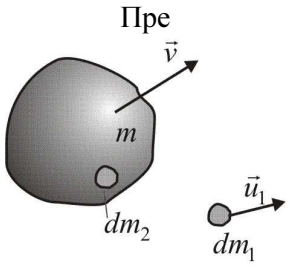
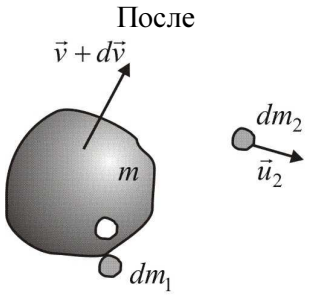
2 ДИНАМИКА ТАЧКЕ ПРОМЕНЉИВЕ МАСЕ

2.1 ЈЕДНАЧИНА МЕШЧЕРСКОГ



Слика 1



Слика 2

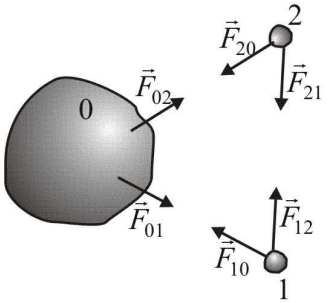
t	$\vec{K}(t)$	$t+dt$	$\vec{K}(t+dt)$
0	$m\vec{v}$		$m(\vec{v}+d\vec{v})$
1	$dm_2\vec{v}$		$dm_2\vec{u}_2$
2	$dm_1\vec{u}_1$		$dm_1(\vec{v}+d\vec{v})$

$$m(\vec{v}+d\vec{v})-m\vec{v}=\left(\vec{F}_0^s+\vec{F}_{01}+\vec{F}_{02}\right)dt$$

$$dm_1(\vec{v}+d\vec{v})-dm_1\vec{u}_1=\left(\vec{F}_1^s+\vec{F}_{10}+\vec{F}_{12}\right)dt$$

$$dm_2\vec{u}_2-dm_2\vec{v}=\left(\vec{F}_2^s+\vec{F}_{20}+\vec{F}_{21}\right)dt$$

$$md\vec{v}+dm_1(\vec{v}-\vec{u}_1)-dm_2(\vec{v}-\vec{u}_2)+dm_1d\vec{v}=\vec{F}^s dt$$



$i, j = 0,1,2 \qquad i \neq j$

$\vec{F}_{ij}=-\vec{F}_{ji}$

Слика 3

$$\vec{v}_{1r}=\vec{u}_1-\vec{v} \qquad \vec{v}_{2r}=\vec{u}_2-\vec{v}$$

$$\vec{F}^s=\vec{F}_0^s+\vec{F}_1^s+\vec{F}_2^s$$

$$m\frac{d\vec{v}}{dt}+\frac{dm_1}{dt}(-\vec{v}_{1r})-\frac{dm_2}{dt}(-\vec{v}_{2r})=\vec{F}^s$$

$$m\frac{d\vec{v}}{dt}=\vec{F}^s+\frac{dm_1}{dt}\vec{v}_{1r}-\frac{dm_2}{dt}\vec{v}_{2r}$$

$$\vec{\Phi}_1=\frac{dm_1}{dt}\vec{v}_{1r} \qquad \vec{\Phi}_2=-\frac{dm_2}{dt}\vec{v}_{2r}$$

$$m\frac{d\vec{v}}{dt}=\vec{F}^s+\vec{\Phi}_1+\vec{\Phi}_2$$

$$\vec{\Phi}_1 + \vec{\Phi}_2 = \vec{\Phi}$$

$$m\frac{d\vec{v}}{dt} = \vec{F}^s + \vec{\Phi}$$

$$m\ddot{x} = X^s + \Phi_x$$

$$m\ddot{y} = Y^s + \Phi_y$$

$$m\ddot{z} = Z^s + \Phi_z$$

$$m = m_0 + m_1 - m_2$$

$$m_0 = const$$

$$\frac{dm}{dt} = \frac{dm_1}{dt} - \frac{dm_2}{dt}$$

2.1.1 Специјални случајеви једначине Мешчерског

а) $dm_2 = 0$ $\qquad \frac{dm}{dt} = \frac{dm_1}{dt}$

$dm_1 = 0$ $\qquad \frac{dm}{dt} = -\frac{dm_2}{dt}$

или

$\vec{v}_{1r} = \vec{v}_{2r} = \vec{v}_r$ $\qquad m\frac{d\vec{v}}{dt} = \vec{F}^s + \frac{dm}{dt}\vec{v}_r$

