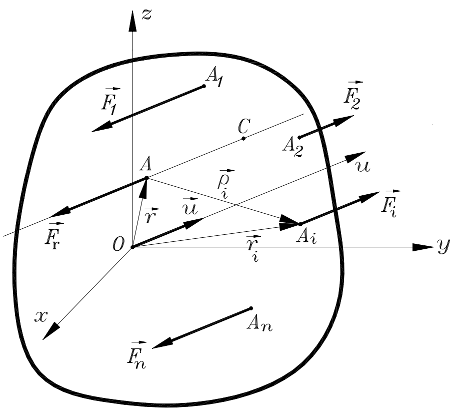
Težište

Određivanje položaja centra (središta) sistema vezanih paralelnih sila



$$(\vec{F}_R \neq 0)$$
$$(\vec{M}_o \perp \vec{F}_R)$$

$$\vec{M}_A(\vec{F}_r) = \sum_i \vec{M}_A(\vec{F}_i) = 0$$
$$\sum_i (\vec{\rho}_i \times \vec{F}_i) = 0$$
$$\vec{r}_i = \vec{r} + \vec{\rho}_i$$
$$\sum_i (\vec{r}_i - \vec{r}) \times \vec{F}_i = 0$$
$$\sum_i \vec{r}_i \times \vec{F}_i - \vec{r} \times \sum_i \vec{F}_i = 0$$
$$\vec{F}_i = F_i \vec{u}$$
$$\sum_i \vec{r}_i \times F_i \vec{u} - \vec{r} \times \sum_i F_i \vec{u} = 0,$$
$$(\sum_i F_i \vec{r}_i - \vec{r} \sum_i F_i) \times \vec{u} = 0$$
$$(\sum_i F_i \vec{r}_i - \vec{r}_C \sum_i F_i) \times \vec{u} = 0$$

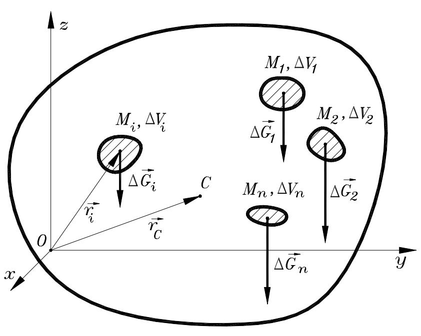
Neka se sada pretpostavi da su sile posmatranog sistema vezane za svoje napadne tačke (sistem vezanih paralelnih sila). Geometrijska tačka kroz koju prolazi napadna linija rezultante sistema vezanih paralelnih sila (tačka *C*), čiji položaj se ne menja, pri bilo kom zaokretanju svih sila oko svojih napadnih tačaka za isti ugao (za bilo koju osu *Ou*), naziva se centar (središte) sistema vezanih paralelnih sila.

$$\vec{r}_C = \frac{\sum_i F_i \vec{r}_i}{\sum_i F_i} \qquad \vec{r}_C = \frac{\sum_i F_i \vec{r}_i}{F_r}$$

$$x_C = \frac{\sum_i F_i x_i}{F_r}, \quad y_C = \frac{\sum_i F_i y_i}{F_r}, \quad z_C = \frac{\sum_i F_i z_i}{F_r}$$

Veličina $\sum_i F_i \vec{r}_i$ naziva se statički moment sistema vezanih paralelnih sila u odnosu na centar *O*. Brojioci u prethodnim izrazima predstavljaju statičke momente sistema vezanih paralelnih sila u odnosu na ravni *Oyz*, *Oxz* i *Oxy*, respektivno.

Težište tela



<u>Geometrijska tačka kroz koju prolazi napadna linija rezultante sila Zemljine teže svih delića tela pri bilo kom položaju tela u prostoru, naziva se težište tela</u>.

$$\vec{G} = \sum_i \Delta\vec{G}_i \qquad \vec{r}_C = \frac{\sum_i \Delta G_i \vec{r}_i}{G}$$

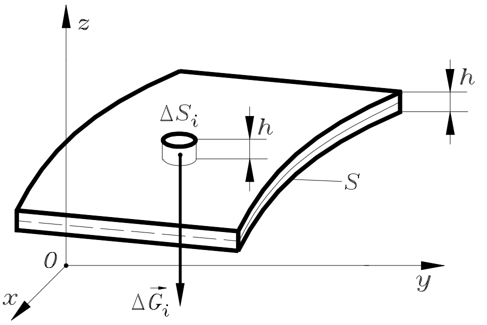
$$x_C = \frac{\sum_i \Delta G_i x_i}{G}, \quad y_C = \frac{\sum_i \Delta G_i y_i}{G}, \quad z_C = \frac{\sum_i \Delta G_i z_i}{G}$$

