + A_2 e^{-\nu t}\right)$$

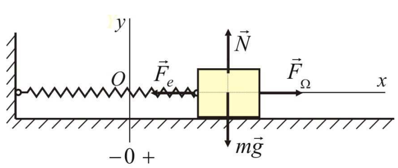
$$e^{\pm \nu t} = ch(\nu t) \pm sh(\nu t)$$

$$x = e^{-\delta t}\left[C_1\, ch(\nu t) + C_2\, sh(\nu t)\right]$$

$$C_1 = A_1 + A_2 \qquad C_2 = A_1 - A_2$$

Карактер $x = x(t)$ као и у случају $\delta = \omega$ (слике а, б, в)

1.4 ПРИНУДНЕ НЕПРИГУШЕНЕ ОСЦИЛАЦИЈЕ МАТЕРИЈАЛНЕ ТАЧКЕ



Слика 10

$$F_{\Omega,x} = F_0 \sin(\Omega t + \beta) \qquad F_{e,x} = -cx$$

$$m\vec{a} = \vec{F}_e + \vec{N} + m\vec{g} + \vec{F}_\Omega$$

$$m\ddot{x} = -cx + F_0 \sin(\Omega t + \beta)$$

$$m\ddot{y} = N - mg = 0 \qquad N = mg$$

$$\ddot{x} + \frac{c}{m}x = \frac{F_0}{m}\sin(\Omega t + \beta) \qquad \frac{F_0}{m} = h \qquad \frac{c}{m} = \omega^2$$

$$\ddot{x} + \omega^2 x = h \sin(\Omega t + \beta)$$

$$x = x_h + x_p$$

$$x_h = C_1 \cos \omega t + C_2 \sin \omega t$$

$$x_p = C\sin\left(\Omega t+\beta\right) \qquad \ddot{x}_p = -C\,\Omega^2\sin\left(\Omega t+\beta\right)$$

$$C\left(\omega^2-\Omega^2\right)\sin\left(\Omega t+\beta\right)\equiv h\sin\left(\Omega t+\beta\right)$$

$$C=\frac{h}{\omega^2-\Omega^2} \qquad x_p=\frac{h}{\omega^2-\Omega^2}\sin\left(\Omega t+\beta\right)$$

$$x=C_1\cos\omega t+C_2\sin\omega t+\frac{h}{\omega^2-\Omega^2}\sin\left(\Omega t+\beta\right)$$

$$t=t_0=0 \quad x(0)=x_0 \quad \dot{x}(0)=\dot{x}_0$$

$$\dot{x}=-C_1\omega\sin\omega t+C_2\omega\cos\omega t+\frac{h\Omega}{\omega^2-\Omega^2}\cos\left(\Omega t+\beta\right)$$

$$x_0=C_1+\frac{h}{\omega^2-\Omega^2}\sin\beta \qquad C_1=x_0-\frac{h}{\omega^2-\Omega^2}\sin\beta$$

$$\dot{x}_0=C_2\omega+\frac{h\Omega}{\omega^2-\Omega^2}\cos\beta \qquad C_2=\frac{\dot{x}_0}{\omega}-\frac{h}{\omega^2-\Omega^2}\frac{\Omega}{\omega}\cos\beta$$

$$x=\left(x_0-\frac{h}{\omega^2-\Omega^2}\sin\beta\right)\cos\omega t+\left(\frac{\dot{x}_0}{\omega}-\frac{h}{\omega^2-\Omega^2}\frac{\Omega}{\omega}\cos\beta\right)\sin\omega t+\frac{h}{\omega^2-\Omega^2}\sin\left(\Omega t+\beta\right)$$

$$x=x_0\cos\omega t+\frac{\dot{x}_0}{\omega}\sin\omega t-\frac{h}{\omega^2-\Omega^2}\left(\cos\omega t\sin\beta+\frac{\Omega}{\omega}\sin\omega t\cos\beta\right)+\frac{h}{\omega^2-\Omega^2}\sin\left(\Omega t+\beta\right)$$

1.4.1 Динамички фактор појачавања код принудних непригушених осцилација

$$F_\Omega=F_0=const \qquad x_p^{(s)}=\frac{h}{\omega^2} \qquad C_{st}=\frac{h}{\omega^2}$$

$$F_\Omega=F_0\sin\left(\Omega t+\beta\right) \qquad x_p^{(d)}=\frac{h}{\omega^2-\Omega^2}\sin\left(\Omega t+\beta\right) \qquad C_d=\frac{h}{\omega^2-\Omega^2}$$

