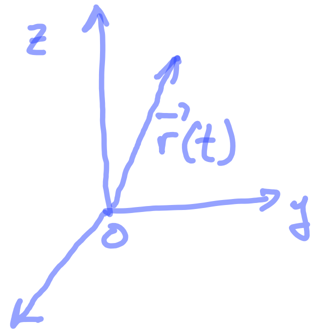
1.2*) $m = 1\,kg$

$\vec{r} = e^{2t}\,\vec{i} + \cos^2 t\,\vec{j} - t^2\,\vec{k}$

$t_0 = 0$

$\vec{F} - ?$



$m\vec{a} = \vec{F}\ ;\quad \dfrac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} = \vec{a}$

$m\ddot{\vec{r}} = \vec{F}$

$\dot{\vec{r}} = 2e^{2t}\,\vec{i} - 2\cos t\sin t\,\vec{j} - 2t\,\vec{k} = \quad / \dfrac{d}{dt}$

$= 2e^{2t}\,\vec{i} - \sin 2t\,\vec{j} - 2t\,\vec{k}$

$\ddot{\vec{r}} = 4e^{2t}\,\vec{i} - 2\cos 2t\,\vec{j} - 2\,\vec{k}$

$\vec{F} = 4e^{2t}\,\vec{i} - 2\cos 2t\,\vec{j} - 2\vec{k} = m\cdot\ddot{\vec{r}} \leftarrow t_0 = 0\,s$

$e^0 = 1$

$\cos 0 = 1$

$\vec{F}(t_0) = 4\,\vec{i} - 2\,\vec{j} - 2\vec{k}$

$F_0 = \sqrt{4^2+2^2+2^2} = \sqrt{24} = 2\sqrt{6}\ N$

1.7*) $m = 1\,kg$

$x = t^3 - 3t^2 = x(t)$

$y = -9t^2 + 1 = y(t)$

$z = 5t + 4 = z(t)$

$[t_0 = 0,\ t_1 = 5s]$

$\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$

$F - F_{min} - F_{max} - ?$

$\vec{r} = (t^3 - 3t^2)\,\vec{i} + (-9t^2 + 1)\,\vec{j} + (5t + 4)\,\vec{k} \quad / \dfrac{d}{dt}$

$\dot{\vec{r}} = (3t^2 - 6t)\,\vec{i} - 18t\,\vec{j} + 5\,\vec{k} \quad / \dfrac{d}{dt}$

$$\ddot{\vec r} = (6t-6)\vec i - 18\vec j = \boxed{6(t-1)\vec i - 18\vec j}$$

$$\vec F = m\cdot\ddot{\vec r}, \quad m = 1\ \text{kg}$$

$$\boxed{F = 6\sqrt{(t-1)^2+9}} \quad *$$

$$\boxed{\frac{dF}{dt} = \dot F = \overset{3}{\cancel{6}}\,\frac{2(t-1)\cdot 1}{\underset{1}{\cancel{2}}\sqrt{(t-1)^2+9}} = 6\,\frac{t-1}{\sqrt{(t-1)^2+9}}}$$

$$\dot F = 0 \Rightarrow t^* = 1\ \text{s}$$

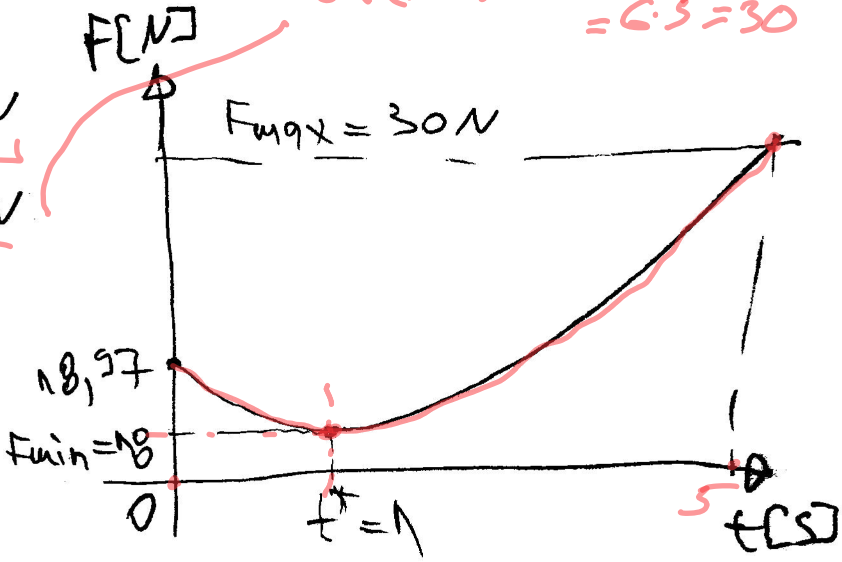
$$\ddot F = \frac{54}{(10-2t+t^2)^{3/2}}$$

$$\ddot F(t^*=1) = 2 > 0$$

$$6\sqrt{(5-1)^2+9} = 6\sqrt{16+9} = 6\cdot 5 = 30$$

$$\boxed{F_{min} = F(t^*=1\,s) = 18\,N}$$

$$F_{max} = F(t=5\,s) = 30\,N$$



1.9*) $m = 1\,kg$

$\boxed{R = 1\,m}$

$s = t^2 - t + 1 = s(t) \quad \Big/ \frac{d}{dt}$

$F = ?$

$$\boxed{\dot s = V = 2t - 1} \quad \Big/ \frac{d}{dt}$$

$$\ddot s = a_t = 2$$

$$\boxed{a_n = \frac{V^2}{R_k}} = \frac{(2t-1)^2}{1} = \boxed{(2t-1)^2}$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4 + (2t-1)^2}$$

$$\vec{F} = m\ddot{\vec{r}} = m\vec{a}_n + m\vec{a}_t + m\vec{a}_b =$$

$$= m(2t-1)^2\vec{n} + 2(m)\vec{t} = \boxed{(2t-1)^2\vec{n} + 2\vec{t} = \vec{F}(t)}$$