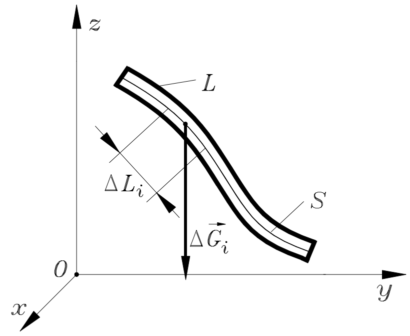
$$\gamma_i = \frac{\Delta G_i}{\Delta V_i} \qquad \vec{r}_C = \frac{\sum_i \gamma_i \Delta V_i \vec{r}_i}{\sum_i \gamma_i \Delta V_i}$$

Kod <u>nehomogenih</u> tela - $\gamma(x,y,z) = \lim\limits_{\Delta V_i \to 0} \frac{\Delta G_i}{\Delta V_i} = \lim\limits_{\Delta V \to 0} \frac{\Delta G}{\Delta V}$

Kod homogenog tela $\gamma_i = \gamma = const.$, $\Delta G_i = \gamma \cdot \Delta V_i$, $\quad G = \gamma \cdot V$

$$\vec{r}_C = \frac{\sum_i \Delta V_i \vec{r}_i}{V} \quad x_C = \frac{\sum_i \Delta V_i x_i}{V}, \quad y_C = \frac{\sum_i \Delta V_i y_i}{V}, \quad z_C = \frac{\sum_i \Delta V_i z_i}{V}$$

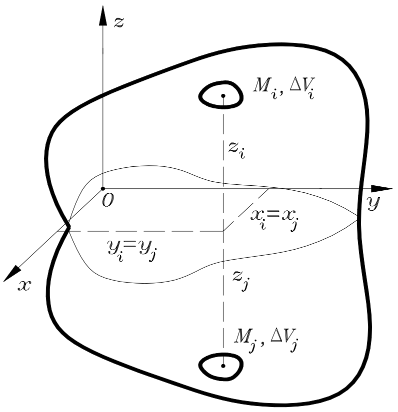




$$\Delta G_i = \gamma \Delta S_i h, \quad G = \gamma S h \quad \vec{r}_C = \frac{\sum_i \Delta S_i \vec{r}_i}{S}$$

$$\Delta G_i = \gamma S \Delta L_i, G = \gamma S L \quad \vec{r}_C = \frac{\sum_i \Delta L_i \vec{r}_i}{L}$$

Metode za određivanje težišta homogenih tela



Metoda simetrije homogenih tela

Određivanje težišta tela koje ima ravan simetrije

Težište homogenog tela, koje ima ravan simetrije, nalazi se u toj ravni.

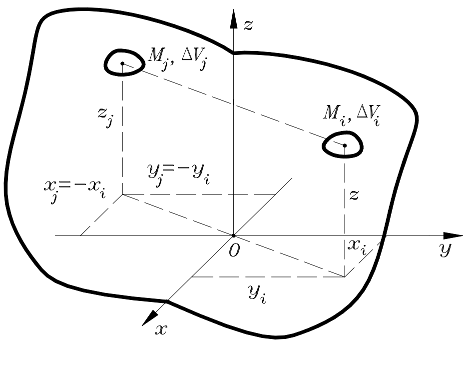
$$x_i = x_j, \quad y_i = y_j, \quad z_i = -z_j$$

$$\Delta V_i z_i + \Delta V_j z_j = 0$$

$$\sum_{k=1}^{n} \Delta V_k z_k = 0$$

$$\sum_{k=1}^{n} \Delta V_k z_k = z_C V$$

$$z_C = 0$$



Određivanje težišta tela koje ima osu simetrije

Tezište homogenog tela koje ima osu simetrije nalazi se na toj osi.

$$x_i = -x_j, \quad y_i = -y_j, \quad z_i = z_j$$

$$\Delta V_i x_i + \Delta V_j x_j = 0, \quad \Delta V_i y_i + \Delta V_j y_j = 0$$

$$x_C = y_C = 0$$

Određivanje težišta tela koje ima centar simetrije

Težište homogenog tela koje ima centar simetrije nalazi se u toj tački.

$$x_i = -x_j, \qquad y_i = -y_j, \qquad z_i = -z_j$$

$$x_C = y_C = z_C = 0$$

Računska metoda određivanja položaja težišta tela

$$\vec{r}_C = \frac{\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\gamma_i \Delta V_i \vec{r}_i}{\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\gamma_i \Delta V_i} \qquad \vec{r}_C = \frac{\int\limits_V \gamma \vec{r}\, dV}{\int\limits_V \gamma\, dV}$$

$$G = \int\limits_V \gamma\, dV \qquad \vec{r}_C = \frac{1}{G}\int\limits_V \gamma \vec{r}\, dV$$

$$x_C = \frac{1}{G}\int\limits_V \gamma x\, dV, \quad y_C = \frac{1}{G}\int\limits_V \gamma y\, dV, \quad z_C = \frac{1}{G}\int\limits_V \gamma z\, dV$$

(γ = const.)

