5 *ДИНАМИКА КРУТОГ ТЕЛА*

ОСНОВНИ ЗАДАЦИ ДИНАМИКЕ КРУТОГ ТЕЛА
ДИФЕРЕНЦИЈАЛНЕ ЈЕДНАЧИНЕ ОПШТЕГ КРЕТАЊА КРУТОГ ТЕЛА

$$M\vec{a}_C = \sum_{i=1}^{n} \vec{F}_i^s = \vec{F}_R^s \qquad (I)$$

$$\frac{d\vec{L}_A}{dt} + \vec{v}_A \times \vec{K} = \sum_{i=1}^{n} \vec{M}_A^{\vec{F}_i^s} \qquad (II)$$

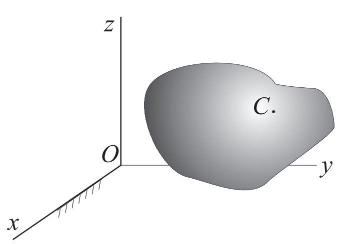
$$\frac{d\vec{L}_O}{dt} = \sum_{i=1}^{n} \vec{M}_O^{\vec{F}_i^s} \qquad \frac{d\vec{L}_C}{dt} = \sum_{i=1}^{n} \vec{M}_C^{\vec{F}_i^s} \qquad (II',\ II'')$$

$$M\vec{a}_C = \sum_{i=1}^{n} \vec{F}_i^s = \vec{F}_R^s \qquad (I)$$

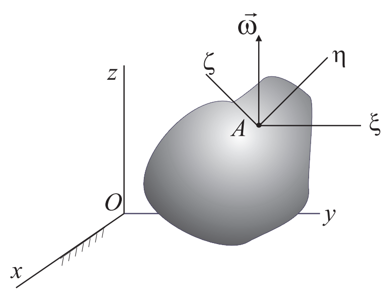
$$Ma_{Cx} = M\ddot{x}_C = \sum_{i=1}^{n} X_i^s = X_R^s \qquad (1)$$

$$Ma_{Cy} = M\ddot{y}_C = \sum_{i=1}^{n} Y_i^s = Y_R^s \qquad (2)$$

$$Ma_{Cz} = M\ddot{z}_C = \sum_{i=1}^{n} Z_i^s = Z_R^s \qquad (3)$$



Слика 1



Слика 2

$$\frac{d\vec{L}_A}{dt} + \vec{v}_A \times \vec{K} = \sum_{i=1}^{n} \vec{M}_A^{\vec{F}_i^s} \qquad (II)$$

$$A \equiv C \qquad \frac{d\vec{L}_C}{dt} = \sum_{i=1}^{n} \vec{M}_C^{\vec{F}_i^s} = \vec{M}_C^s$$

$$\frac{d\vec{L}_C}{dt} = \frac{\tilde{d}\vec{L}_C}{dt} + \vec{\omega} \times \vec{L}_C$$

$$\{L_C\} = [J_C]\{\omega\} \qquad \textbf{\textit{односно}} \qquad \begin{Bmatrix} L_{C\xi} \\ L_{C\eta} \\ L_{C\zeta} \end{Bmatrix} = \begin{bmatrix} J_\xi & -J_{\xi\eta} & -J_{\xi\zeta} \\ -J_{\eta\xi} & J_\eta & -J_{\eta\zeta} \\ -J_{\zeta\xi} & -J_{\zeta\eta} & J_\zeta \end{bmatrix} \begin{Bmatrix} \omega_\xi \\ \omega_\eta \\ \omega_\zeta \end{Bmatrix}$$

$$L_{C\xi} = J_\xi \omega_\xi - J_{\xi\eta}\omega_\eta - J_{\xi\zeta}\omega_\zeta$$
$$L_{C\eta} = -J_{\eta\xi}\omega_\xi + J_\eta \omega_\eta - J_{\eta\zeta}\omega_\zeta$$
$$L_{C\zeta} = -J_{\zeta\xi}\omega_\xi - J_{\zeta\eta}\omega_\eta + J_\zeta \omega_\zeta$$

ξ, η, ζ - главне централне осе: $L_{C\xi} = J_\xi \omega_\xi \qquad L_{C\eta} = J_\eta \omega_\eta \qquad L_{C\zeta} = J_\zeta \omega_\zeta$

$$\frac{dL_{C\xi}}{dt} + \left(\omega_\eta L_{C\zeta} - \omega_\zeta L_{C\eta}\right) = \sum_{i=1}^{n} M_{C\xi}^{\vec{F}_i^s} = M_{C\xi}^s \tag{4'}$$

$$\frac{dL_{C\eta}}{dt} + \left(\omega_\zeta L_{C\xi} - \omega_\xi L_{C\zeta}\right) = \sum_{i=1}^{n} M_{C\eta}^{\vec{F}_i^s} = M_{C\eta}^s \tag{5'}$$

$$\frac{dL_{C\zeta}}{dt} + \left(\omega_\xi L_{C\eta} - \omega_\eta L_{C\xi}\right) = \sum_{i=1}^{n} M_{C\zeta}^{\vec{F}_i^s} = M_{C\zeta}^s \tag{6'}$$

