6 ТЕОРИЈА УДАРА

ПОЈАВА УДАРА. УДАРНА СИЛА. УДАРНИ ИМПУЛС

$$\vec{I}^{ud} = \int_{T_1}^{T_2} \vec{F}^{ud} dt = \int_{T_1}^{T_1+\tau} \vec{F}^{ud} dt \qquad \vec{F}_{sr} = \frac{1}{\tau}\int_{T_1}^{T_1+\tau} \vec{F}^{ud} dt$$

$$m\frac{d\vec{v}}{dt} = \vec{F}^{ud}$$

$$\vec{v}\,' = \vec{v}(T_2), \qquad \vec{v} = \vec{v}(T_1)$$

$$\vec{v}\,' = \vec{v} + \frac{1}{m}\int_{T_1}^{T_1+\tau} \vec{F}^{ud} dt$$

$$\vec{v}(t) = \vec{v} + \frac{1}{m}\int_{T_1}^{T_1+t} \vec{F}^{ud} dt = \frac{d\vec{r}}{dt}, \quad t \geq t_1$$

$$d\vec{r} = \vec{v}dt + \frac{1}{m}\left[\int_{T_1}^{T_1+t} \vec{F}^{ud} dt\right]dt$$

$$\vec{r}_2 = \vec{r}_1 + \left(\vec{v} + \frac{1}{m}\vec{I}_{sr}^{ud}\right)\tau$$

$$\vec{I}_{sr}^{ud} = \frac{1}{\tau}\int_{T_1}^{T_1+\tau} \vec{I}^{ud} dt$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \left(\vec{v} + \frac{1}{m}\vec{I}_{sr}^{ud}\right)\tau \approx 0$$

$$m\vec{v} = \vec{K} \qquad m\vec{v}\,' = \vec{K}\,'$$

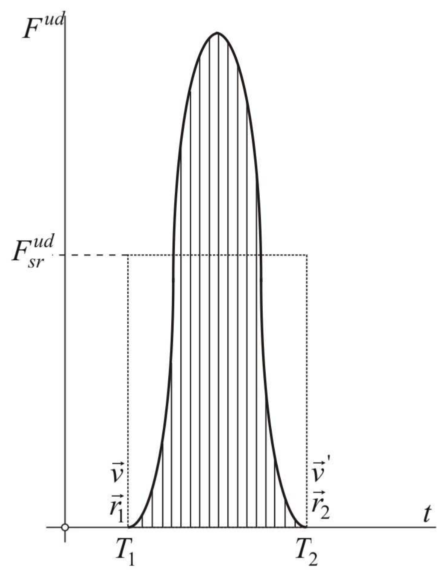
$$d\left(mv\right) = d\vec{I}$$

$$m\vec{v}\,' - m\vec{v} = \vec{I}^{ud} = \vec{F}_{sr}^{ud}\tau$$

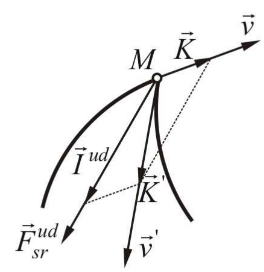
$$mv_x' - mv_x = \int_{T_1}^{T_2} X^{ud} dt = I_x^{ud}$$

$$mv_y' - mv_y = \int_{T_1}^{T_2} Y^{ud} dt = I_y^{ud}$$

$$mv_z' - mv_z = \int_{T_1}^{T_2} Z^{ud} dt = I_z^{ud}$$



Слика 1



Слика 2

ТЕОРЕМА О ПРОМЕНИ КОЛИЧИНЕ КРЕТАЊА МАТЕРИЈАЛНОГ СИСТЕМА ПРИ УДАРУ (СУДАРУ)

$$m_i\vec{v}_i^{\,'} - m_i\vec{v}_i = \vec{I}_i^{\,s} + \vec{I}_i^{\,u} \qquad i = 1, 2, ..., n$$