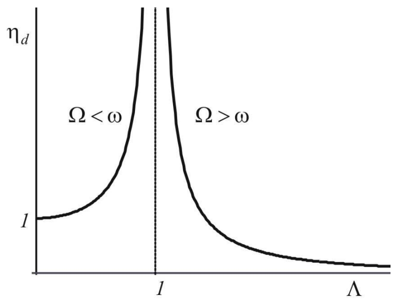
$$\eta_d=\left|\frac{C_d}{C_{st}}\right|=\left|\frac{\omega^2}{\omega^2-\Omega^2}\right|=\frac{1}{\left|1-\Lambda^2\right|} \qquad \Lambda=\frac{\Omega}{\omega}$$

η_d - динамички фактор појачавања

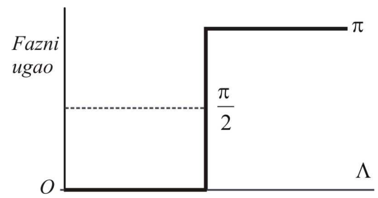
Λ - коефицијент поремећаја

$\eta_d\to\infty$ - резонанција за $\Lambda=1$

$$x_p=\eta_d C_{st}\sin\left(\Omega t+\beta\right)$$



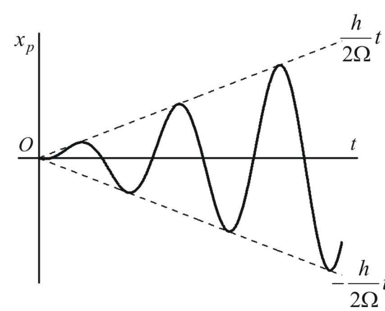
Слика 11



Слика 12

1.4.2 Резонанција

$$\Omega = \omega$$
$$x_p = Bt\cos(\Omega t + \beta)$$
$$\dot{x}_p = B\cos(\Omega t + \beta) - Bt\Omega\sin(\Omega t + \beta)$$
$$\ddot{x}_p = -2B\Omega\sin(\Omega t + \beta) - Bt\Omega^2\cos(\Omega t + \beta)$$
$$-2B\Omega\sin(\Omega t + \beta) - Bt\Omega^2\cos(\Omega t + \beta) + Bt\omega^2\cos(\Omega t + \beta) \equiv h\sin(\Omega t + \beta)$$
$$B = -\frac{h}{2\Omega} \qquad x_p = -\frac{h}{2\Omega}t\cos(\Omega t + \beta)$$
$$x_p = \frac{h}{2\Omega}t\sin\left[(\Omega t + \beta) - \frac{\pi}{2}\right]$$



Слика 13

1.4.3 Подрхтавање (бијење)

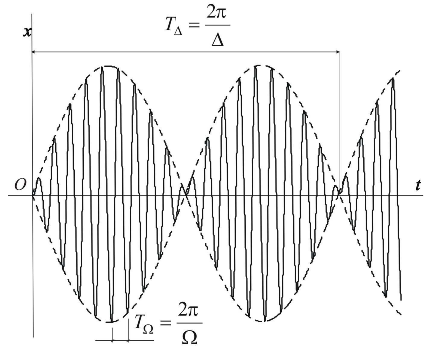
$$x_0 = \dot{x}_0 = 0 \quad \frac{\Omega}{\omega} \approx 1 \quad \text{тј.} \quad \Omega - \omega = 2\Delta$$
$$x \cong -\frac{h}{\omega^2 - \Omega^2}(\cos\omega t\sin\beta + \sin\omega t\cos\beta) + \frac{h}{\omega^2 - \Omega^2}\sin(\Omega t + \beta)$$
$$x \cong \frac{h}{\omega^2 - \Omega^2}(\sin(\Omega t + \beta) - \sin(\omega t + \beta))$$
$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$
$$\sin(x + y) - \sin(x - y) = 2\cos x\sin y$$

$$x + y = \Omega t + \beta \qquad x = \frac{\Omega + \omega}{2} t + \beta \approx \Omega t + \beta$$

$$x - y = \omega t + \beta \qquad y = \frac{\Omega - \omega}{2} t = \Delta\, t$$

$$x \cong \frac{2h}{\omega^2 - \Omega^2} \cos\left(\Omega t + \beta\right) \sin\left(\Delta\, t\right) = \frac{2h}{\omega^2 - \Omega^2} \sin\left(\Delta\, t\right) \cos\left(\Omega t + \beta\right)$$