1.11*)

$m = 10\,kg$

$r = t \Rightarrow \dot{r} = 1;\quad \ddot{r} = 0$

$\varphi = 2t \Rightarrow \dot{\varphi} = 2;\quad \ddot{\varphi} = 0$

$$a_r = \ddot{r} - r\dot{\varphi}^2 = -4t$$

$$a_\varphi = r\ddot{\varphi} + 2\dot{r}\dot{\varphi} = 2\cdot 1\cdot 2 = 4$$

$$F_r \overset{a_r m}{=} -40t\,N,\quad F_\varphi \overset{a_\varphi m}{=} 40\,N$$

$$F = \sqrt{F_r^2 + F_\varphi^2} = 40\sqrt{1+t^2}\ N$$

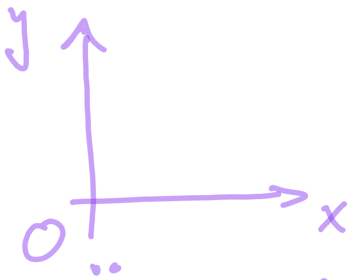
1.14*)

m

$x^2 = by$

$v = v_0 = const.$

$F(x) = ?$



$$m\vec{a} = m\ddot{\vec{r}} = m\ddot{x}\vec{i} + m\ddot{y}\vec{j} = \vec{F}$$

$$\vec{F} = m\ddot{x}\vec{i} + m\ddot{y}\vec{j}$$

$$\dot{x} = \frac{dx}{dt},\quad \dot{y} = \frac{dy}{dt},\quad v^2 = v_0^2 = \dot{x}^2 + \dot{y}^2$$

$$\boxed{y = \frac{1}{b}x^2} \Big| \frac{d}{dt} \qquad \left(\frac{dy}{dx}\right.$$

$$\dot y = \frac{dy}{dt}\frac{dx}{dx} = \boxed{\dot x}\, y' = \dot x\,\frac{2}{b}x = \frac{2}{b}x\dot x \Big| \frac{d}{dt}$$

$$\boxed{\ddot y} = \frac{d\dot y}{dt} = \frac{2}{b}\dot x^2 + \frac{2}{b}x\ddot x = \frac{2}{b}\left(\dot x^2 + x\ddot x\right)$$

$$\boxed{V_0^2} = \dot x^2 + \dot y^2 = \dot x^2 + \frac{4}{b^2}x^2\dot x^2 =$$

$$= \dot x^2\left(1 + \frac{4}{b^2}x^2\right) \Big| \frac{d}{dt} \qquad (1)$$

$$0 = 2\dot x\ddot x\left(1 + \frac{4}{b^2}x^2\right) + \dot x^2\frac{8}{b^2}x\dot x \qquad (2)$$

$$(1) \Rightarrow \dot x^2 = \frac{V_0^2}{1 + \frac{4}{b^2}x^2} = \frac{b^2V_0^2}{b^2 + 4x^2}$$

$$(2) \Rightarrow \ddot x = -\frac{\frac{8}{b^2}x\dot x^2}{2\dot x\left(1 + \frac{4}{b^2}x^2\right)} =$$

$$= -\frac{4x\dot x^2 b^2}{b^2\left(b^2 + 4x^2\right)} = -\frac{4x\dot x^2}{b^2 + 4x^2}$$

$$\ddot x = -\frac{4x\frac{b^2V_0^2}{b^2+4x^2}}{b^2 + 4x^2} = -\frac{4xb^2V_0^2}{(b^2+4x^2)^2}$$

$$\ddot y = \frac{2}{b}\left(\frac{b^2V_0^2}{b^2+4x^2} - \frac{4x^2b^2V_0^2}{(b^2+4x^2)^2}\right)$$

$$\ddot{y} = \frac{2}{b}\frac{b^2 v_0^2}{b^2+4x^2}\left(1-\frac{4x^2}{b^2+4x^2}\right) =$$

$$= \frac{2}{b}\frac{b^2 v_0^2}{b^2+4x^2}\cdot\frac{b^2}{b^2+4x^2} = \frac{2b^3 v_0^2}{(b^2+4x^2)^3}$$

$$\left.\begin{aligned} X &= -4m\frac{b^2 v_0^2 x}{(b^2+4x^2)^2} = m\ddot{x} \\ Y &= 2m\frac{b^3 v_0^2}{(b^2+4x^2)^3} = m\ddot{y} \end{aligned}\right\} \vec{F} = X\vec{i} + Y\vec{j}$$

