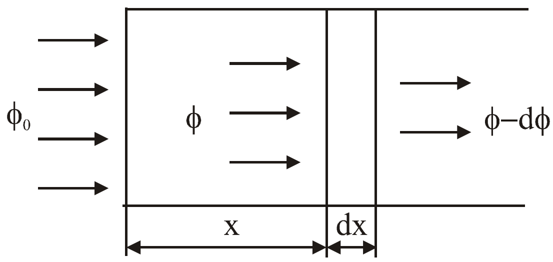
Difuzija neutrona



broj sudara u jedinici zapremine $S = \Sigma_S \phi$

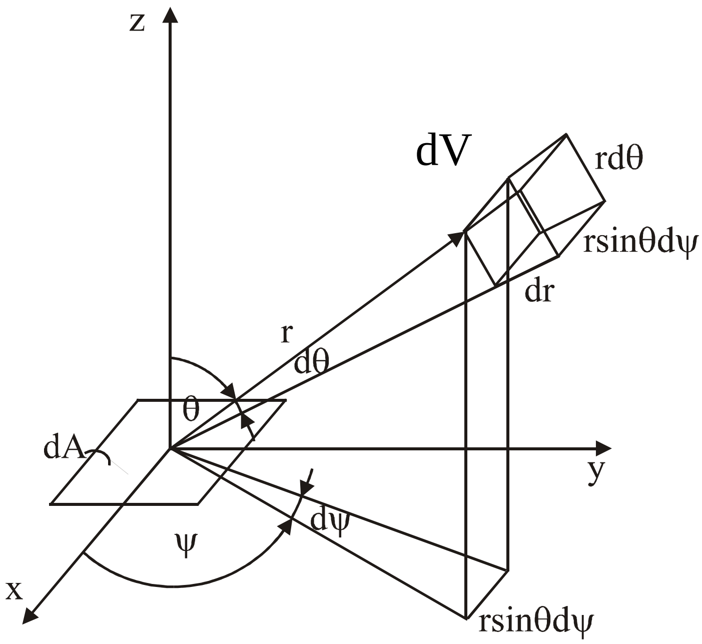
$$-\frac{d\phi}{dx} = \Sigma_S \phi$$

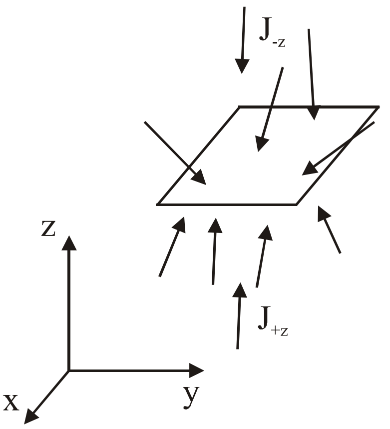
$$\frac{d\phi}{\phi} = -\Sigma_S dx$$

$$\phi = \phi_0 e^{-\Sigma_S x}$$

Pretpostavke:

1. homogena sredina
2. $\Sigma_A << \Sigma_S$
3. ukoliko su sudari anizotropni uvodi se λ_{tr}
4. male promene ϕ sa rastojanjem





- broj neutrona koji posle sudara u dV prolazi kroz dA

$$\frac{\mathrm{d}A\cos\theta}{4\pi r^2}\Sigma_{S,tr}\phi e^{-\Sigma_{S,tr}\cdot r}\mathrm{d}V=\mathrm{d}J_{-z}\mathrm{d}A$$

$$\mathrm{d}V=r^2\sin\theta\mathrm{d}r\mathrm{d}\theta\mathrm{d}\psi$$

$$\phi(r)=\phi_0+x\left(\frac{\partial\phi}{\partial x}\right)_0+y\left(\frac{\partial\phi}{\partial y}\right)_0+z\left(\frac{\partial\phi}{\partial z}\right)_0+\cdots$$

$$=\phi_0+r\sin\theta\cos\psi\left(\frac{\partial\phi}{\partial x}\right)_0+r\sin\theta\sin\psi\left(\frac{\partial\phi}{\partial y}\right)_0+r\cos\theta\left(\frac{\partial\phi}{\partial z}\right)_0\cdots$$

$$J_{-z}=\frac{\Sigma_{S,tr}}{4\pi}\int_{\theta=0}^{\pi/2}\int_{\psi=0}^{2\pi}\int_{r=0}^{\infty}\left(\phi_0+r\cos\theta\left(\frac{\partial\phi}{\partial z}\right)_0\right)e^{-\Sigma_{S,tr}\cdot r}\cos\theta\sin\theta\mathrm{d}r\mathrm{d}\psi\mathrm{d}\theta$$

$$J_{-z}=\frac{\phi_0}{4}+\frac{1}{6\Sigma_{S,tr}}\left(\frac{\partial\phi}{\partial z}\right)_0$$