m_i

$\vec{v}_i$ - брзина тачке пре удара

$\vec{v}_i^{\,'}$ - брзина тачке после удара

$$\sum_{i=1}^{n} m_i\vec{v}_i^{\,'} - \sum_{i=1}^{n} m_i\vec{v}_i = \sum_{i=1}^{n}\vec{I}_i^{\,s} + \sum_{i=1}^{n}\vec{I}_i^{\,u}$$

$$\vec{I}^{\,s} = \sum_{i=1}^{n}\vec{I}_i^{\,s}$$

$$\vec{I}^{\,u} = \sum_{i=1}^{n}\vec{I}_i^{\,u} = 0$$

$$\vec{K}^{'} = \sum_{i=1}^{n} m_i\vec{v}_i^{\,'}$$

$$\vec{K} = \sum_{i=1}^{n} m_i\vec{v}_i$$

$$\vec{K}^{'} - \vec{K} = \vec{I}^{\,s} = \sum_{i=1}^{n}\vec{I}_i^{\,s}$$

$$K_x^{'} - K_x = \sum_{i=1}^{n} I_{ix}^{s} \qquad mv_{Cx}^{'} - mv_{Cx} = \sum_{i=1}^{n} I_{ix}^{s}$$

$$K_y^{'} - K_y = \sum_{i=1}^{n} I_{iy}^{s} \qquad mv_{Cy}^{'} - mv_{Cy} = \sum_{i=1}^{n} I_{iy}^{s}$$

$$K_z^{'} - K_z = \sum_{i=1}^{n} I_{iz}^{s} \qquad mv_{Cz}^{'} - mv_{Cz} = \sum_{i=1}^{n} I_{iz}^{s}$$

☺

$$\vec{I}^{\,s} = \sum_{i=1}^{n}\vec{I}_i^{\,s} = 0$$

$\vec{K}^{'} = \vec{K}$ или $\vec{v}_C^{\,'} = \vec{v}_C$ јер је $\vec{K} = m\vec{v}_C$ и $\vec{K}^{'} = m\vec{v}_C^{\,'}$

☺

$$I_x^{s} = \sum_{i=1}^{n} I_{ix}^{s} = 0 \qquad K_x^{'} = K_x \qquad v_{Cx}^{'} = v_{Cx}$$

ТЕОРЕМА О ПРОМЕНИ МОМЕНТА КОЛИЧИНЕ КРЕТАЊА МАТЕРИЈАЛНОГ СИСТЕМА ПРИ УДАРУ (СУДАРУ

$$\vec{r}_i \times m_i \vec{v}_i^{'} - \vec{r}_i \times m_i \vec{v}_i = \vec{r}_i \times \vec{I}_i^s + \vec{r}_i \times \vec{I}_i^u \qquad i = 1, 2, ..., n$$

$$\sum_{i=1}^{n} \vec{r}_i \times m_i \vec{v}_i^{'} - \sum_{i=1}^{n} \vec{r}_i \times m_i \vec{v}_i = \sum_{i=1}^{n} \vec{r}_i \times \vec{I}_i^s + \sum_{i=1}^{n} \vec{r}_i \times \vec{I}_i^u$$

$$\vec{L}_A^{'} = \sum_{i=1}^{n} \vec{r}_i \times m_i \vec{v}_i^{'} \qquad \vec{L}_A = \sum_{i=1}^{n} \vec{r}_i \times m_i \vec{v}_i$$

$$\sum_{i=1}^{n} \vec{r}_i \times \vec{I}_i^s = \sum_{i=1}^{n} \vec{r}_i \times \int_{T_1}^{T_2} \vec{F}_i^s dt = \int_{T_1}^{T_2} \left(\sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i^s \right) dt = \int_{T_1}^{T_2} M_A^s dt$$

$$\sum_{i=1}^{n} \vec{r}_i \times \vec{I}_i^u = \sum_{i=1}^{n} \vec{r}_i \times \int_{T_1}^{T_2} \vec{F}_i^u dt = \int_{T_1}^{T_2} \left(\sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i^u \right) dt = \int_{T_1}^{T_2} M_A^u dt = 0$$

$$\vec{L}_A^{'} - \vec{L}_A = \sum_{i=1}^{n} \vec{r}_i \times \vec{I}_i^s = \int_{T_1}^{T_2} M_A^s dt$$