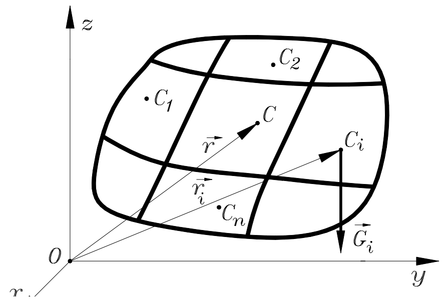
$$x_C = \frac{1}{V}\int\limits_V x\, dV, \quad y_C = \frac{1}{V}\int\limits_V y\, dV, \quad z_C = \frac{1}{V}\int\limits_V z\, dV$$

$$x_C = \frac{1}{S}\int\limits_S x\, dS, \quad y_C = \frac{1}{S}\int\limits_S y\, dS, \quad z_C = \frac{1}{S}\int\limits_S z\, dS$$

$$x_C = \frac{1}{S}\int\limits_S x\, dS, \quad y_C = \frac{1}{S}\int\limits_S y\, dS$$

$$x_C = \frac{1}{L}\int\limits_L x\, dL, \quad y_C = \frac{1}{L}\int\limits_L y\, dL, \quad z_C = \frac{1}{L}\int\limits_L z\, dL$$

Metoda rastavljanja



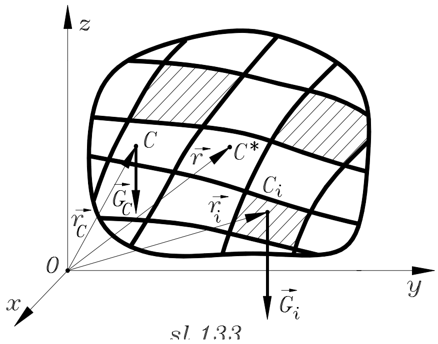
$$\vec{r}_C = \frac{1}{G}\sum_{i=1}^{n} G_i \vec{r}_i$$

$$x_C = \frac{1}{G}\sum_i G_i x_i, \quad y_C = \frac{1}{G}\sum_i G_i y_i, \quad z_C = \frac{1}{G}\sum_i G_i z_i$$

$$x_C = \frac{1}{V}\sum_i V_i x_i, \quad y_C = \frac{1}{V}\sum_i V_i y_i, \quad z_C = \frac{1}{V}\sum_i V_i z_i$$

$$x_C = \frac{1}{S}\sum_i S_i x_i, \quad y_C = \frac{1}{S}\sum_i S_i y_i, \quad z_C = \frac{1}{S}\sum_i S_i z_i$$

$$x_C = \frac{1}{L}\sum_i L_i x_i, \quad y_C = \frac{1}{L}\sum_i L_i y_i, \quad z_C = \frac{1}{L}\sum_i L_i z_i$$



sl. 13.3

Metoda "negativnih" težina

$$\vec{r} = \frac{G_C \vec{r}_C + \sum\limits_{i=1}^{k} G_i \vec{r}_i}{G_C + \sum\limits_{i=1}^{k} G_i}$$

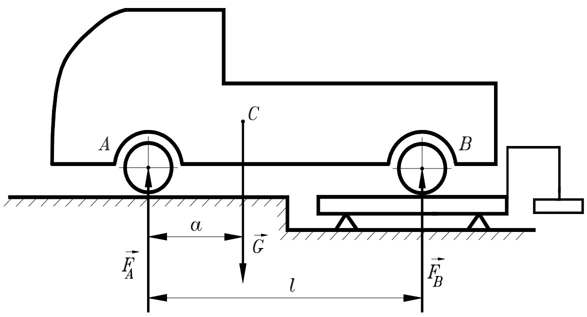
$$\vec{r}_C = \frac{G\vec{r} + \sum\limits_{i=1}^{k} (-G_i)\vec{r}_i}{G + \sum\limits_{i=1}^{k} (-G_i)}$$