ξ, η, ζ - главне централне осе:

$$J_\xi \frac{d\omega_\xi}{dt} + \left(J_\zeta - J_\eta\right)\omega_\eta \omega_\zeta = M_{C\xi}^s \tag{4}$$

$$J_\eta \frac{d\omega_\eta}{dt} + \left(J_\xi - J_\zeta\right)\omega_\zeta \omega_\xi = M_{C\eta}^s \tag{5}$$

$$J_\zeta \frac{d\omega_\zeta}{dt} + \left(J_\eta - J_\xi\right)\omega_\xi \omega_\eta = M_{C\zeta}^s \tag{6}$$

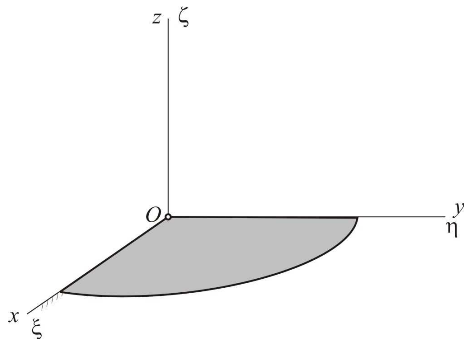
$$\omega_\xi = \dot{\psi}\sin\theta\sin\varphi + \dot{\theta}\cos\varphi \tag{7}$$

$$\omega_\eta = \dot{\psi}\sin\theta\cos\varphi - \dot{\theta}\sin\varphi \tag{8}$$

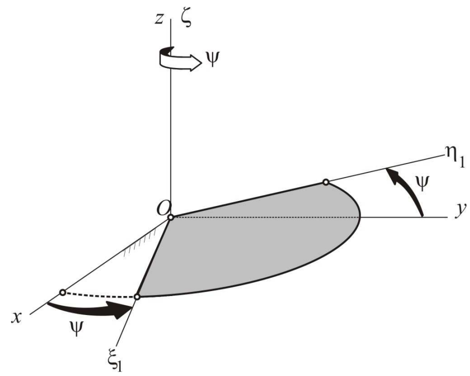
$$\omega_\zeta = \dot{\varphi} + \dot{\psi}\cos\theta \tag{9}$$

Ојлерове кинематичке једначине

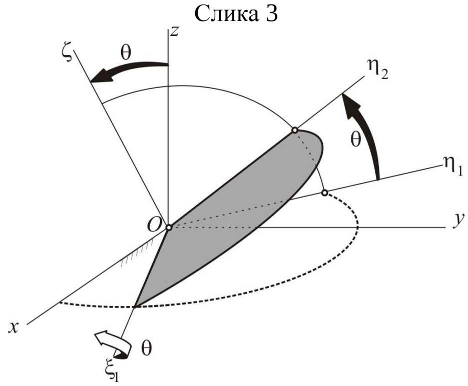
9 једначина ((1)-(9)),
9 непознатих: $x_C, y_C, z_C, \quad \omega_\xi, \omega_\eta, \omega_\zeta, \quad \varphi, \theta, \psi$



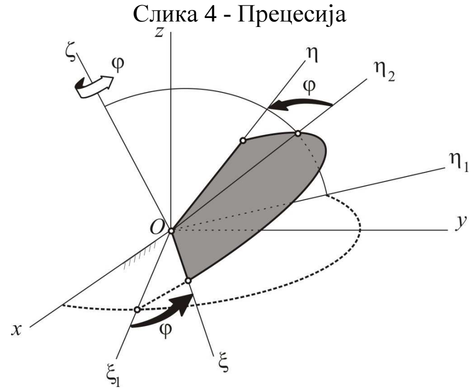
Слика 3



Слика 4 - Прецесија



Слика 5 - Нутација



Слика 6 – Сопствена ротација

ОЈЛЕРОВ СЛУЧАЈ ОБРТАЊА КРУТОГ ТЕЛА ОКО НЕПОМИЧНЕ ТАЧКЕ

$$M\vec{a}_C = \vec{F}_R^s = \sum_{i=1}^{n} \vec{F}_i^s = m\vec{g} + \vec{F}_o$$

$$C \equiv O$$

$$\vec{a}_C = 0 \quad \Rightarrow \quad \vec{F}_o = -m\vec{g}$$

$$\frac{d\vec{L}_o}{dt} = \sum_{i=1}^{n} M_o^{\vec{F}_i^s} = \vec{M}_o^s = \vec{M}_o^{\vec{F}_o} + \vec{M}_o^{m\vec{g}} = 0$$