$$L_{Ax}^{'} - L_{Ax} = \sum_{i=1}^{n} M_{Ax}^{\vec{I}_i^s}$$

$$L_{Ay}^{'} - L_{Ay} = \sum_{i=1}^{n} M_{Ay}^{\vec{I}_i^s}$$

$$L_{Az}^{'} - L_{Az} = \sum_{i=1}^{n} M_{Az}^{\vec{I}_i^s}$$

☺

$$\sum_{i=1}^{n} \vec{r}_i \times \vec{I}_i^s = \sum_{i=1}^{n} M_A^{\vec{I}_i^s} = 0 \qquad \Rightarrow \qquad \vec{L}_A^{'} = \vec{L}_A$$

☺

$$\sum_{i=1}^{n} M_{Ax}^{\vec{I}_i^s} = 0 \qquad \Rightarrow \qquad L_{Ax}^{'} = L_{Ax}$$

УДАР МАТЕРИЈАЛНЕ ТАЧКЕ У НЕПОКРЕТНУ ПОВРШ. КОЕФИЦИЈЕНТ РЕСТИТУЦИЈЕ (УСПОСТАВЉАЊА)

$$m\vec{v}\,' - m\vec{v} = \vec{I}_N$$

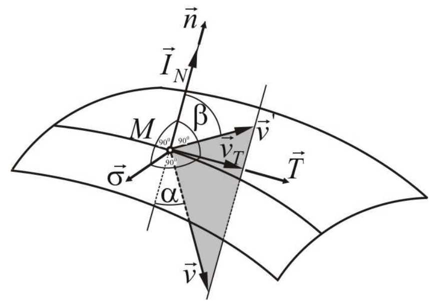
$$mv_n' - mv_n = I_N$$

$$mv_T' - mv_T = 0 \qquad \Rightarrow \qquad v_T' = v_T$$

$$mv_\sigma' - mv_\sigma = 0 \qquad \Rightarrow \qquad v_\sigma' = v_\sigma = 0$$

$$-mv_n = I_{N_1}$$

$$mv_n' = I_{N_2} \qquad I_N = I_{N_1} + I_{N_2}$$



Слика 3

Прва фаза – компресија
Друга фаза - успостављање

$$k = \frac{I_{N_2}}{I_{N_1}} = -\frac{v_n'}{v_n} \qquad \Rightarrow \qquad v_n' = -kv_n$$

$$I_N = -mv_n\left(1+k\right) \qquad v_n = -v\cos\alpha$$

$$I_N = mv\left(1+k\right)\cos\alpha$$

$$v\sin\alpha = v'\sin\beta$$

$$v = \frac{|v_n|}{\cos\alpha} \qquad v' = \frac{|v_n'|}{\cos\beta} \qquad v' = \frac{|kv_n|}{\cos\beta} = \frac{|kv\cos\alpha|}{\cos\beta} \qquad \frac{v\sin\alpha}{\sin\beta} = \frac{kv\cos\alpha}{\cos\beta}$$

$$tg\alpha = k\,tg\beta$$

$$tg\beta = \frac{1}{k}tg\alpha$$

УДАР ТАЧКЕ О ПОКРЕТНУ ПОВРШ

$$f(x,y,z;t)\geq 0$$

$$x=x(t),\;\; y=y(t),\;\; z=z(t)$$

$$f\left[x(t),y(t),z(t);t\right]=0 \qquad \Rightarrow \qquad t=T_1$$

$$\left(\frac{df}{dt}\right)_{t=T_1}=0\,,\;\left(\frac{d^2f}{dt^2}\right)_{t=T_1}=0\,,...,\left(\frac{d^nf}{dt^n}\right)_{t=T_1}=0\,,... \text{ услов останка на вези}$$

$$\left(\frac{df}{dt}\right)_{t=T_1}<0 \qquad \text{појава удара}$$

$$\left(\frac{df}{dt}\right)_{t=T_2}>0 \qquad \text{услов одвајања од везе}$$

$$\left(\frac{df}{dt}\right)_{t=T_m}=0 \qquad \text{крај прве фазе удара}$$

$$\frac{df}{dt}=\left(\frac{\partial f}{\partial t}\right)+\frac{\partial f}{\partial x}\dot{x}+\frac{\partial f}{\partial y}\dot{y}+\frac{\partial f}{\partial z}\dot{z}=$$