- struje neutrona

$$J_{-z}=\frac{\phi}{4}+\frac{\lambda_{tr}}{6}\frac{\partial\phi}{\partial z}\quad(\text{neutrona}/\,m^2\,s)$$

$$J_{+z}=\frac{\phi}{4}-\frac{\lambda_{tr}}{6}\frac{\partial\phi}{\partial z}\quad(\text{neutrona}/\,m^2\,s)$$

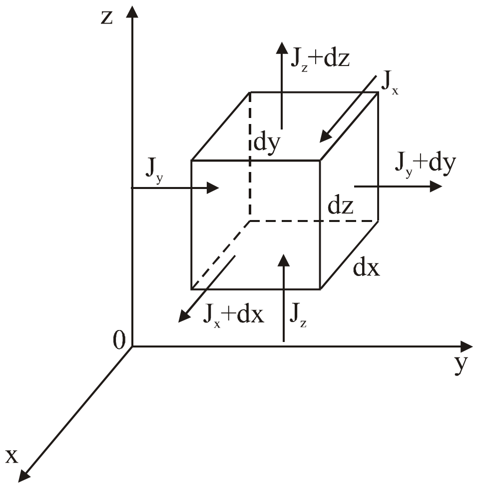
$$J_z=J_{+z}-J_{-z}=-\frac{\lambda_{tr}}{3}\frac{\partial\phi}{\partial z}=-\frac{1}{3\Sigma_{tr}}\frac{\partial\phi}{\partial z}$$

$$J_z=-D\frac{\partial\phi}{\partial z}$$: Fick-ov zakon difuzije

$$D=\frac{\lambda_{tr}}{3}=\frac{1}{3\Sigma_{tr}}$$

- umicanje neutrona u z-pravcu

$$J_{z+dz}-J_z=\left(-D\frac{\partial\phi}{\partial z}-\frac{\partial}{\partial z}\left(D\frac{\partial\phi}{\partial z}\right)\mathrm{d}z\right)-\left(-D\frac{\partial\phi}{\partial z}\right)=-D\frac{\partial^2\phi}{\partial z^2}\mathrm{d}z$$



- umicanje neutrona

$$j_x = -D\frac{\partial^2\phi}{\partial x^2}\mathrm{d}x\mathrm{d}y\mathrm{d}z \quad (\text{neutrona}/s)$$

$$j_y = -D\frac{\partial^2\phi}{\partial y^2}\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$j_z = -D\frac{\partial^2\phi}{\partial z^2}\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

- ukupno umicanje iz zapremine dV

$$j = -D\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$j = -D\nabla^2\phi\mathrm{d}V$$

∇^2 - Laplasijan

S – izvor neutrona (neutrona/ $m^3 s$)

apsorpcija neutrona: $N\sigma_A\phi dV = \Sigma_A\phi dV$

<u>Jednačina difuzije neutrona</u>

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\text{ (umicanje) – (apsorpcija) + (izvor) } = D\nabla^2\phi - \Sigma_A\phi + S$$