1.19*) m

$r = a(1+\cos\varphi) = r(t)$

$v = \mathrm{const.}$

$F_r - F_p - ?$

$$F_r = m a_r = m\underbrace{(\ddot{r} - r\dot{\varphi}^2)}_{a_r}$$

$$F_p = m a_p = m\underbrace{(r\ddot{\varphi} + 2\dot{r}\dot{\varphi})}_{a_p}$$

$$v^2 = \mathrm{const.} = \dot{r}^2 + r^2\dot{\varphi}^2$$

$v^2 = v_r^2 + v_p^2$, $v_r = \dot{r}$, $v_p = r\cdot\dot{\varphi}$

$$\dot{r} = \frac{dr}{dt}\frac{d\varphi}{d\varphi} = \dot{\varphi}\, \underbrace{r'}_{=\frac{dr}{d\varphi}} = -\dot{\varphi}\, a\sin\varphi$$

$$v^2 = a^2\dot\varphi^2\sin^2\varphi + \dot\varphi^2 a^2(1+2\cos\varphi+\cos^2\varphi) =$$

$$= a^2\dot\varphi^2 + a^2\dot\varphi^2 + 2a^2\dot\varphi^2\cos\varphi =$$

$$= 2a^2\dot\varphi^2(1+\cos\varphi)$$

$$\dot\varphi^2 = \frac{v^2}{2a^2(1+\cos\varphi)} \quad \Big/ \frac{d}{dt}$$

$$\cancel{2\dot\varphi}\,\ddot\varphi = \frac{0 + v^2\cancel{2a^2}\sin\varphi\,\cancel{\dot\varphi}}{4a^{4/2}(1+\cos\varphi)^2}$$

$$\boxed{\ddot\varphi = \frac{v^2\sin\varphi}{4a^2(1+\cos\varphi)^2}}$$

$$\dot v = -a\sin\varphi\,\dot\varphi \quad \Big/\frac{d}{dt}$$

$$\ddot v = -a\dot\varphi^2\cos\varphi - a\sin\varphi\,\ddot\varphi =$$

$$= -\cancel{a}\cos\varphi\frac{v^2}{2a^{\cancel{2}}(1+\cos\varphi)} - \cancel{a}\sin\varphi\frac{v^2\sin\varphi}{4a^2(1+\cos\varphi)^2}$$

$$= -\frac{\cos\varphi\, v^2}{2a(1+\cos\varphi)} - \frac{\sin^2\varphi\, v^2}{4a(1+\cos\varphi)^2} =$$

$$= -\frac{v^2}{2a(1+\cos\varphi)}\left(\cos\varphi + \frac{\sin^2\varphi}{2(1+\cos\varphi)}\right) =$$

$$\ddot{r} = -\frac{v^2}{2a(1+\cos\varphi)}\left(\frac{2\cos\varphi+2\cos^2\varphi+\sin^2\varphi}{2(1+\cos\varphi)}\right)$$

$$= -\frac{v^2}{4a(1+\cos\varphi)^2}\left(2\cos\varphi+\cos^2\varphi+1\right) =$$

$$= -\frac{v^2}{4a\cancel{(1+\cos\varphi)^2}}\cancel{(1+\cos\varphi)^2} = -\frac{v^2}{4a}$$

$$F_r = m\left(-\frac{v^2}{4a} - \cancel{a(1+\cos\varphi)}\frac{v^2}{2a^{\cancel{2}}\cancel{(1+\cos\varphi)}}\right) =$$

$$= -\frac{mv^2}{2a}\left(\frac{1}{2}+1\right) = \boxed{-\frac{3mv^2}{4a}}$$

$$F_\varphi = m\left(\cancel{a(1+\cos\varphi)}\frac{v^2\sin\varphi}{4a^2(1+\cos\varphi)^2}\right.$$

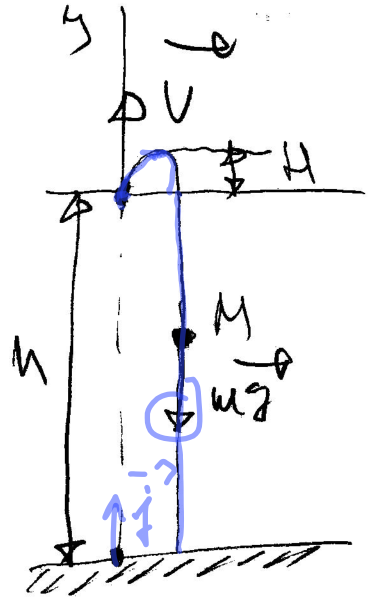
$$\left.-\cancel{2}\cancel{a}\sin\varphi\frac{v^2}{\cancel{2}a^{\cancel{2}}(1+\cos\varphi)}\right) =$$

$$= \frac{mv^2\sin\varphi}{a(1+\cos\varphi)}\left(\frac{1}{4}-1\right) = -\frac{3mv^2\sin\varphi}{4a(1+\cos\varphi)}$$

1.24*)

$v = 10\,m/s$

$h = 200\,m$



$m\ddot{\vec{r}} = \vec{F} \quad | \cdot \vec{j}$

$y: m\ddot{y} = -mg$

$\ddot{y} = -g \quad | \int$

$\dot{y} = \dot{y}_0 - gt \quad | \int$

$y = y_0 + \dot{y}_0 t - \frac{1}{2} g t^2$

$y_0 = 200\,m, \quad \dot{y}_0 = 10\,m/s$

a) $y = 200 + 10t - 4{,}905t^2$

b) $y(t^*) = 0 \Rightarrow 200 + 10t^* - 4{,}905t^{*2} = 0$ $\quad t \geq 0$

$t_1^* = 7{,}486\,s, \quad t_2^* = \cancel{-5{,}447\,s < 0}$

$\dot{y}(t_1^*) = 10 - 9{,}81 \cdot 7{,}486 = -63{,}438\,m/s$

$v = 63{,}438\,\frac{m}{s}$

c) $t_1^* = 7{,}486\,s$

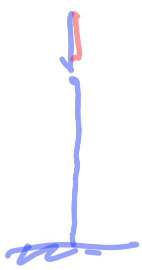
$\boxed{S = 200 + 2H = 210{,}19\,m}$

$\dot{y} = 0 \Rightarrow t_2 = 1{,}019\,s$

$\dot{y}_0 - gt$

$y_{max} = 205{,}097\,m$

$H = y_{max} - 200 = 5{,}097\,m$



1.29*)