$$=\left(\frac{\partial f}{\partial t}\right)+\vec{v}\cdot gradf=\left(\frac{\partial f}{\partial t}\right)+\left|gradf\right|v_n$$

$$\left(\frac{df}{dt}\right)_{t=T_m}=\left(\frac{\partial f}{\partial t}\right)_{t=T_m}+\left|gradf\right|_{t=T_m}u_n=0$$

$$\left(\frac{\partial f}{\partial t}\right)_{t=T_1}+\left|gradf\right|_{t=T_1}u_n\approx 0$$

$$mu_n-mv_n=I_{N_1} \tag{1}$$

$$mv_n^{'}-mu_n=I_{N_2} \tag{2}$$

$$\left(\frac{\partial f}{\partial t}\right)_{t=T_1}+\left|gradf\right|_{t=T_1}u_n=0 \tag{3}$$

$$I_{N_2}=kI_{N_1} \qquad \Rightarrow \qquad k=\frac{v_n^{'}-u_n}{u_n-v_n} \tag{4}$$

☺☺

судар две тачке

$$k = \frac{v_{2n}^{'} - v_{1n}^{'}}{v_{2n} - v_{1n}}$$

ОДРЕЂИВАЊЕ РЕАКТИВНИХ УДАРНИХ РЕАКЦИЈА ТЕЛА КОЈЕ СЕ ОБРЋЕ ОКО НЕПОКРЕТНЕ ОСЕ

$$K\left(x_K, y_K, z_K\right)$$

$$\vec{v}^{\,'}\left(-y_C\omega^{'}, 0, 0\right)$$

$$\vec{v}\left(-y_C\omega, 0, 0\right)$$

$$L_{Ox}^{'} = -J_{xz}\omega^{'} \qquad L_{Oy}^{'} = -J_{yz}\omega^{'} \qquad L_{Oz}^{'} = J_z\omega^{'}$$

$$L_{Ox} = -J_{xz}\omega \qquad L_{Oy} = -J_{yz}\omega \qquad L_{Oz} = J_z\omega$$

$$K_x^{'} = -my_C\omega^{'} \qquad K_y^{'} = K_z^{'} = 0$$

$$K_x = -my_C\omega \qquad K_y = K_z = 0$$

$$\vec{K}^{\,'} - \vec{K} = \vec{I}^{\,s} + \vec{I}_A + \vec{I}_B \qquad (I)$$

$$\vec{L}_O^{'} - \vec{L}_O = \vec{M}_O^{\vec{I}^s} + \vec{M}_O^{\vec{I}_A} + \vec{M}_O^{\vec{I}_B} \qquad (II)$$

$$-my_C\left(\omega^{'} - \omega\right) = I_x^s + I_{Ax} + I_{Bx}$$

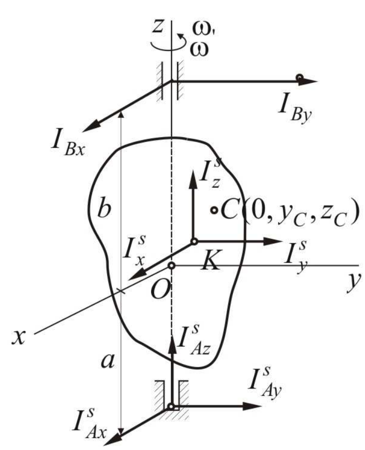
$$0 = I_y^s + I_{Ay} + I_{By}$$

$$0 = I_z^s + I_{Az}$$

$$-J_{xz}\left(\omega^{'} - \omega\right) = -I_y^s\, z_K + I_z^s\, y_K + I_{Ay}a - I_{By}b$$

$$-J_{yz}\left(\omega^{'} - \omega\right) = -I_z^s\, x_K + I_x^s\, z_K + I_{Ax}a - I_{Bx}b$$

$$J_z\left(\omega^{'} - \omega\right) = -I_x^s\, y_K + I_y^s\, x_K$$



Слика 4

ЦЕНТАР УДАРА

$$I_{Ax} = I_{Ay} = I_{Az} = 0 \qquad I_{Bx} = I_{By} = 0$$

(1) $\quad -my_C\left(\omega^{'} - \omega\right) = I_x^s \quad \Rightarrow \quad \omega^{'} - \omega = -\dfrac{I_x^s}{my_C}$