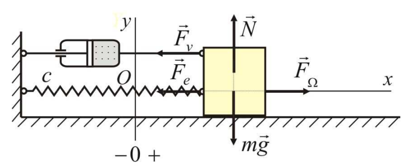
$$x = D\left(t\right) \cos\left(\Omega t + \beta\right) \qquad D\left(t\right) = \frac{2h}{\omega^2 - \Omega^2} \sin\left(\Delta\, t\right)$$



Слика 14

1.5 ПРИНУДНЕ ПРИГУШЕНЕ ОСЦИЛАЦИЈЕ МАТЕРИЈАЛНЕ ТАЧКЕ

1.5.1 Принудне пригушене осцилације материјалне тачке под дејством силе отпора пропорционалне првом степену брзине тачке



Слика 15

$$\vec{F}_v = -b\vec{v} \qquad b > 0$$

$$m\vec{a} = \vec{F}_v + \vec{F}_e + \vec{F}_\Omega + \vec{N} + m\vec{g}$$

$$m\ddot{x} = F_{v,x} + F_{e,x} + F_{\Omega,x}$$

$$F_{e,x} = -cx \qquad F_{v,x} = -b\dot{x}$$

$$m\ddot{y} = N - mg = 0 \qquad N = mg$$

$$m\ddot{x} = -b\dot{x} - cx + F_0 \sin\left(\Omega t + \beta\right)$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{c}{m}x = \frac{F_0}{m}\sin\left(\Omega t + \beta\right)$$

$$\frac{b}{m} = 2\delta \qquad \frac{c}{m} = \omega^2 \qquad \frac{F_0}{m} = h$$

$$\ddot{x} + 2\delta\dot{x} + \omega^2 x = h\sin\left(\Omega t + \beta\right)$$

$$x_h = \begin{cases} e^{-\delta t}\left(C_1 \cos pt + C_2 \sin pt\right) & \delta < \omega \quad (\textit{мало пригушење}) \\ e^{-\delta t}\left(C_1 + C_2 t\right) & \delta = \omega \quad (\textit{гранично пригушење}) \\ e^{-\delta t}\left(C_1 ch\nu t + C_2 sh\nu t\right) & \delta > \omega \quad (\textit{велико пригушење}) \end{cases}$$

$$x_p = A\sin\left(\Omega t + \beta - \gamma\right)$$

$$\dot{x}_p = A\Omega\cos\left(\Omega t + \beta - \gamma\right)$$

$$\ddot{x}_p = A\Omega^2 \sin\left(\Omega t + \beta - \gamma\right)$$

$$A\left(\omega^2 - \Omega^2\right)\sin\left(\Omega t + \beta - \gamma\right) + 2\delta A\Omega\cos\left(\Omega t + \beta - \gamma\right) \equiv h\sin\left[\left(\Omega t + \beta - \gamma\right) + \gamma\right]$$

$$A\left(\omega^2 - \Omega^2\right)\sin\left(\Omega t + \beta - \gamma\right) + 2\delta A\Omega\cos\left(\Omega t + \beta - \gamma\right) \equiv$$

$$\equiv h\sin\left(\Omega t + \beta - \gamma\right)\cos\gamma + h\cos\left(\Omega t + \beta - \gamma\right)\sin\gamma$$

$$A\left(\omega^2 - \Omega^2\right) = h\cos\gamma$$

$$2\delta A\Omega = h\sin\gamma$$

$$tg\gamma = \frac{2\delta\Omega}{\omega^2 - \Omega^2} \qquad A = \frac{h}{\sqrt{\left(\omega^2 - \Omega^2\right)^2 + 4\delta^2\Omega^2}}$$

$\psi = \dfrac{\delta}{\omega}$ - бездимензиони коефицијент пригушења

$\Lambda = \dfrac{\Omega}{\omega}$ - коефицијент поремећаја

$$A = \frac{h/\omega^2}{\sqrt{\left(1 - \Lambda^2\right)^2 + 4\psi^2\Omega^2}} \qquad tg\gamma = \frac{2\psi\Lambda}{1 - \Lambda^2}$$