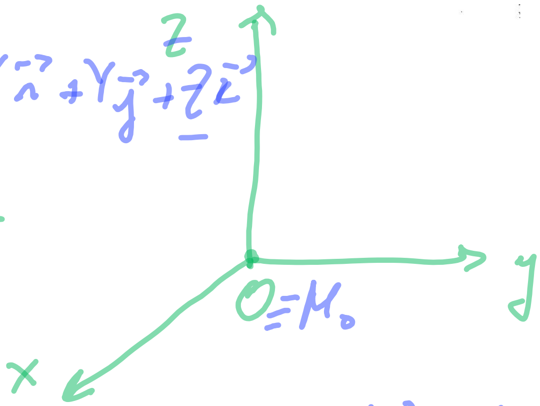
$m = 3\,kg$

$\vec{F} = 9t^2\,\vec{i} + 12\,\vec{j} = X\vec{i} + Y\vec{j} + Z\vec{k}$

$t_0 = 0 \qquad M_0(0, 0, 0)$

$\vec{V}_0 = 6\,\vec{j} + 5\,\vec{k}$

$\vec{r} - ?$



$$\boxed{m\ddot{\vec{r}} = \vec{F}} \qquad m\ddot{x}\vec{i} + m\ddot{y}\vec{j} + m\ddot{z}\vec{k} = X\vec{i} + Y\vec{j} + Z\vec{k}$$

$$x:\ m\ddot{x} = X \ ;\quad y:\ m\ddot{y} = Y \ ;\quad z:\ m\ddot{z} = Z \qquad) : 3$$

$$3\ddot{x} = 9t^2 \ ;\quad 3\ddot{y} = 12 \ ;\quad 3\ddot{z} = 0$$

$x(t) = ?\ \ y(t) = ?\ \ z(t) = ?$

$$\ddot{x} = 3t^2 \Big/ \int \ ;\quad \ddot{y} = 4 \Big/ \int \ ;\quad \ddot{z} = 0 \Big/ \int$$

$x_0 = 0;\ y_0 = 0;\ z_0 = 0$

$\dot{x}_0 = 0;\ \dot{y}_0 = 6$

$\dot{z}_0 = 5$

$$\dot{x} = \cancel{\dot{x}_0}^{=0} + t^3 \ ;\quad \dot{y} = \underset{\llcorner 6}{\dot{y}_0} + 4t \ ;\quad \dot{z} = \underset{\llcorner 5}{\dot{z}_0}$$

$$\dot{x} = t^3 \Big/ \int \ ;\quad \dot{y} = 6 + 4t \Big/ \int \ ;\quad \dot{z} = 5 \Big/ \int$$

$$x = \underset{\llcorner 0}{\cancel{x_0}} + \frac{t^4}{4} \ ;\quad y = \underset{\llcorner 0}{\cancel{y_0}} + 6t + 2t^2 \ ;\quad z = \underset{\llcorner 0}{\cancel{z_0}} + 5t$$

$$x(t) = \frac{t^4}{4} \ ;\quad y(t) = 6t + 2t^2 \ ;\quad z(t) = 5t$$

$$\boxed{\vec{r}(t) = \frac{t^4}{4}\vec{i} + (2t^2 + 6t)\vec{j} + 5t\vec{k}}$$

$$= x\vec{i} + y\vec{j} + z\vec{k}$$

1.37*)

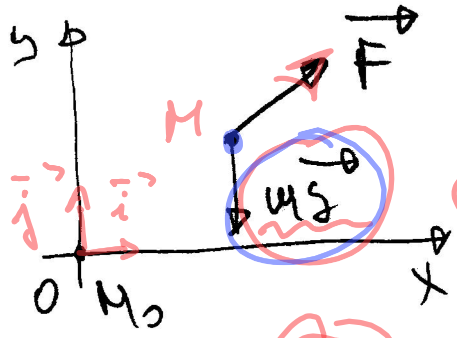
$\vec F = 3my\,\vec i + 3mg\,\vec j$

$t_0 = 0 \quad M_0(0,0) \quad v_0(0,0)$

$y = y(x) - ?$

$\vec a = \ddot{\vec r} = \ddot x\,\vec i + \ddot y\,\vec j$

$m\ddot{\vec r} = \vec F + m\vec g \quad /\cdot\vec i \quad /\cdot\vec j$



$x:\quad m\ddot x = 3my + 0 \quad /:m$

$y:\quad m\ddot y = 3mg - mg \quad /:m$

$\dot y - \dot y_0 = 2gt$

$\boxed{\ddot x = 3y}$

$\ddot y = 3g - g = 2g \Rightarrow \dot y = \dot y_0 + 2gt \Rightarrow \dots$

$\Rightarrow \boxed{y(t)} = \cancel{y_0}^{=0} + \cancel{\dot y_0}^{=0} t + gt^2 = \boxed{gt^2}$