(2) $\quad 0 = I_y^s$

(3) $\quad 0 = I_z^s$

(4) $\quad -J_{xz}\left(\omega^{'} - \omega\right) = 0$

(5) $\quad -J_{yz}\left(\omega^{'} - \omega\right) = I_x^s\, z_K \quad \Rightarrow \quad z_K = \dfrac{J_{yz}}{my_C}$

(6) $\quad J_z\left(\omega^{'} - \omega\right) = -I_x^s\, y_K \quad \Rightarrow \quad y_K = \dfrac{J_z}{my_C} = \dfrac{J_{Cz} + my_C^2}{my_C} = y_C + \dfrac{J_{Cz}}{my_C}$

(2), (3) $\quad \Rightarrow \quad I_y^s = I_z^s = 0$

(4) $\quad \Rightarrow \quad J_{xz} = 0$

(5), (6) $\quad \Rightarrow \quad x_K$ - произвољно

$$y_K = y_C + \frac{J_{Cz}}{my_C}$$

$$z_K = \frac{J_{yz}}{my_C} \qquad \left(z_K = 0 \quad \Rightarrow \quad J_{yz} = 0\right)$$

ПРИМЕНА ЛАНГРАНЖЕВИХ ЈЕДНАЧИНА ДРУГЕ ВРСТЕ У ТЕОРИЈИ УДАРА

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) = \frac{\partial T}{\partial q_j} + Q_j \qquad (j = 1,2,\ldots,N)$$

$$\left(\frac{\partial T}{\partial \dot{q}_j}\right)' - \left(\frac{\partial T}{\partial \dot{q}_j}\right) = S_j$$

$$\left(\frac{\partial T}{\partial \dot{q}_j}\right)' = \left(\frac{\partial T}{\partial \dot{q}_j}\right)_{t=T_2} \qquad \left(\frac{\partial T}{\partial \dot{q}_j}\right) = \left(\frac{\partial T}{\partial \dot{q}_j}\right)_{t=T_1}$$

$$S_j = \int_{T_1}^{T_2} Q_j dt \qquad \int_{T_1}^{T_2} \frac{\partial T}{\partial q_j} dt = 0$$

☺

За случај незадржавајуће везе $f(t; q_1, q_2, \ldots, q_N) = 0$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) = \frac{\partial T}{\partial q_j} + Q_j + \lambda \frac{\partial f}{\partial q_j} \qquad (j = 1,2,\ldots,N)$$

Када је $Q_j = 0$:

$$\left(\frac{\partial T}{\partial \dot{q}_j}\right)_m - \left(\frac{\partial T}{\partial \dot{q}_j}\right) = \chi_1 \left(\frac{\partial f}{\partial q_j}\right)_{t=T_1} \tag{1}$$

$$\left(\frac{\partial T}{\partial \dot{q}_j}\right)' - \left(\frac{\partial T}{\partial \dot{q}_j}\right)_m = \chi_2 \left(\frac{\partial f}{\partial q_j}\right)_{t=T_1} \tag{2}$$

$$\left(\frac{df}{dt}\right)_{t=T_m} = \left(\frac{\partial f}{\partial t}\right)_{t=T_1} + \sum_{j=1}^{N} \left(\frac{\partial f}{\partial q_j}\right) \dot{q}_j \Big|_{t=T_1} = 0 \tag{3}$$

$$\chi_2 = k\chi_1, \tag{4}$$

при чему су:

$$\chi_1 = \int_{T_1}^{T_m} \lambda dt \qquad \text{и} \qquad \chi_2 = \int_{T_m}^{T_2} \lambda dt$$