- u stacionarnim uslovima $\frac{\mathrm{d}n}{\mathrm{d}t} = 0$

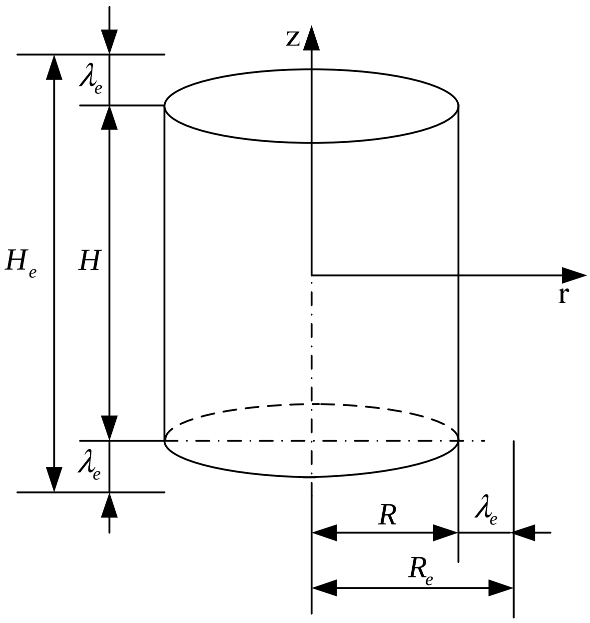
$$\nabla^2\phi - \frac{\Sigma_A}{D}\phi = -\frac{S}{D};\ \nabla^2\phi - \left(3\Sigma_{tr}\Sigma_A\right)\phi = -\frac{S}{D};\ \nabla^2\phi - \frac{3}{\lambda_{tr}\lambda_A}\phi = -\frac{S}{D}$$

$$\nabla^2\phi - \frac{\Sigma_A}{D}\phi = -\frac{k_\infty\phi\Sigma_A}{D};\ \nabla^2\phi + \frac{\Sigma_A(k_\infty - 1)}{D}\phi = 0;\ \nabla^2\phi + B^2\phi = 0;$$

$$B^2 = \frac{\Sigma_A(k_\infty - 1)}{D} = \frac{k_\infty - 1}{L_{th}^2};\ L_{th}^2 = \frac{D}{\Sigma_A}$$

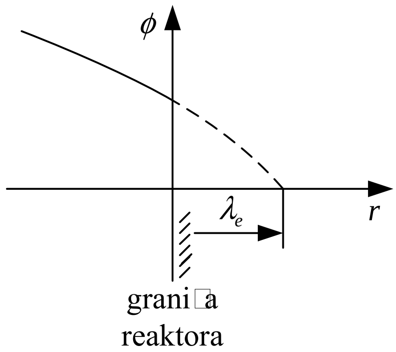
Rešavanje jednačine reaktora za cilindričnu geometriju

- jednačina reaktora $\nabla^2\phi + B^2\phi = 0$
- rešavanje za cilindrično jezgro
    - pretpostavke: a) "goli" reaktor
        b) homogeni reaktor ili parametri ćelije se ne menjaju po zapremini tako da je $B^2 = const$.
    - osnosimetrični problem



- granični uslovi:

1) $r = 0 : \phi = \hat{\phi}$ konačna vrednost
2) $r = R_e : \phi = 0$
3) $z = \dfrac{\pm H_e}{2} : \phi = 0$



$$\left(J_{+r}\right)_{gr} = \frac{\phi_{gr}}{4} - \frac{\lambda_{tr}}{6}\left(\frac{\partial\phi}{\partial r}\right)_{gr}$$

$$\left(J_{-r}\right)_{gr} = \frac{\phi_{gr}}{4} + \frac{\lambda_{tr}}{6}\left(\frac{\partial\phi}{\partial r}\right)_{gr} = 0$$

$$\left(\frac{\partial\phi}{\partial r}\right)_{gr} \approx -\frac{\phi_{gr}}{\lambda_e}$$

λ_e - ekstrapolisana dužina,

$$\left(J_{-r}\right)_{gr} = \frac{\phi_{gr}}{4} + \frac{\lambda_{tr}}{6}\left(-\frac{\phi_{gr}}{\lambda_e}\right) = 0$$

$$\phi_{gr}\left(\frac{1}{4} - \frac{\lambda_{tr}}{6\lambda_e}\right) = 0$$

$$\lambda_e = \frac{2}{3}\lambda_{tr}$$

- transportna teorija ("tačniji proračun") daje: $\lambda_e = 0,71\lambda_{tr}$

$R_e = R + \lambda_e$: ekstrapolisani radijus,

$H_e = H + 2\lambda_e$: ekstrapolisana visina.