$M_0(0,0)$

$\vec v_0 = 0\vec i + 0\vec j$

$\dot y_0$

$\ddot x = 3gt^2 \Rightarrow \dot x = \cancel{\dot x_0}_{=0} + gt^3 = gt^3$

$x = \cancel{x_0}_{=0} + \frac{1}{4}gt^4$

$y(t) = gt^2$

$x(t) = \frac{1}{4}gt^2$

$\boxed{x(t) = \frac{1}{4}gt^4} \Rightarrow gt^4 = 4x$

$y^2 = g^2t^4 = g\cdot gt^4 = 4gx$

$\boxed{y^2 = 4gx}$, како је $y = gt^2$, а $t \geq 0$, следи да је $y \geq 0$

$$\sqrt{\frac{k}{m}} = \frac{g}{V_0} \checkmark$$

$$x(t) = V_0 \frac{V_0}{g} \sin \frac{g}{V_0} t = \frac{V_0^2}{g} \sin \frac{g}{V_0} t$$

$$y(t) = \frac{\cancel{mg}}{\frac{g^2 \cancel{m}}{V_0^2}} \left(1 - \cos \frac{g}{V_0} t\right) = \frac{V_0^2}{g} \left(1 - \cos \frac{g}{V_0} t\right)$$

$$\left.\begin{aligned} \frac{gx}{V_0^2} &= \sin \frac{g}{V_0} t \quad \Big|^2 \\ \frac{gy}{V_0^2} - 1 &= -\cos \frac{g}{V_0} t \quad \Big|^2 \end{aligned}\right\} +$$

$$\frac{g^2 x^2}{V_0^4} + \left(\frac{gy}{V_0^2} - 1\right)^2 = 1 \quad \Big| \cdot \frac{V_0^4}{g^2}$$

$$\boxed{x^2 + \left(y - \frac{V_0^2}{g}\right)^2 = \frac{V_0^4}{g^2}}$$

1.41*) $O \downarrow y$

$\vec{F} = -k(x\vec{i} + y\vec{j})$

m

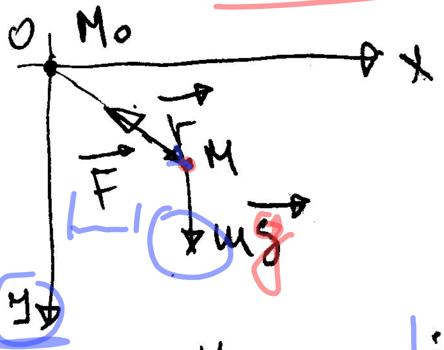
$\vec{F} = -k\vec{r}$, $k>0$ $\quad x_0 \quad y_0$

$t_0 = 0$ $\quad M_0(0,0)$ $\quad V_0(V_0, 0) \Leftarrow \vec{v}_0 = v_0 \vec{i}$

$V = const.$ $\quad \vec{r} = x\vec{i} + y\vec{j}$ $\quad \vec{r} = \vec{OM}$

$\vec{F} = -k\vec{OM} = k\vec{MO}$

$y - y(x) - ?$



$m\ddot{\vec{r}} = \vec{F} + m\vec{g}$ $/ \cdot \vec{i} / \cdot \vec{j}$

$x:$ $m\ddot{x} = -kx$ $/ : m$

$y:$ $m\ddot{y} = -ky + mg$ $/ : m$

$\ddot{x} = -\frac{k}{m}x$, $\quad \ddot{y} = -\frac{k}{m}y + g$

$\ddot{x} + \frac{k}{m}x = 0$, $\quad \ddot{y} + \frac{k}{m}y = g$ $\quad$ I

$t_0 = 0$

$x_0 = 0$, $y_0 = 0$

$\dot{x}_0 = v_0$; $\dot{y}_0 = \cdot$

$x = x_h$, $\quad y = y_h + y_p$

$\lambda^2 + \frac{k}{m} = 0 \Rightarrow \lambda = \pm\sqrt{\frac{k}{m}}\, i$ $\quad i = \sqrt{-1}$

$x = C_1 \cos\sqrt{\frac{k}{m}}t + C_2 \sin\sqrt{\frac{k}{m}}t$

C_1

$y_h = C_3 \cos\sqrt{\frac{k}{m}}t + C_4 \sin\sqrt{\frac{k}{m}}t$

C_2

C_3

$y_p = A$, $\quad \dot{y}_p = 0$, $\quad \ddot{y}_p = 0$

C_4

$\frac{k}{m}A = g \Rightarrow \boxed{A = \frac{mg}{k}}$

$y = C_3 \cos\sqrt{\frac{k}{m}}t + C_4 \sin\sqrt{\frac{k}{m}}t + \frac{mg}{k} = y$

$$\dot{x} = -\sqrt{\frac{k}{m}}\,C_1 \sin\sqrt{\frac{k}{m}}\,t + \sqrt{\frac{k}{m}}\,C_2 \cos\sqrt{\frac{k}{m}}\,t \quad (*)$$

$t_0 = 0: \quad x_0 = 0, \quad \dot{x}_0 = V_0$

$$\boxed{0 = C_1} \qquad V_0 = \sqrt{\frac{k}{m}}\,C_2 \Rightarrow \boxed{C_2 = V_0\sqrt{\frac{m}{k}}}$$

$$\boxed{x(t) = V_0\sqrt{\frac{m}{k}}\sin\sqrt{\frac{k}{m}}\,t}$$

$$\dot{y} = -\sqrt{\frac{k}{m}}\,C_3 \sin\sqrt{\frac{k}{m}}\,t + \sqrt{\frac{k}{m}}\,C_4 \cos\sqrt{\frac{k}{m}}\,t$$