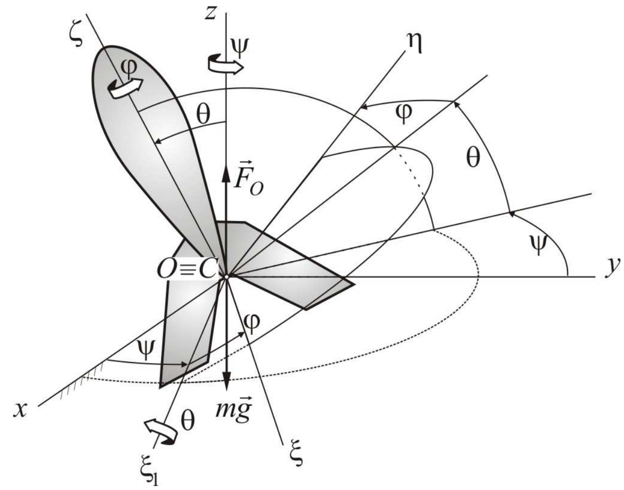
$$\vec{L}_o = const = \left(\vec{L}_o\right)_0 = L_{Oz}\vec{k}$$

$$\vec{L}_o = L_{O\xi}\vec{\lambda} + L_{O\eta}\vec{\mu} + L_{O\zeta}\vec{\nu}$$

$$\frac{dL_{O\xi}}{dt}+\left(\omega_\eta L_{O\zeta}-\omega_\zeta L_{O\eta}\right)=0 \qquad (1) \qquad\qquad \omega_\xi=\dot\psi\sin\theta\sin\varphi+\dot\theta\cos\varphi \qquad (4)$$

$$\frac{dL_{O\eta}}{dt}+\left(\omega_\zeta L_{O\xi}-\omega_\xi L_{O\zeta}\right)=0 \qquad (2) \qquad\qquad \omega_\eta=\dot\psi\sin\theta\cos\varphi+\dot\theta\sin\varphi \qquad (5)$$

$$\frac{dL_{O\zeta}}{dt}+\left(\omega_\xi L_{O\eta}-\omega_\eta L_{O\xi}\right)=0 \qquad (3) \qquad\qquad \omega_\eta=\dot\psi\cos\theta+\dot\varphi \qquad (6)$$



Слика 7

ξ,η,ζ - главне осе:

$$L_{O\xi}=J_{O\xi}\omega_\xi=A\,\omega_\xi$$
$$L_{O\eta}=J_{O\eta}\omega_\eta=B\,\omega_\eta$$
$$L_{O\zeta}=J_{O\zeta}\omega_\zeta=C\,\omega_\zeta$$

$$A\frac{d\omega_\xi}{dt}+(C-B)\,\omega_\eta\omega_\zeta=0 \qquad (1')$$

$$B\frac{d\omega_\eta}{dt}+(A-C)\,\omega_\zeta\omega_\xi=0 \qquad (2')$$

$$C\frac{d\omega_\zeta}{dt}+(B-A)\,\omega_\xi\omega_\eta=0 \qquad (3')$$

$$A\omega_\xi\frac{d\omega_\xi}{dt}+B\,\omega_\eta\frac{d\omega_\eta}{dt}+C\,\omega_\zeta\frac{d\omega_\zeta}{dt}=\frac{d}{dt}\left[\frac{1}{2}\left(A\,\omega_\xi^2+B\,\omega_\eta^2+C\,\omega_\zeta^2\right)\right]=0$$

$$T=\frac{1}{2}\left(A\,\omega_\xi^2+B\,\omega_\eta^2+C\,\omega_\zeta^2\right)=const=T_0 \qquad \left(A^s=A^u=0\right)$$

$$A^2\omega_\xi \frac{d\omega_\xi}{dt} + B^2\omega_\eta \frac{d\omega_\eta}{dt} + C^2\omega_\zeta \frac{d\omega_\zeta}{dt} = \frac{d}{dt}\left(\frac{1}{2}\left(A^2\omega_\xi^2 + B^2\omega_\eta^2 + C^2\omega_\zeta^2\right)\right) = 0$$

$$L_0^2 = A^2\omega_\xi^2 + B^2\omega_\eta^2 + C^2\omega_\zeta^2 = const \qquad \left(M_0^s = 0\right)$$

☺

$$A = B \qquad L_0 = L_{0z} = const$$

$$A\frac{d\omega_\xi}{dt} + \left(C - A\right)\omega_\eta\omega_\zeta = 0$$

$$A\frac{d\omega_\eta}{dt} + \left(A - C\right)\omega_\zeta\omega_\xi = 0$$

$$C\frac{d\omega_\zeta}{dt} = 0 \qquad \Rightarrow \qquad \omega_\zeta = \omega_{\zeta 0} = const$$

$$L_{O\xi} = L_O \sin\theta \sin\varphi; \qquad L_{O\eta} = L_O \sin\theta\cos\varphi; \qquad L_{O\zeta} = L_O\cos\theta;$$

