1. Please send solutions to the email address:

   numericke.metode.metode@gmail.com

   till 23:59 hours on 25.12.2022. Solutions that comes after given moment are not going to be considered regardless of the excuse.

2. In a subject of an email you should write the following:

   KRAUF.NM.999/GG

   where:

   - KRAUF-marks Department of computer science which should exist in the near future
   - NM-marks Numerical methods
   - 999/GG-student index number with possible leading zero

   For example, if your index number is 23 and you started your studies in 2011., then your subject should be:

   KRAUF.NM.023/11

   Similarly, if your index number is 124 and you started your studies in 2011., then your subject should be:

   KRAUF.NM.124/11

3. Solutions of the problems: Matlab script, figures as ilustrations in JPEG format, text typed in Word and then exported into PDF, or scanned solutions writen on a paper, send as an attachment of your email, such that all the files connected with one problem are zipped into an archive with name

   problem01.zip, problem02.zip, problem03.zip, problem04.zip

4. Last email you send is the one to be assumed is valid and it is going to be examined, so it has to include solutions to all problems you want to submit.

5. Every cheating is going to be sanctioned, and all that is submitted is going to be subject of the oral exam.

6. Solution to every problem brings 33%, so 32% bonus points can be collected.

University of Belgrade
Faculty of Mechanical Engineering

23.12.2022.

Numerical Methods, written exam

1. Let function f in the neighbourhood of number 0, be defined by potential series

$$f(x) = \sum_{k=0}^{+\infty} \frac{k+2}{k!} x^k.$$

a) Determine radius of cenvergence of the potential series and find closed expression for the function f.

b) Using potential series representation of the function f write `script` in `Matlab` which computes value of the function f at the point x.

c) Using `script` from the part b), calculate value of the function f at the point $x = -50.1$ (value has to be real number) and determine the number of significant digits using closed expression for the function f.

d) Explain the result of the part c).

Remark: problem is completely solved if radius of convergence is determined, closed form expression of the function f is found a), `script` is written which sums potential series representation of the function f b), number of significant digits of the result is computed of the sumation of the potential series c), and an explanation of the previous result is given d).

2. Determine number of significant digits in the value of the function $f(x, y) = \sin(2\pi xy)$ defined over area $[0, 1] \times [0, 1]$ under assumption that x and y has three significant digits.

a) Determine theoretical bound for the number of significant digits in the value of the function f using number of significant digits in its arguments.

b) Implement `script` in `Matlab` and demonstrate grafically dependence of the number of significant digits in the value of the function f as a function of the value of the its arguments in the area $[0, 1] \times [0, 1]$, with at least 1000 points in grid for every variable. Two pictures are expected here one which presents theoretical bound from the part a) and one obtained from the experiment.

c) Explain the reason why in some specific areas of the set $[0, 1] \times [0, 1]$ value of the function f is determined with higger number of significant digits with respect to the other parts. Identify part of the set $[0, 1] \times [0, 1]$ where value of the function f has the smallest number of significant digits.

Remark: problem is solved completly if thoetical bound is determined a), `script` file and pictures are provided b), and coments is provided c).

3. Examine the system of linear equations $Ax = b$, where

```
A=-diag(ones(6,1),-4)-diag(ones(9,1),-1)+diag(4*ones(10,1))...
-diag(ones(6,1),4)-diag(ones(9,1),1)
```

and

$$b = [2111001112]^T$$

a) Using function `linsolve` solve the system of linear equations.

b) Implements `script` in `Matlab` which solves given system of linear equations using Jacobi method. Determine the number of iterations needed to obtain at least 15 significant digits in the solution.

b) Implement `script` in `Matlab` which solve the previous system of linear equation using iterative method

$$x^{k+1} = Bx^k + \beta, \quad B = -(D+2L)^{-1}(-L+U), \quad \beta = (D+2L)^{-1}b,$$

where `D=diag(diag(A))`, `L=tril(A,-1)`, `U=triu(A,1)`. Show that iterative process is convergent and converges to the solution of the system $Ax = b$. Determine the number of iterations needed to determine solution of linear equations with at least 15 significant digits.

d) Explain the difference in the number of iterations in parts b) and c).

Remark: problem is completely solved if `script` file is provided a), `script` file and needed number of iterations is provided b), `script` file and needed number of iterations is provided c), explanations is provided d).

4. Assume we are solving Cauchy problem

$$y' = f(x, y) = -40y, \quad y(0) = 1.$$

a) Find closed from solution of the problem.

b) Implement `script` in `Matlab` which solves Cauchy problem using Adams-Moulton method

$$y_{n+2} - y_{n+1} = h\left(\frac{5}{12}f_{n+2} + \frac{2}{3}f_{n+1} - \frac{1}{12}f_n\right).$$

Starting value y_1 calculate using exact solution obtained in part a). Solve Cauchy problem, with $h = .001$ and draw the number of significant digits as a function of argument of the function y on the interval $[0, 10]$. Explain the shape of the curve.

c) For $h_i = 2^{-i}$, $i = 5, \ldots, 10$, draw the number of significant digits in the solution y as a function of the argument of the function y. Using picture determine which values of the step h_i cause huge loss of significant digits with the increase of the argument x, and which values of the step h_i have siginificantly smaller loss in the significant digits with the increase of the argument x. Explain previous using stability of the method.

Remark: problem is completely solved if solution of the Cauchy problem is provided a), `script` and picture is provided b) with the explanation of the picture, picture and explanation of the behaviour of the method for different values of step is provided c).

prof. dr Aleksandar Cvetković