$t_0 = 0: \quad y_0 = 0, \quad \dot{y}_0 = 0$

$$0 = C_3 + \frac{mg}{k} \Rightarrow C_3 = -\frac{mg}{k}$$

$$\boxed{0 = C_4}$$

$$y(t) = -\frac{mg}{k}\cos\sqrt{\frac{k}{m}}\,t + \frac{mg}{k} = \frac{mg}{k}\left(1 - \cos\sqrt{\frac{k}{m}}\,t\right)$$

$$\dot{x} = V_0\cos\sqrt{\frac{k}{m}}\,t, \quad \dot{y} = \frac{mg}{k}\sqrt{\frac{k}{m}}\sin\sqrt{\frac{k}{m}}\,t = g\sqrt{\frac{m}{k}}\sin\sqrt{\frac{k}{m}}\,t$$

$$\dot{x}^2 + \dot{y}^2 = V_0^2$$

$$V_0^2\cos^2\sqrt{\frac{k}{m}}\,t + g^2\frac{m}{k}\sin^2\sqrt{\frac{k}{m}}\,t = V_0^2 \quad |:V_0^2$$

$$\cos^2\sqrt{\frac{k}{m}}\,t + \frac{g^2 m}{kV_0^2}\sin^2\sqrt{\frac{k}{m}}\,t = 1$$

$$\frac{g^2 m}{kV_0^2} = 1 \Rightarrow \boxed{k = \frac{g^2 m}{V_0^2}}$$

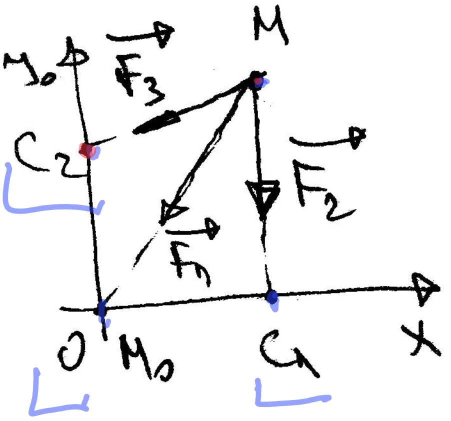
$(*)\ \cos^2\alpha + \sin^2\alpha = 1$

1.44*) M, m

$$\vec{F_1} = 3mk^2\,\vec{MO}$$
$$\vec{F_2} = 3mk^2\,\vec{MC_1}$$
$$\vec{F_3} = 3mk^2\,\vec{MC_2}, \quad k = const.$$
$$O(0,0)$$
$$\overline{OC_1} = b\cos\omega t, \quad (b, \omega - const.)$$
$$\overline{OC_2} = bt$$
$$t_0 = 0 \quad M_0(0,0), \; V_0(0,0)$$

$x(t) - y(t) - ?$

$$\boxed{m\ddot{\vec{r}} = \vec{F_1} + \vec{F_2} + \vec{F_3}} = \sum_{i=1}^{3} \vec{F_i}$$



$$\vec{F_1} = 3mk^2\,\vec{MO} = 3mk^2[(x_0 - x)\vec{i} + (y_0 - y)\vec{j}] = -3mk^2 x\vec{i} - 3mk^2 y\vec{j}$$

$$\vec{F_2} = 3mk^2[(x_{C_1} - x)\vec{i} + (\overset{=0}{y_{C_1}} - y)\vec{j}] = 3mk^2(b\cos\omega t - x)\vec{i} - 3mk^2 y\vec{j}$$

$$\vec{F_3} = 3mk^2[(\overset{=0}{\cancel{x_{C_2}}} - x)\vec{i} + (y_{C_2} - y)\vec{j}] = -3mk^2 x\vec{i} + 3mk^2(bt - y)\vec{j}$$

$$m\ddot{\vec{r}} = \sum_i \vec{F_i} \;\Big/\cdot\vec{i} \;\Big/\cdot\vec{j}$$

x: $m\ddot{x} = -3mK^2x + 3mK^2(b\cos\omega t - x) - $

$-3mK^2x$ OK

y: $m\ddot{y} = -3mK^2y - 3mK^2y + 3mK^2(bt - y)$

$m\ddot{x} = -9mK^2x + 3mK^2b\cos\omega t \quad |:m$

$m\ddot{y} = -9mK^2y + 3mK^2bt \quad |:m$

$$\boxed{\ddot{x} + 9K^2x = 3K^2b\cos\omega t} \quad (1)$$

$$\ddot{y} + 9K^2y = 3K^2bt \quad (2)$$

$\boxed{x = x_h + x_p}$

$y = y_h + y_p$

$\lambda^2 + 9K^2 = 0 \Rightarrow \boxed{\lambda = \pm 3Ki}$ $\quad i = \sqrt{-1}$

$x_h = C_1\cos 3Kt + C_2\sin 3Kt$

$\boxed{x_p = A\cos\omega t + B\sin\omega t}$ $\Big|\frac{d}{dt}$

$\dot{x}_p = -A\omega\sin\omega t + B\omega\cos\omega t$ $\Big|\frac{d}{dt}$

$\ddot{x}_p = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$

(1) $-A\omega^2\cos\omega t - B\omega^2\sin\omega t + 9K^2A\cos\omega t +$

$+9K^2B\sin\omega t = 3K^2b\cos\omega t$

$A(9K^2-\omega^2)\cos\omega t + B(9K^2-\omega^2)\sin\omega t = 3K^2b\cos\omega t$