$$L_{O\zeta} = L_O\cos\theta = C\omega_\zeta = C\omega_{\zeta 0} \qquad \Rightarrow \qquad \cos\theta = \frac{C\omega_{\zeta 0}}{L_O} = const$$

$$\cos\theta = const \qquad \Rightarrow \qquad \theta = \theta_0 = const$$

$$\omega_\xi = \dot\psi \sin\theta_0 \sin\varphi$$

$$\Rightarrow \qquad L_O \sin\theta_0 \sin\varphi = A\dot\psi \sin\theta_0\sin\varphi \;\; \Rightarrow \;\; \dot\psi = \frac{L_O}{A} = const = \dot\psi_0$$

$$\psi = \psi_0 + \dot\psi_0 t$$

$$\omega_\eta = \dot\psi_0 \sin\theta_0 \cos\varphi$$

$$\omega_\zeta = \dot\psi_0\cos\theta_0 + \dot\varphi \qquad \Rightarrow \qquad \dot\varphi = \omega_{\zeta 0} - \dot\psi_0\cos\theta_0 = const = \dot\varphi_0$$

$$\varphi = \varphi_0 + \dot\varphi_0 t$$

$$\theta = \theta_0$$
$$\psi = \psi_0 + \dot\psi_0 t$$
$$\varphi = \varphi_0 + \dot\varphi_0 t$$

регуларна прецесија

$$\frac{d\omega_\xi}{dt} = \dot\psi_0\dot\varphi_0 \sin\theta_0 \cos\varphi$$
$$\frac{d\omega_\eta}{dt} = -\dot\psi_0\dot\varphi_0 \sin\theta_0 \sin\varphi$$
$$\frac{d\omega_\zeta}{dt} = 0$$
$$A\dot\psi_0\dot\varphi_0 \sin\theta_0 \cos\varphi + (C-A)(\dot\psi_0 \sin\theta_0 \cos\varphi)(\dot\psi_0\cos\theta_0 + \dot\varphi_0) = M^s_{O\xi}$$
$$-A\dot\psi\dot\varphi_0 \sin\theta_0 \sin\varphi + (A-C)(\dot\psi_0 \cos\theta_0 + \dot\varphi_0)(\dot\psi_0 \sin\theta_0 \sin\varphi) = M^s_{O\eta}$$

$$M^s_{O\xi} = \left[A\dot\psi_0\dot\varphi_0 \sin\theta_0 + (C-A)\dot\psi_0 \sin\theta_0 (\dot\psi_0\cos\theta_0 + \dot\varphi_0)\right]\cos\varphi$$
$$M^s_{O\eta} = -\left[A\dot\psi_0\dot\varphi_0 \sin\theta_0 + (C-A)\dot\psi_0 \sin\theta_0 (\dot\psi_0\cos\theta_0 + \dot\varphi_0)\right]\sin\varphi$$
$$M^s_{O\zeta} = 0$$
$$M^s_O = \sqrt{\left(M^s_{O\xi}\right)^2 + \left(M^s_{O\eta}\right)^2 + \left(M^s_{O\zeta}\right)^2} = \left|A\dot\psi_0\dot\varphi_0 \sin\theta_0 + (C-A)\dot\psi_0 \sin\theta_0 (\dot\psi_0\cos\theta_0 + \dot\varphi_0)\right|$$

$$\cos\alpha = \cos\left(\vec M^s_O, \vec\lambda\right) = \pm\cos\varphi$$
$$\cos\beta = \cos\left(\vec M^s_O, \vec\mu\right) = \mp\sin\varphi$$
$$\cos\gamma = \cos\left(\vec M^s_O, \vec\nu\right) = 0$$

☺

Приближна теорија гироскопа

$$\dot\varphi_0 >> \dot\psi_0$$
$$M^s_O = \left|C\dot\psi_0\dot\varphi_0 \sin\theta_0\right|$$

СТАБИЛНОСТ ОБРТАЊА КРУТОГ ТЕЛА ОКО ГЛАВНИХ ОСА ИНЕРЦИЈЕ

$$\omega_\xi = \omega_\eta = 0 \qquad \omega_\zeta = \Omega = const \qquad x \equiv \xi, \quad y \equiv \eta, \quad z \equiv \zeta$$

$$\omega_\xi = \omega_1 \qquad \omega_\eta = \omega_2 \qquad \omega_\zeta = \omega_3 + \Omega$$
$$A\frac{d\omega_1}{dt} + (C-B)\omega_2\omega_3 + (C-B)\omega_2\Omega = 0$$

$$B\frac{d\omega_2}{dt}+\left(A-C\right)\omega_1\omega_3+\left(A-C\right)\omega_1\Omega=0$$