1.5.2 Развијање произвољне функције принудне силе у Фуријеов ред

$$F_\Omega\left(t\right) = F_{\Omega 0} + \sum_{n=1}^{\infty}\left(A_n \cos n\Omega t + B_n \sin n\Omega t\right)$$

$F_{\Omega 0}, A_n, B_n$ - константне величине

$$F_{\Omega 0} = \frac{1}{T_\Omega}\int_0^{T_\Omega} F_\Omega(t)dt$$

$$A_n = \frac{2}{T_\Omega}\int_0^{T_\Omega} F_\Omega(t)\cos n\Omega t dt$$

$$B_n = \frac{2}{T_\Omega}\int_0^{T_\Omega} F_\Omega(t)\sin n\Omega t dt$$

$A_n \cos n\Omega t + B_n \sin n\Omega t$ - $n-$ти хармоник реда функције $F_\Omega(t)$

$$A_n = C_n \cos\rho_n \qquad B_n = C_n \sin\rho_n$$

$$C_n = \sqrt{A_n^2 + B_n^2} \qquad tg\rho_n = \frac{B_n}{A_n}$$

$$F_\Omega(t) = F_{\Omega 0} + \sum_{n=1}^{\infty} C_n \cos(n\Omega t - \rho_n)$$

$$\ddot{x} + 2\delta\dot{x} + \omega^2 x = \frac{F_{\Omega 0}}{m} + \frac{1}{m}\sum_{n=1}^{\infty}\left(A_n \cos n\Omega t + B_n \sin n\Omega t\right)$$

$$x_p = x_{\Omega 0} + \sum_{n=1}^{\infty}\left(a_n \cos n\Omega t + b_n \sin n\Omega t\right)$$

$$\omega^2 x_{\Omega 0} + \sum_{n=1}^{\infty}\left\{\left[\left(\omega^2 - n^2\Omega^2\right)a_n + 2\delta n\Omega b_n\right]\cos n\Omega t + \left[\left(\omega^2 - n^2\Omega^2\right)b_n - 2\delta n\Omega a_n\right]\sin n\Omega t\right\} =$$

$$= \frac{F_{\Omega 0}}{m} + \frac{1}{m}\sum_{n=1}^{\infty}\left(A_n \cos n\Omega t + B_n \sin n\Omega t\right)$$

$$\omega^2 x_{\Omega 0} = \frac{F_{\Omega 0}}{m}$$

$$\left(\omega^2 - n^2\Omega^2\right)a_n + 2\delta n\Omega b_n = \frac{A_n}{m} \qquad \left(\omega^2 - n^2\Omega^2\right)b_n - 2\delta n\Omega a_n = \frac{B_n}{m}$$

$$x_{\Omega 0} = \frac{F_{\Omega 0}}{m\omega^2}$$

$$a_n = \frac{\left(\omega^2 - n^2\Omega^2\right)A_n - 2\delta n\Omega B_n}{m\left[\left(\omega^2 - n^2\Omega^2\right)^2 + \left(2\delta n\Omega\right)^2\right]} \qquad b_n = \frac{\left(\omega^2 - n^2\Omega^2\right)B_n + 2\delta n\Omega A_n}{m\left[\left(\omega^2 - n^2\Omega^2\right)^2 + \left(2\delta n\Omega\right)^2\right]}$$

1.5.3 Динамички фактор појачавања код принудних пригушених осцилација

$$\eta_d = \frac{A_d}{A_{st}} \qquad A_{st} = \frac{h}{\omega^2} \qquad A_d = \frac{h/\omega^2}{\sqrt{\left(1-\Lambda^2\right)^2 + 4\psi^2\Lambda^2}}$$

$$\eta_d = \frac{1}{\sqrt{\left(1-\Lambda^2\right)^2 + 4\psi^2\Lambda^2}} \qquad y = \frac{1}{\eta_d^2} = \left(1-\Lambda^2\right)^2 + 4\psi^2\Lambda^2$$

$$y' = -4\Lambda\left(1-\Lambda^2\right) + 8\psi^2\Lambda \qquad y' = -4\Lambda\left(1-\Lambda^2-2\psi^2\right)$$