$+0\cdot\sin\omega t$

$$A(9K^2-w^2) = 3K^2 b \Rightarrow A = \frac{3K^2 b}{9K^2-w^2}$$

$$B(9K^2-w^2) = 0 \Rightarrow B=0$$

$$\boxed{X_P = \frac{3K^2 b}{9K^2-w^2} \cos wt}$$

$$\boxed{X = C_1 \cos 3Kt + C_2 \sin 3Kt + \frac{3K^2 b}{9K^2-w^2} \cos wt}$$

$$\dot{X} = -3K C_1 \sin 3Kt + 3K C_2 \cos 3Kt - \frac{3K^2 b w}{9K^2-w^2} \sin wt$$

$t_0 = 0$

$t_0 = 0$, $\dot{x}_0 = 0$, $x_0 = 0$

$t_0 = 0: \; X_0 = 0, \; \dot{X}_0 = 0$

x_0: $0 = C_1 \cdot 1 + \cancel{C_2 \sin 0 = 0} + \frac{3K^2 b}{9K^2-w^2} \Rightarrow \boxed{C_1 = -\frac{3K^2 b}{9K^2-w^2}}$

$\dot{x}_0$: $0 = 3K C_2 \Rightarrow C_2 = 0$

$$\boxed{X(t) = \frac{3K^2 b}{9K^2-w^2}(\cos wt - \cos 3Kt)}$$

$w \neq 3K$

$$\boxed{y_h = C_3 \cos 3\cancel{K}t + C_4 \sin 3Kt}$$

$\boxed{y_P = Dt} \Rightarrow \dot{y}_P = D, \; \ddot{y}_P = 0$

$$9k^2 D t = 3k^2 b t$$

$$D = \frac{b}{3}, \quad y_p = \frac{b}{3} t$$

$$y = C_3 \cos 3kt + C_4 \sin 3kt + \frac{b}{3} t$$

$$\dot{y} = -3kC_3 \sin 3kt + 3kC_4 \cos 3kt + \frac{b}{3}$$

$$t_0 = 0: \quad y_0 = 0, \quad \dot{y}_0 = 0$$

$$\boxed{0 = C_3}$$

$$0 = 3kC_4 + \frac{b}{3} \Rightarrow \boxed{C_4 = -\frac{b}{9k}}$$

$$\boxed{y(t) = \frac{b}{3}\left(t - \frac{1}{3k} \sin 3kt\right)}$$

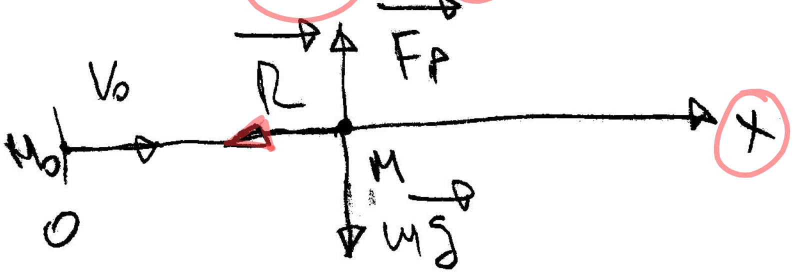
1.46*) $m = 2000\ kg$

$V_0 = 1{,}5\ m/s$

$R = 50\ V$

$\boxed{V_1 = \frac{V_0}{4}}$

$t_1 - S_1 - ?$



$m\ddot{\vec{r}} = \sum \vec{F_i}$

$x:\ \overset{2000}{m}\ddot{x} = -50\,\dot{x} \quad /:m$

$\ddot{x} + \frac{1}{40}\dot{x} = 0$

$\lambda^2 + \frac{1}{40}\lambda = 0$

$\lambda_1 = 0,\ \lambda_2 = -\frac{1}{40}$ $\quad e^0 = 1$

$x = C_1 + C_2 e^{-\frac{t}{40}} \quad / \frac{d}{dt}$

$\dot{x} = -\frac{1}{40} C_2 e^{-\frac{t}{40}}$

$t_0 = 0:\ x_0 = 0,\ \dot{x}_0 = 1{,}5$

$0 = C_1 + C_2$

$1{,}5 = -\frac{C_2}{40} \Rightarrow \boxed{C_2 = -60}$

$\dot{x}_0 = -\frac{1}{40} C_2 e^0$

$C_1 = -C_2 = 60$

$x = 60\left(1 - e^{-\frac{t}{40}}\right) = x(t)$

$\dot{x} = 1{,}5\, e^{-\frac{t}{40}}$

$*\ t_1 \quad \dot{x}_1 = 1{,}5/4$

$\boxed{\frac{1{,}5}{4}} = 1{,}5\, e^{-\frac{t_1}{40}}$

$\frac{1}{4} = e^{-\frac{t_1}{40}}$

$e^{\frac{t_1}{40}} = 4$

$\ln 4 = \frac{t_1}{40} \Rightarrow \boxed{t_1 = 40 \ln 4}$

$x(t_1)$

$x_1 = 60\left(1 - \frac{1}{4}\right) = 60\,\frac{3}{4}$

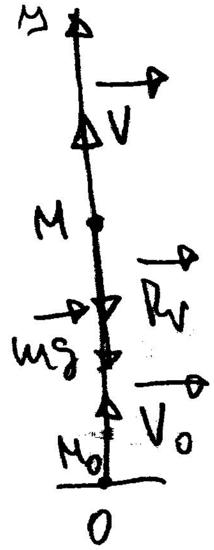
$\boxed{x_1 = 45\ m}$

1.51*) m

V_0

$R_v = m k^2 V^2, \quad k = \text{const}$

$H, T - ?$



$$m\ddot{\vec{r}} = m\vec{g} + \vec{R_v}$$

$$y: \quad m\ddot{y} = -mg - mk^2\dot{y}^2 \quad |:m$$

$$\ddot{y} + k^2\dot{y}^2 = -g$$

$$\ddot{y} = \frac{d\dot{y}}{dt}\frac{dy}{dy} = \frac{\dot{y}\,d\dot{y}}{dy} = \frac{1}{2}\frac{d}{dy}(\dot{y}^2)$$