$$C\frac{d\omega_3}{dt}+\left(B-A\right)\omega_2\omega_1=0$$

$$\Omega >> \omega_1 \qquad \Omega >> \omega_2 \qquad \Omega >> \omega_3$$

$$A\frac{d\omega_1}{dt}+\left(C-B\right)\omega_2\Omega=0$$

$$B\frac{d\omega_2}{dt}+\left(A-C\right)\omega_1\Omega=0$$

$$C\frac{d\omega_3}{dt}=0$$

$$A\frac{d^2\omega_1}{dt^2}+\left(C-B\right)\frac{d\omega_2}{dt}\Omega=0 \qquad A\frac{d^2\omega_1}{dt^2}+\frac{\left(C-A\right)\left(C-B\right)}{B}\Omega^2\omega_1=0$$

$$B\frac{d^2\omega_2}{dt^2}+\left(A-C\right)\frac{d\omega_1}{dt}\Omega=0 \qquad B\frac{d^2\omega_2}{dt^2}+\frac{\left(A-C\right)\left(B-C\right)}{A}\Omega^2\omega_2=0$$

$$\frac{d^2\omega_1}{dt^2}+\frac{\left(C-A\right)\left(C-B\right)}{AB}\Omega^2\omega_1=0 \qquad \frac{d^2\omega_2}{dt^2}+\frac{\left(A-C\right)\left(B-C\right)}{AB}\Omega^2\omega_2=0$$

$$\left(A-C\right)\left(B-C\right)>0 \qquad \left(C-A\right)\left(C-B\right)>0$$

$$\begin{matrix} A<C \\ B<C \end{matrix} \quad \textit{или} \quad \begin{matrix} A>C \\ B>C \end{matrix}$$

C ***је највећи или најмањи момент инерције од*** $A, B, C \Rightarrow$ ***кретање је стабилно***

ЛАНГРАНЖЕВ СЛУЧАЈ ОБРТАЊА ОКО НЕПОМИЧНЕ ТАЧКЕ

$J_\xi = J_\eta = A = B$ *обртно тело*

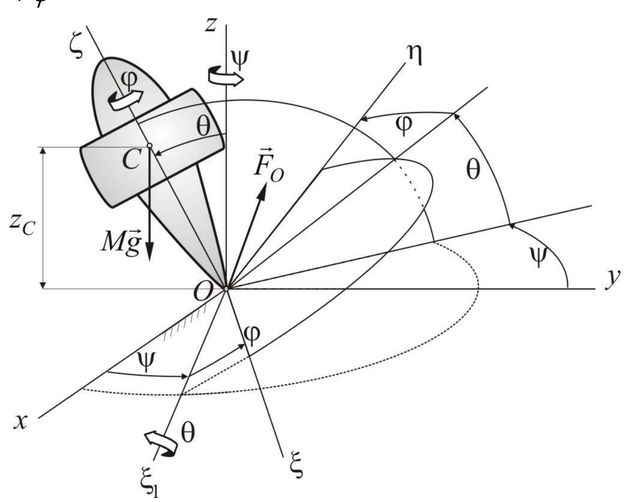
$J_\zeta = C$

$\xi,\ \eta,\ \zeta$ *- главне осе инерције*

ζ *- оса симетрије тела*

$$T=\frac{1}{2}A\omega_\xi^2+\frac{1}{2}B\omega_\eta^2+\frac{1}{2}C\omega_\zeta^2$$

$$T=\frac{1}{2}A\left(\omega_\xi^2+\omega_\eta^2\right)+\frac{1}{2}C\omega_\zeta^2$$
$$\omega_\xi=\dot\psi\sin\theta\sin\varphi+\dot\theta\cos\varphi$$
$$\omega_\eta=\dot\psi\sin\theta\cos\varphi-\dot\theta\sin\varphi$$
$$\omega_\zeta=\dot\psi\cos\theta+\dot\varphi$$



Слика 8

$$T=\frac{1}{2}A\left(\dot\psi^2\sin^2\theta+\dot\theta^2\right)+\frac{1}{2}C\left(\dot\psi\cos\theta+\dot\varphi\right)^2$$
$$T+\Pi=h \qquad z_C=\zeta_C\cos\theta$$
$$\frac{1}{2}A\left(\dot\psi^2\sin^2\theta+\dot\theta^2\right)+\frac{1}{2}C\left(\dot\psi\cos\theta+\dot\varphi\right)^2+Mg\zeta_C\cos\theta=h \qquad (1)$$
$$\frac{dL_{Oz}}{dt}=M_{Oz}^s=M_{Oz}^{M\vec g}+M_{Oz}^{\vec F_O}=0 \quad L_{Oz}=const=C_1$$
$$L_{Oz}=L_{O\xi}\sin\theta\sin\varphi+L_{O\eta}\sin\theta\cos\varphi+L_{O\zeta}\cos\theta$$
$$A\omega_\xi\sin\theta\sin\varphi+A\omega_\eta\sin\theta\cos\varphi+C\omega_\zeta\cos\theta=C_1$$
$$A\left(\dot\psi\sin\theta\sin\varphi+\dot\theta\cos\varphi\right)\sin\theta\sin\varphi+A\left(\dot\psi\sin\theta\cos\varphi-\dot\theta\sin\varphi\right)\sin\theta\cos\varphi+$$
$$+C\left(\dot\psi\cos\theta+\dot\varphi\right)\cos\theta=C_1$$
$$A\dot\psi\sin^2\theta+C\left(\dot\psi\cos\theta+\dot\varphi\right)\cos\theta=C_1 \qquad (2)$$
$$\frac{dL_{O\zeta}}{dt}=M_{O\zeta}^s=M_{O\zeta}^{M\vec g}+M_{O\zeta}^{\vec F_O}=0 \quad\Rightarrow\quad C\omega_\zeta=const$$