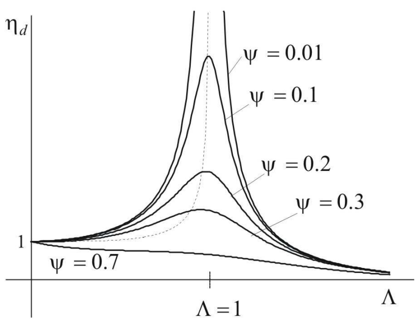
$$y' = 0 \;\Rightarrow\; y = y_e \quad \Lambda_1 = 0, \quad \Lambda_2 = \sqrt{1-2\psi^2}, \quad \Lambda_3 = -\sqrt{1-2\psi^2} \quad \psi \le \frac{\sqrt{2}}{2}$$

$$\Lambda = \Lambda_2 \qquad y'' = \frac{d^2 y}{d\Lambda^2} = -4 + 12\Lambda^2 + 8\psi^2 \quad y''\left(\Lambda_2\right) = -4 + 12\left(1-2\psi^2\right) + 8\psi^2$$

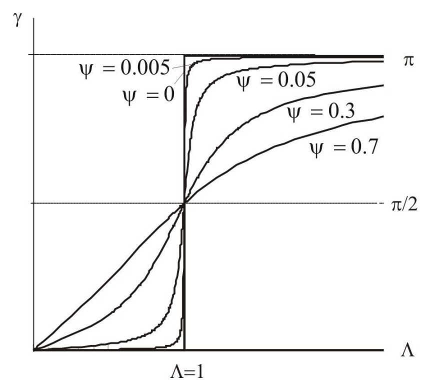
$$y''\left(\Lambda_2\right) = 8 - 16\psi^2 = 8\left(1-2\psi^2\right) > 0$$

$$y\left(\Lambda_2\right) = \left[1-\left(1-2\psi^2\right)\right]^2 + 4\psi^2\left(1-2\psi^2\right) = 4\psi^2\left(1-\psi^2\right)$$

$$\eta_{d,\max} = \frac{1}{2\psi\sqrt{1-\psi^2}}$$



Слика 16



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1.6 ОСЦИЛОВАЊЕ МЕХАНИЧКИХ СИСТЕМА СА ЈЕДНИМ СТЕПЕНОМ СЛОБОДЕ КРЕТАЊА

ДИФЕРЕНЦИЈАЛНЕ ЈЕДНАЧИНЕ КРЕТАЊА

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)-\frac{\partial T}{\partial q}=Q\left(q,\dot{q},t\right),\quad \vec{r}_i=\vec{r}_i\left(q,t\right),\quad i=1,2,...,N$$

$$T=\sum_{i=1}^{N}T_i=\sum_{i=1}^{N}\frac{1}{2}m_iV_i^2$$

$$\vec{V}_i=\frac{d\vec{r}_i}{dt}=\frac{\partial\vec{r}_i}{\partial q}\dot{q}+\frac{\partial\vec{r}_i}{\partial t}$$

$$V_i^2=\vec{V}_i\cdot\vec{V}_i=\left(\frac{\partial\vec{r}_i}{\partial q}\cdot\frac{\partial\vec{r}_i}{\partial q}\right)\dot{q}^2+2\frac{\partial\vec{r}_i}{\partial q}\cdot\frac{\partial\vec{r}_i}{\partial t}\dot{q}+\frac{\partial\vec{r}_i}{\partial t}\cdot\frac{\partial\vec{r}_i}{\partial t}=\left(\frac{\partial\vec{r}_i}{\partial q}\right)^2\dot{q}^2+2\left(\frac{\partial\vec{r}_i}{\partial q}\cdot\frac{\partial\vec{r}_i}{\partial t}\right)\dot{q}+\left(\frac{\partial\vec{r}_i}{\partial t}\right)^2$$

$$T=\frac{1}{2}\sum_{i=1}^{N}m_i\left[\left(\frac{\partial\vec{r}_i}{\partial q}\right)^2\dot{q}^2+2\frac{\partial\vec{r}_i}{\partial q}\cdot\frac{\partial\vec{r}_i}{\partial t}\dot{q}+\left(\frac{\partial\vec{r}_i}{\partial t}\right)^2\right]=$$

$$=\frac{1}{2}\left\{\left[\sum_{i=1}^{N}m_i\left(\frac{\partial\vec{r}_i}{\partial q}\right)^2\right]\dot{q}^2+2\left[\sum_{i=1}^{N}m_i\frac{\partial\vec{r}_i}{\partial q}\cdot\frac{\partial\vec{r}_i}{\partial t}\right]\dot{q}+\sum_{i=1}^{N}m_i\left(\frac{\partial\vec{r}_i}{\partial t}\right)^2\right\}$$