$$\frac{1}{2}\frac{d}{dy}(\dot{y}^2) + k^2\dot{y}^2 = -g, \quad \text{смена} \quad \dot{y}^2 = z$$

$$\frac{1}{2}\frac{dz}{dy} + k^2 z = -g \,|\cdot 2, \quad \frac{dz}{dy} = z'$$

$$z' + 2k^2 z = -2g$$

$$z = z_n + z_p$$

$$\lambda + 2k^2 = 0 \Rightarrow \lambda = -2k^2$$

$$z_n = C e^{-2k^2 y}$$

$$z_p = A, \quad z_p' = 0$$

$$\cancel{2}k^2 A = -\cancel{2}g \Rightarrow A = -\frac{g}{k^2}$$

$$z_p = -\frac{g}{k^2}$$

$$z = C e^{-2k^2 y} - \frac{g}{k^2}$$

$$\dot{y}^2 = C e^{-2k^2 y} - \frac{g}{k^2}$$

$t_0 = 0: \quad y_0 = 0 \quad \dot{y} = V_0$

$$V_0^2 = C - \frac{g}{k^2} \Rightarrow C = V_0^2 + \frac{g}{k^2}$$

$$\dot{y}^2 = \left(V_0^2 + \frac{g}{k^2}\right) e^{-2k^2 y} - \frac{g}{k^2}$$

$\dot{y} = 0 \rightarrow$ ΓΛΟΒ

$$\frac{g}{k^2} = \left(V_0^2 + \frac{g}{k^2}\right) e^{-2k^2 H}$$

$$e^{2k^2 H} = \frac{V_0^2 + \frac{g}{k^2}}{\frac{g}{k^2}} = \frac{V_0^2 k^2 + g}{g}$$

$$2k^2 H = \ln \frac{V_0^2 k^2 + g}{g}$$

$$\boxed{H = \frac{1}{2k^2} \ln \frac{k^2 V_0^2 + g}{g}}$$

II НАЧИН

$$m\dot{v} = -mg - mk^2v^2$$

$$\dot{v} = -g - k^2v^2$$

$$\dot{v} = \frac{dv}{dt}\frac{dy}{dy} = \frac{v\,dv}{dy}$$

$$\frac{v\,dv}{dy} = -(g + k^2v^2)$$

$$\frac{v\,dv}{g + k^2v^2} = -dy \quad /\cdot\int$$

$$\int_{v_0}^{v} \frac{v\,dv}{g + k^2v^2} = -\int_{y_0}^{y} dy \,, \qquad g + k^2v^2 = z$$

$$2k^2v\,dv = dz$$

$$v\,dv = \frac{1}{2k^2}\,dz$$

$$\frac{1}{2k^2}\int \frac{dz}{z} = y_0 - y$$

$$\frac{1}{2k^2}\ln z \Big|_{z_0}^{z} = \overset{\to 0}{y_0} - y$$

$$\frac{1}{2k^2}\ln\frac{z}{z_0} = -y \Rightarrow \quad -y = \frac{1}{2k^2}\ln\frac{g + k^2v^2}{g + k^2v_0^2}$$

$$y = \frac{1}{2k^2}\ln\frac{g + k^2v_0^2}{g + k^2v^2}$$

ЗА $v = 0$, $y = H$

$$H = \frac{1}{2k^2}\ln\frac{g + k^2v_0^2}{g}$$

$$\dot{v} = -(g + k^2 v^2)$$

$$\frac{dv}{dt} = \dot{v}$$

$$\frac{dv}{dt} = -(g + k^2 v^2)$$

$$\frac{dv}{g + k^2 v^2} = -dt \quad \Big| \cdot \int$$

$$\int_{v_0}^{v} \frac{dv}{g + k^2 v^2} = -\int_{t_0}^{t} dt \quad (*)$$

$$\int_{v_0}^{v} \frac{dv}{g + k^2 v^2} = \frac{1}{k^2} \int_{v_0}^{v} \frac{dv}{\frac{g}{k^2} + v^2} = \frac{1}{k^2} \int \frac{dv}{\left(\frac{\sqrt{g}}{k}\right)^2 + v^2}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg}\left(\frac{x}{a}\right), \quad a = \frac{\sqrt{g}}{k}$$

$(*)$

$$\frac{k}{\sqrt{g}} \frac{1}{k^2} \operatorname{arctg}\left(\frac{v}{\left(\frac{\sqrt{g}}{k}\right)}\right) \Bigg|_{v_0}^{v} = -t$$

$$\frac{1}{k\sqrt{g}} \left(\operatorname{arctg} \frac{kv}{\sqrt{g}} - \operatorname{arctg} \frac{kv_0}{\sqrt{g}} \right) = -t$$

$$t = \frac{1}{k\sqrt{g}} \left(\operatorname{arctg} \frac{kv_0}{\sqrt{g}} - \operatorname{arctg} \frac{kv}{\sqrt{g}} \right)$$

за $v = 0 \quad t = T$

$$\boxed{T = \frac{1}{k\sqrt{g}} \operatorname{arctg} \frac{kv_0}{\sqrt{g}}}$$