$$C\omega_{\zeta} = C\left(\dot{\psi}\cos\theta + \dot{\varphi}\right) = C_2 \tag{3}$$

$$\dot{\psi}\cos\theta + \dot{\varphi} = \frac{C_2}{C}$$

(2), (3) $\rightarrow$ $A\dot{\psi}\sin^2\theta + C_2\cos\theta = C_1$

$$A\dot{\psi}\sin^2\theta = C_1 - C_2\cos\theta$$

$$\dot{\psi} = \frac{C_1 - C_2\cos\theta}{A\sin^2\theta} \tag{4}$$

(1), (2), (3), (4) $\rightarrow$

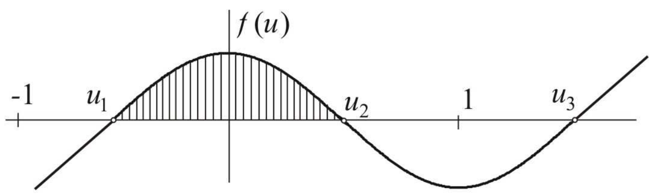
$$\frac{1}{2}A\dot{\theta}^2 + \frac{1}{2}A\left(\frac{C_1 - C_2\cos\theta}{A\sin^2\theta}\right)^2\sin^2\theta + \frac{1}{2}C\frac{C_2^2}{C^2} + Mg\zeta_C\cos\theta = h$$

$$\frac{1}{2}A\dot{\theta}^2 + \frac{1}{2}\frac{\left(C_1 - C_2\cos\theta\right)^2}{A\sin^2\theta} + Mg\zeta_C\cos\theta = C_3$$

$$C_3 = h - \frac{1}{2}\frac{C_2^2}{C}$$

$$A^2\dot{\theta}^2\sin^2\theta = -\left(C_1 - C_2\cos\theta\right)^2 - 2AMg\zeta_C\cos\theta\sin^2\theta + 2C_3A\sin^2\theta$$

$$\cos\theta = u \qquad -\dot{\theta}\sin\theta = \dot{u}$$



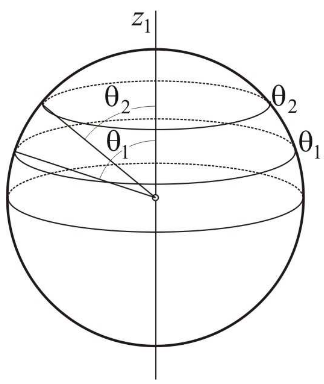
Слика 9

$$A^2\dot{u}^2 = -\left(C_1 - C_2u\right)^2 - 2AMg\zeta_C u\left(1-u^2\right) + 2C_3A\left(1-u^2\right) \tag{5}$$

$$f(u) = -\left(C_1 - C_2u\right)^2 - 2AMg\zeta_C u\left(1-u^2\right) + 2C_3A\left(1-u^2\right)$$

$$A^2\dot{u}^2 = f(u)$$

$$A\frac{du}{dt} = \pm\sqrt{f(u)} \qquad dt = \pm A\frac{du}{\sqrt{f(u)}} \qquad t = \pm A\int\frac{du}{\sqrt{f(u)}}$$



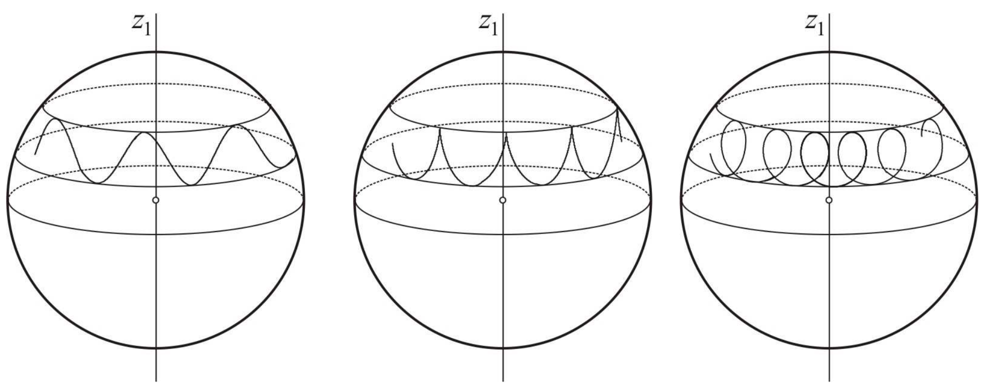
$u_1 < u < u_2$

$\theta_2 < \theta < \theta_1$

$f(-\infty) = -\infty \qquad f(+\infty) = +\infty$

$f(-1) < 0 \qquad f(+1) < 0$

Слика 10



Слика 11

а) $\dot{\psi}$ ***не мења знак*** ***б)*** $\dot{\psi} = 0$ ***б)*** $\dot{\psi}$ ***мења знак***