- u cilindričnim koordinatama jednačina reaktora glasi:

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} + B^2\phi = 0$$

- rešavanje pomoću "Furijeovog metoda razdvajanja promenljivih" (Fourier)

$$\phi(r,z) = F(r)\cdot G(z)$$

$$\frac{1}{F}\frac{\mathrm{d}^2F}{\mathrm{d}r^2} + \frac{1}{rF}\frac{\mathrm{d}F}{\mathrm{d}r} + \frac{1}{G}\frac{\mathrm{d}^2G}{\mathrm{d}z^2} + B^2 = 0$$

$$\left.\begin{aligned} &\frac{1}{F}\frac{\mathrm{d}^2F}{\mathrm{d}r^2} + \frac{1}{rF}\frac{\mathrm{d}F}{\mathrm{d}r} = -\alpha^2 \\ &\frac{1}{G}\frac{\mathrm{d}^2G}{\mathrm{d}z^2} = -\beta^2 \end{aligned}\right\} \quad \text{pošto je } B^2 = const.$$

- rešenje jednačine (2)

$$\frac{\mathrm{d}^2G}{\mathrm{d}z^2} + \beta^2 G = 0$$

$$G = A\sin(\beta z) + C\cos(\beta z)$$

- $G(-z) = G(z)$: iz uslova simetričnosti G je parna funkcija;
- pošto je sinus neparna funkcija $A = 0$;

$$G = C\cos(\beta z)$$

$$G(z) = 0 \text{ za } z = \pm\frac{H_e}{2}$$

$$\beta\frac{H_e}{2} = n\frac{\pi}{2};\ n = 1,3,5,\ldots$$

$$\beta\frac{H_e}{2} = \frac{\pi}{2} \Rightarrow \beta = \frac{\pi}{H_e}$$

$$G(z) = C\cos\left(\frac{\pi z}{H_e}\right)$$

- rešenje jednačine (1)

$$\frac{\mathrm{d}^2F}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}F}{\mathrm{d}r} + \alpha^2 F = 0$$

(obična Bessel-ova diferencijalna jednačina nultog reda)

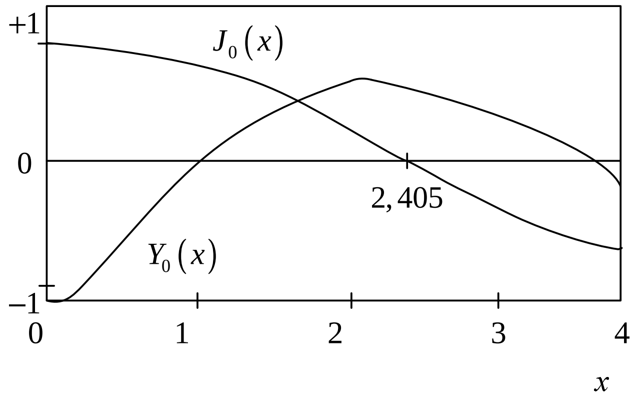
$$F(r) = DJ_0(\alpha r) + EY_0(\alpha r)$$

$$\left.\begin{aligned} &J_0 \\ &Y_0 \end{aligned}\right\} \quad \text{obične Bessel-ove funkcije nultog reda}$$

J_0 - prve vrste

Y_0 - druge vrste

$F = x^{\rho} \sum_{k=0}^{\infty} a_k x^k$: Bessel-ove funkcije predstavljeju stepene redove



$r \to 0 : Y_0(\alpha r) \to -\infty \Rightarrow E = 0$

$r \to R_e : J_0(\alpha R_e) = 0 \Rightarrow \alpha R_e = 2,405$

$$\alpha = \frac{2,405}{R_e}$$

$$F(r) = D J_0\left(\frac{2,405 r}{R_e}\right)$$

- konačno rešenje

$$\phi(r,z) = A' \cos\left(\frac{\pi z}{H_e}\right) J_0\left(\frac{2,405 r}{R_e}\right)$$

$r = 0,\ z = 0: \quad \phi(0,0) = \phi_{\max} = \hat{\phi}$

$$\phi(r,z) = \hat{\phi} \cos\left(\frac{\pi z}{H_e}\right) J_0\left(\frac{2,405 r}{R_e}\right)$$

$$B^2 = \alpha^2 + \beta^2 = \left(\frac{2,405}{R_e}\right)^2 + \left(\frac{\pi}{H_e}\right)^2$$

$B_m^2 = \frac{k_\infty - 1}{M^2}$: materijalni bakling

$B_g^2 = \left(\frac{\pi}{H_e}\right)^2 + \left(\frac{2,405}{R_e}\right)^2$: geometrijski bakling

- uslov kritičnosti

$B_m^2 = B_g^2$