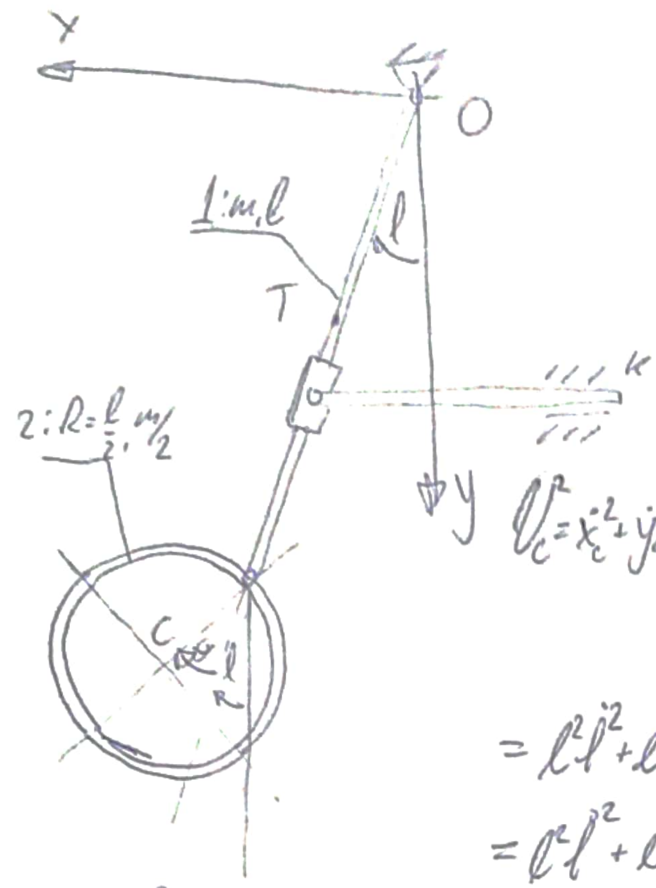

$$y_T = \frac{l}{2}\cos\varphi$$

$$x_T = \frac{l}{2}\sin\varphi$$

$$y_C = l\cos\varphi + \frac{l}{2}\cos(\varphi+\theta);\quad \dot y_C = -l\sin\varphi\cdot\dot\varphi - \frac{l}{2}\sin(\varphi+\theta)(\dot\varphi+\dot\theta)$$

$$x_C = l\sin\varphi + \frac{l}{2}\sin(\varphi+\theta);\quad \dot x_C = l\cos\varphi\cdot\dot\varphi + \frac{l}{2}\cos(\varphi+\theta)(\dot\varphi+\dot\theta)$$

$$v_C^2 = \dot x_C^2 + \dot y_C^2 = l^2\cos^2\varphi\,\dot\varphi^2 + l^2\cos\varphi\,\dot\varphi\cos(\varphi+\theta)(\dot\varphi+\dot\theta) + \frac{l^2}{4}\cos^2(\varphi+\theta)(\dot\varphi+\dot\theta)^2$$
$$+ l^2\sin^2\varphi\,\dot\varphi^2 + l^2\sin\varphi\,\dot\varphi\sin(\varphi+\theta)(\dot\varphi+\dot\theta) + \frac{l^2}{4}\sin^2(\varphi+\theta)(\dot\varphi+\dot\theta)^2 =$$

$$= l^2\dot\varphi^2 + l^2(\dot\varphi+\dot\theta)\dot\varphi\left(\cos\varphi\cos(\varphi+\theta) + \sin\varphi\sin(\varphi+\theta)\right) + \frac{l^2}{4}(\dot\varphi+\dot\theta)^2$$

$$= l^2\dot\varphi^2 + l^2(\dot\varphi+\dot\theta)\dot\varphi\cos(\theta) + \frac{l^2}{4}(\dot\varphi+\dot\theta)^2$$

$$\omega_1 = \dot\varphi,\quad \omega_2 = \dot\varphi + \dot\theta$$

$$E_k = \frac{1}{2}J_{O_z}^{(1)}\omega_1^2 + \frac{1}{2}m_2 v_C^2 + \frac{1}{2}J_{C_z}^{(2)}\omega_2^2 = \frac{1}{2}\frac{1}{3}ml^2\dot\varphi^2 + \frac{1}{2}\frac{m}{2}\cdot\left(l^2\dot\varphi^2 + l^2(\dot\varphi+\dot\theta)\dot\varphi\cos\theta + \frac{l^2}{4}(\dot\varphi+\dot\theta)^2\right) +$$

$$+ \frac{1}{2}\frac{m}{2}\cdot\left(\frac{l}{2}\right)^2\cdot(\dot\varphi+\dot\theta)^2 = ml^2\left(\frac{1}{6}\dot\varphi^2 + \frac{2\cdot 1}{2\cdot 4}\dot\varphi^2 + \frac{1}{4}\dot\varphi^2\cos\varphi + \frac{1}{4}\dot\varphi\dot\theta\cos\theta + \frac{1}{16}\dot\varphi^2 + \frac{1}{8}\dot\varphi\dot\theta + \frac{1}{16}\dot\theta^2\right.$$

$$\left. + \frac{1}{16}\dot\varphi^2 + \frac{1}{8}\dot\varphi\dot\theta + \frac{1}{16}\dot\theta^2\right) = ml^2\left(\frac{13}{24}\dot\varphi^2 + \frac{1}{4}\dot\varphi^2\cos\varphi + \frac{1}{4}\dot\varphi\dot\theta(1+\cos\theta) + \frac{1}{8}\dot\theta^2\right)$$

$$= ml^2\left(\frac{1}{4}\dot\varphi^2\left(\frac{13}{6}+\cos\varphi\right) + \frac{1}{4}\dot\varphi\dot\theta(1+\cos\theta) + \frac{1}{8}\dot\theta^2\right) = \frac{ml^2}{4}\left(\dot\varphi^2\left(\frac{13}{6}+\cos\varphi\right) + \dot\varphi\dot\theta(1+\cos\theta) + \frac{1}{2}\dot\theta^2\right)$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial\dot\varphi}\right) = \frac{ml^2}{4}\left(\frac{d}{dt}\left(2\dot\varphi\left(\frac{13}{6}+\cos\varphi\right) + \dot\theta(1+\cos\theta)\right)\right) = \frac{ml^2}{4}\left(2\ddot\varphi\left(\frac{13}{6}+\cos\varphi\right) - 2\dot\varphi^2\sin\varphi + \ddot\theta(1+\cos\theta) - \dot\theta^2\sin\theta\right)$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial\dot\theta}\right) = \frac{ml^2}{4}\left(\frac{d}{dt}\left(\dot\varphi(1+\cos\theta) + \dot\theta\right)\right) = \frac{ml^2}{4}\left(\ddot\varphi(1+\cos\theta) - \dot\varphi\dot\theta\sin\theta + \ddot\theta\right)$$

$$\frac{\partial E_k}{\partial\varphi} = \frac{ml^2}{4}\left(-\dot\varphi^2\sin\varphi\right)\qquad \frac{\partial E_k}{\partial\theta} = -\frac{ml^2}{4}\dot\varphi\dot\theta\sin\theta$$

desna strana Lagranževih j-na je ista kao sa zosa