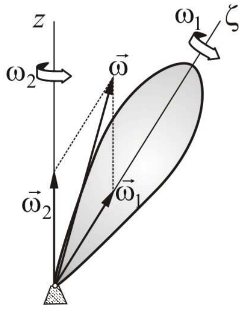
Задатак: ***<u>Могућа регуларна прецесија</u>***

$u = u_0 = \cos\theta_0$

$\theta = \theta_0 \qquad \dot{\varphi} = n \qquad \dot{\psi} = n_1$

Показати: $(A - C)n^2 \cos\theta_0 - Cnn_1 + Mg\zeta_C = 0$

ПРИБЛИЖНА ТЕОРИЈА ГИРОСКОПСКИХ ПОЈАВА. КИНЕТИЧКИ МОМЕНТ ГИРОСКОПА



Слика 12

$$\dot{\varphi} >> \dot{\psi} \qquad \dot{\varphi} >> \dot{\theta}$$

$$\omega_\xi = \dot{\psi}\sin\theta\sin\varphi + \dot{\theta}\cos\varphi$$

$$|\omega_\zeta| >> |\omega_\xi| \qquad |\omega_\zeta| > |\omega_\eta|$$

$$\omega_\eta = \dot{\psi}\sin\theta\cos\varphi - \dot{\theta}\sin\varphi$$

$$L_{O\xi} = A\omega_\xi \qquad L_{O\eta} = A\omega_\eta \qquad L_{O\zeta} = C\omega_\zeta$$

$$\omega_\zeta = \dot{\psi}\cos\theta + \dot{\varphi}$$

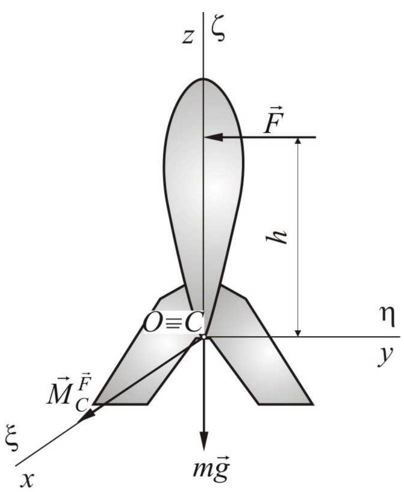
$$|L_{O\zeta}| >> |L_{O\xi}| \qquad |L_{O\zeta}| >> |L_{O\eta}|$$

$$\vec{L}_o = L_{O\xi}\vec{\lambda} + L_{O\eta}\vec{\mu} + L_{O\zeta}\vec{v} \approx L_{O\zeta}\vec{v}$$

$$\vec{\omega} = \omega_\xi\vec{\lambda} + \omega_\eta\vec{\mu} + \omega_\zeta\vec{v} \approx \omega_\zeta\vec{v} = \omega_1\vec{v}$$

$$\vec{L}_o = L_{O\zeta}\frac{\vec{\omega}}{\omega_1} \qquad \vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 \approx \vec{\omega}_1$$

УРАВНОТЕЖЕНИ ГИРОСКОП СА ТРИ СТЕПЕНА СЛОБОДЕ



Слика 13

(1) $$A\frac{d\omega_\xi}{dt} + (C-B)\,\omega_\eta\omega_\zeta = Fh$$

(2) $$B\frac{d\omega_\eta}{dt} + (A-C)\,\omega_\zeta\omega_\xi = 0$$

(3)
$$C\frac{d\omega_\zeta}{dt} + (B-A)\,\omega_\xi\omega_\eta = 0 \quad \Rightarrow \quad \omega_\zeta = const = \omega_{\zeta 0} = 0$$