б) $dm_1 = dm_2$

$$m\frac{d\vec{v}}{dt} = \vec{F}^s + \frac{dm_1}{dt}\left(\vec{u}_1 - \vec{u}_2\right)$$

в) $\vec{v}_{1r} = \vec{v}_{2r} = 0$

$$m\frac{d\vec{v}}{dt} = \vec{F}^s$$

г) $\dot{m}_1 = \dot{m}_2 = 0$

$$m\frac{d\vec{v}}{dt} = \vec{F}^s$$

д) $\vec{u}_1 = \vec{u}_2 = 0$ $\qquad \frac{d}{dt}\left(m\vec{v}\right) = \vec{F}^s$

2.2 ЗАДАТАК ЦИОЛКОВСКОГ

$$m\frac{d\vec{v}}{dt}=\frac{dm}{dt}\vec{v}_r$$

$$m\frac{d\dot{x}}{dt}=\frac{dm}{dt}v_{rx}$$

$$m\frac{d\dot{x}}{dt}=-\frac{dm}{dt}v_r$$

$$d\dot{x}=-dm\,v_r$$

$$\int_{v_0}^{\dot{x}} d\dot{x}=-v_r\int_{m(0)}^{m(t)} dm$$

$$\dot{x}-v_0=-v_r\ln m\Big|_{m(0)}^{m(t)}$$

$$\dot{x}-v_0=v_r\ln\frac{m(0)}{m(t)}$$

$$m(t)=m_0+m_g(t)$$

$$m(0)=m_0+m_{g0}$$

$$\dot{x}=v_0+v_r\ln\frac{m_0+m_{g0}}{m_0+m_g(t)}$$

$$\dot{x}_{\max}=\dot{x}\Big|_{m_g(t)=0}$$

$$\dot{x}_{\max}=v_0+v_r\ln\frac{m_0+m_{g0}}{m_0}$$

$$\dot{x}_{\max}=v_0+v_r\ln\left(1+\frac{m_{g0}}{m_0}\right)$$

Слика 4

$z=\dfrac{m_{g0}}{m_0}$ - релативна залиха горива (број Циолковског)

$$\dot{x}_{\max}=v_0+v_r\ln(1+z)$$

$$\Delta v_{\max}=\dot{x}_{\max}-v_0=v_r\ln(1+z)$$

$$v_{1\max}=v_0+v_{r1}\ln(1+z_1)$$

$$v_{2\max} = v_{1\max} + v_{r2}\ln(1+z_2) = \underbrace{v_0 + v_{r1}\ln(1+z_1)}_{v_{1\max}} + v_{r2}\ln(1+z_2)$$

$$\vdots$$

$$v_{n\max} = v_{(n-1)\max} + v_{rn}\ln(1+z_n) = v_0 + v_{r1}\ln(1+z_1) + \ldots + v_{rn}\ln(1+z_n)$$

$$v_{\max} = v_0 + \sum_{i=1}^{n} v_{ri}\ln(1+z_i)$$

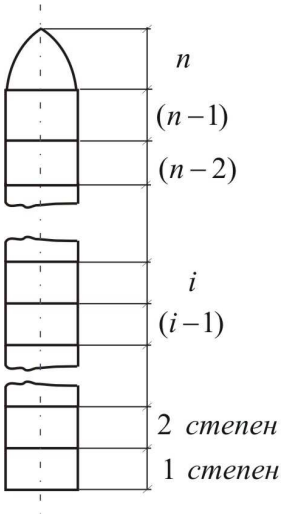
$$v_{r1} = v_{r2} = \ldots = v_{ri} = \ldots = v_{rn} = v_n$$

$$z_1 = z_2 = \ldots = z_i = \ldots = z_n = z$$

$$v_{\max} = v_0 + v_r \sum_{i=1}^{n} \ln(1+z)$$

$$v_{\max} = v_0 + nv_r\ln(1+z)$$

$$\Delta v_{\max} = nv_r\ln(1+z)$$



Слика 5