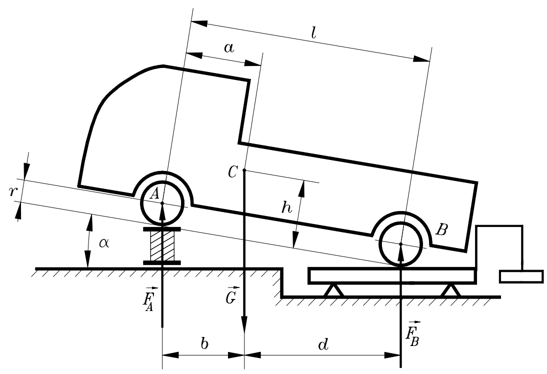
$$G = G_C + \sum_{i=1}^{k} G_i$$

Eksperimentalne metode za određivanje težišta tela

- metoda vešanja.
- metoda merenja vagom. $\sum_i M_A(\vec{F}_i) = 0 \quad a = \dfrac{F_B l}{G}$

$$a = \frac{F_B l}{F_A + F_B}$$





sl.135.

$$\sum_i M_A(\vec{F}_i) = 0\,,\ F_B(b+d) - Gb = 0$$

$$b = a\cos\alpha + (h-r)\sin\alpha\,,$$

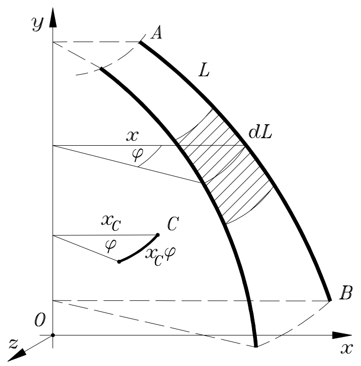
$$b + d = l\cos\alpha$$

$$h = r + \frac{F_B l - Ga}{G} ctg\alpha$$

Guldinove teoreme

<u>*Teorema 1*</u>: *Površina koja nastaje obrtanjem luka ravne krive oko ose koju luk ne preseca, a koja pripada ravni krive, jednaka je proizvodu dužine datog luka i dužine puta koji pređe njegovo težište pri tom obrtanju.*

<u>*Dokaz*</u>:



$$dS = x\varphi\, dL$$

$$S = \varphi \int_L x dL$$

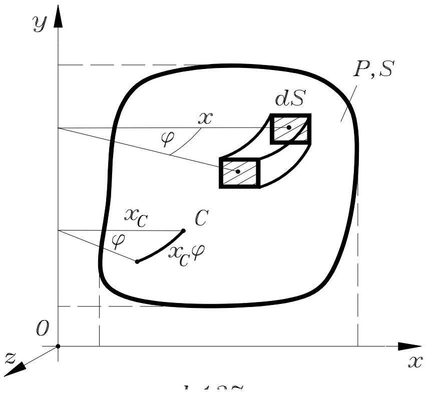
$$\int_L x dL = L x_C$$

$$S = L x_C \varphi$$

$$S = L O_C$$

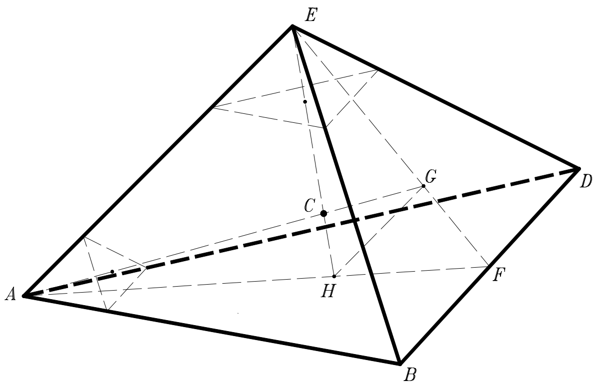
Teorema 2: *Zapremina koja nastaje obrtanjem ravne figure oko ose koja je ne preseca, a koja pripada ravni figure, jednaka je proizvodu površine ravne figure i dužine puta koji pređe težište ravne figure pri tom obrtanju.*

Dokaz:



$$dV = x\varphi\, dS$$
$$V = \varphi\int_S x dS$$
$$\int_S x dS = S x_C$$
$$V = S x_C \varphi$$
$$V = S O_C$$

Težište tetraedra



$$\frac{\overline{AH}}{\overline{HF}} = \frac{\overline{EG}}{\overline{GF}} = \frac{2}{1}$$
$$\frac{\overline{AF}}{\overline{HF}} = \frac{\overline{AE}}{\overline{GH}} = \frac{3}{1}$$
$$\frac{\overline{AE}}{\overline{GH}} = \frac{\overline{EC}}{\overline{CH}}$$
$$\frac{\overline{EC}}{\overline{CH}} = \frac{3}{1}$$
$$\overline{EH} = \overline{EC} + \overline{CH}$$
$$\overline{CH} = \frac{1}{4}\overline{EH}$$