$$(B-A=0)$$

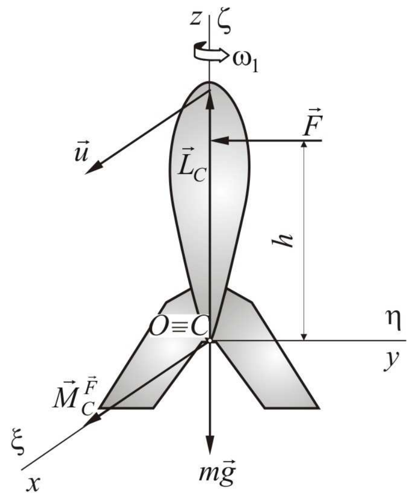
(2) $$\omega_\eta = const$$

(1) $$A\frac{d\omega_\xi}{dt} = Fh = const$$

$$\omega_\xi = Fht + \omega_{\xi 0} = Fht$$

$$\vec{L}_C = J_\zeta \vec{\omega}_1$$

$$\frac{d\vec{L}_C}{dt} = \vec{u} = \vec{M}_C^{\vec{F}}$$



Слика 14

РЕГУЛАРНА ПРЕЦЕСИЈА ТЕШКОГ ГИРОСКОПА

$$\omega_1 >> \omega_2$$

$$\vec{L}_o = J_\zeta \omega_1 \vec{v} \qquad L_{o\zeta} = J_\zeta \omega_1 = const$$

$$\vec{v}_A = \vec{\omega}_2 \times \vec{L}_O = \vec{\omega}_2 \times J_\zeta \vec{\omega}_1$$

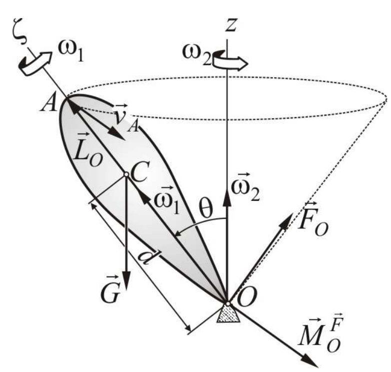
$$v_A = \omega_1 \omega_2 J_\zeta \sin\theta$$

$$\vec{v}_A = \vec{M}_O^{\vec{F}} = \vec{M}_O^{\vec{G}} = \vec{r}_C \times \vec{G}$$

$$M_O^{\vec{G}} = Gd\sin\theta$$

$$\omega_1 \omega_2 J_\zeta \sin\theta = Gd\sin\theta$$

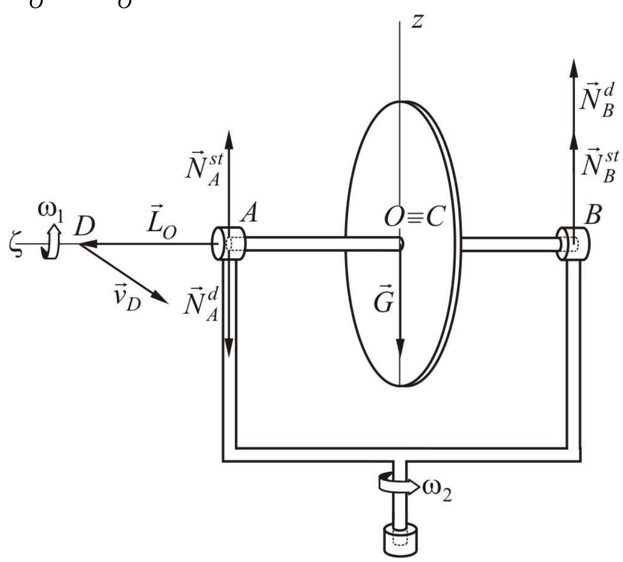
$$\omega_2 = \frac{Gd}{J_\zeta \omega_1}$$



Слика 15

ГИРОСКОП СА ДВА СТЕПЕНА СЛОБОДЕ. ГИРОСКОПСКИ ЕФЕКАТ.

$$\vec{N}_A = \vec{N}_A^{st} + \vec{N}_A^{d}$$
$$\vec{N}_B = \vec{N}_B^{st} + \vec{N}_B^{d}$$
$$M\vec{a}_C = \vec{G} + \vec{N}_A + \vec{N}_B = 0 \qquad \vec{a}_C = 0$$
$$\vec{v}_D = \vec{M}_O^{\vec{F}} = \vec{M}_O^{\vec{N}_A} + \vec{M}_O^{\vec{N}_B}$$



Слика 16

$$\vec{N}_A \,||\, \vec{N}_B$$
$$\vec{N}_A + \vec{N}_B = -\vec{G}$$
$$\vec{M}_O^{\vec{F}} = \vec{r}_A \times \vec{N}_A + \vec{r}_B \times \vec{N}_B = \vec{r}_A \times \left(\vec{N}_A - \vec{N}_B\right) = \vec{v}_D$$
$$N_A^{st} = N_B^{st} = \frac{G}{2}$$
$$\vec{M}_O^{\vec{N}_A^{st}} + \vec{M}_O^{\vec{N}_B^{st}} = 0 \quad \vec{N}_A^d = -\vec{N}_B^d$$
$$N_A^d = N_B^d = \frac{J_C \omega_1 \omega_2}{\overline{AB}} \qquad N_A = \frac{G}{2} - \frac{J_C \omega_1 \omega_2}{\overline{AB}} \qquad N_B = \frac{G}{2} + \frac{J_C \omega_1 \omega_2}{\overline{AB}}$$