$$a_2\left(q,t\right)=\sum_{i=1}^{N}m_i\left(\frac{\partial\vec{r}_i}{\partial q}\right)^2;\quad a_1\left(q,t\right)=\sum_{i=1}^{N}m_i\frac{\partial\vec{r}_i}{\partial q}\cdot\frac{\partial\vec{r}_i}{\partial t};\quad a_0\left(q,t\right)=\sum_{i=1}^{N}m_i\left(\frac{\partial\vec{r}_i}{\partial t}\right)^2$$

$$T=\frac{1}{2}\left(a_2\dot{q}^2+2a_1\dot{q}+a_0\right)$$

$$\frac{\partial T}{\partial\dot{q}}=a_2\dot{q}+a_1\qquad\frac{\partial T}{\partial q}=\frac{1}{2}\frac{\partial a_2}{\partial q}\dot{q}^2+\frac{\partial a_1}{\partial q}\dot{q}+\frac{1}{2}\frac{\partial a_0}{\partial q}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{q}}\right)=\frac{da_2}{dt}\dot{q}+a_2\ddot{q}+\frac{da_1}{dt}=\frac{\partial a_2}{\partial q}\dot{q}^2+\frac{\partial a_2}{\partial t}\dot{q}+a_2\ddot{q}+\frac{\partial a_1}{\partial q}\dot{q}+\frac{\partial a_1}{\partial t}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{q}}\right)-\frac{\partial T}{\partial q}=a_2\ddot{q}+\frac{1}{2}\frac{\partial a_2}{\partial q}\dot{q}^2+\frac{\partial a_2}{\partial t}\dot{q}+\frac{\partial a_1}{\partial t}-\frac{1}{2}\frac{\partial a_0}{\partial q}$$

$$\Psi\left(q,\dot{q},t\right)=\frac{1}{2}\frac{\partial a_0}{\partial q}-\frac{\partial a_2}{\partial t}\dot{q}-\frac{\partial a_1}{\partial t}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)-\frac{\partial T}{\partial q}=a_2\ddot{q}+\frac{1}{2}\frac{\partial a_2}{\partial q}\dot{q}^2-\Psi\left(q,\dot{q},t\right)$$

$$a_2\ddot{q}+\frac{1}{2}\frac{\partial a_2}{\partial q}\dot{q}^2=Q\left(q,\dot{q},t\right)+\Psi\left(q,\dot{q},t\right)$$

- *за стационарне везе*

$$\frac{\partial \vec{r}_i}{\partial t}=0 \quad \Rightarrow \left\{\begin{array}{l} a_1\equiv 0 \\ a_0\equiv 0 \\ \dfrac{\partial a_2}{\partial t}\equiv 0 \end{array}\right\} \quad \Rightarrow \quad \Psi\left(q,\dot{q},t\right)\equiv 0$$

$$a_2\ddot{q}=Q\left(q,\dot{q},t\right)+\Psi\left(q,\dot{q},t\right)-\frac{1}{2}\frac{\partial a_2}{\partial q}\dot{q}^2=\bar{Q}\left(q,\dot{q},t\right)$$

$$\bar{Q}\left(q,\dot{q},t\right)=Q\left(q,\dot{q},t\right)+\Psi\left(q,\dot{q},t\right)-\frac{1}{2}\frac{\partial a_2}{\partial q}\dot{q}^2$$

$a_2\ddot{q}=\bar{Q}\left(q,\dot{q},t\right)$ $\qquad$ $m\ddot{x}=\mathrm{X}\left(x,\dot{x},t\right)$ - *диференцијална једначина праволинијског кретања материјалне тачке*

ако је

$$a=a_2\left(q_0\right)=\sum_{i=1}^{N}m_i\left(\frac{\partial \vec{r}_i}{\partial q}\right)^2\Big|_{q_0}\cong \text{const}>0$$

$$\Psi\left(q,\dot{q},t\right)\equiv 0 \quad \left(\text{\textit{стационаран систем}}\right)$$

$$a\ddot{q}=Q\left(q,\dot{q},t\right)$$

$$Q=\begin{cases} Q\left(q\right)=-cq, & c=\text{const}>0 \\ Q\left(\dot{q}\right)=-b\dot{q}, & b=\text{const}>0 \\ Q\left(t\right) \end{cases}$$