1. Ukoliko srednja vrednost zapreminskog udela pare, u vertikalnoj cevi dužine 10 m, iznosi 0,3, odrediti promenu hidrostatičkog pritiska duž te cevi. Gustina vode je $\rho_1 = 740\ kg/m^3$, a pare $\rho_2 = 36\ kg/m^3$.

L=10 m

$\alpha_2 = 0.3$

$\rho_1 = 740\ \frac{kg}{m^3}$

$\rho_2 = 36\ \frac{kg}{m^3}$

$\Delta p = ?$

$\Delta p = -\overline{\rho} g h$

$\overline{\rho} = \frac{m}{V} = \frac{m_1 + m_2}{V} = \frac{\rho_1 V_1 + \rho_2 V_2}{V} = \rho_1 \alpha_1 + \rho_2 \alpha_2 = \rho_1 (1 - \alpha_2) + \rho_2 \alpha_2$

$h = L \sin\theta, \theta = 90^0 \rightarrow \sin\theta = 1$

$\Delta p = -\overline{\rho} g L = -(\rho_1 (1 - \alpha_2) + \rho_2 \alpha_2) g L$

$\Delta p = -51.875 \cdot 10^3$ Pa

2. Kolika je prividna brzina pare u dvofaznom toku, ukoliko je masena brzina (maseni fluks) dvofazne mešavine $G = 100\ kg/(m^2 s)$, masena protočna koncentracija $\chi = 0{,}1$, a gustina pare $\rho_2 = 1\ kg/m^3$?

$G = 100\ kg/(m^2 s)$

$\chi = 0{,}1$

$\rho_2 = 1\ kg/m^3$

$j_2 = ?$

$j_2 = \frac{\dot{V}_2}{A} = \frac{\dot{m}_2}{\rho_2 A} = \frac{\rho_2 u_2 A_2}{\rho_2 A} = \alpha_2 u_2$

$j_2 = \alpha_2 u_2$

$G = \frac{\dot{m}}{A} = \frac{\dot{m}_1 + \dot{m}_2}{A} = \frac{\rho_1 u_1 A_1 + \rho_2 u_2 A_2}{A} = \underbrace{\rho_1 u_1 \alpha_1}_{G_1} + \underbrace{\rho_2 u_2 \alpha_2}_{G_2} = G_1 + G_2$

$G_2 = \rho_2 u_2 \alpha_2 = j_2 \rho_2 \rightarrow j_2 = \frac{G_2}{\rho_2}$

$\chi = \frac{\dot{m}_2}{\dot{m}} = \frac{\frac{\dot{m}_2}{A}}{\frac{\dot{m}}{A}} = \frac{G_2}{G} \rightarrow G_2 = \chi G$

$j_2 = \frac{\chi G}{\rho_2} = 10\ \frac{m}{s}$

3. Voda i vazduh formiraju mehurasti dvofazni tok u cevi unutrašnjeg prečnika 30 mm. Zapreminski protok vazduha je $5{,}3 \cdot 10^{-5}\ m^3/s$, a snimanjem kamerom je utvrđeno da je srednja brzina mehura 0,75

m/s. a) Odrediti zapreminski udeo vazduha. b) Kolika je brzina vode ako je protok vode $2,7\cdot10^{-4}\ m^3/s$. c) Odrediti relativnu brzinu i faktor klizanja.

$d = 30\ mm$

$\dot{V}_{vaz} = 5.3\cdot10^{-5}\ \frac{m^3}{s}$

$u_{vaz} = 0.75\frac{m}{s}$

$\dot{V}_{H_2O} = 2.7\cdot10^{-4}\ \frac{m^3}{s}$

$\alpha_2 = ?, u_{H_2O} = ?, u_r = ?, S = ?$

$$a)\quad \dot{m} = \rho u A,\ \dot{V} = uA \rightarrow \dot{V}_{vaz} = u_{vaz}A_{vaz} = \left\{\alpha_2 = \frac{A_2}{A} = \frac{A_{vaz}}{A} = \frac{\dot{V}_{vaz}/u_{vaz}}{A}\right\} = u_{vaz}\alpha_2 A = u_{vaz}\alpha_2\frac{d^2\pi}{4}$$

$$\alpha_2 = \frac{\dot{V}_{vaz}}{u_{vaz}A} = \frac{\dot{V}_{vaz}}{u_{vaz}\frac{d^2\pi}{4}} = 0.1$$

$$b)\quad \dot{V}_{H_2O} = u_{H_2O}\alpha_1 A = u_{H_2O}\left(1-\alpha_2\right)A \rightarrow u_{H_2O} = \frac{\dot{V}_{H_2O}}{\left(1-\alpha_2\right)\frac{d^2\pi}{4}} = \frac{2.7\cdot10^{-4}}{\left(1-0.1\right)\frac{\left(30\cdot10^{-3}\right)^2\pi}{4}} = 0.424\ \frac{m}{s}$$

$$c)\quad u_r = u_{vaz} - u_{H_2O} = 0.326$$

$$S = \frac{u_2}{u_1} = \frac{u_{vaz}}{u_{H_2O}} = \frac{0.75}{0.424} = 1.77$$

4. Dvofazna mešavina vode i pare struji u cevi. Zapreminski udeo pare je 80%, a maseni protočni stepen suvoće 1%. Odrediti faktor klizanja za pritisak od a) 0,1 MPa, b) 10 MPa.

$\alpha_2 = 80\% = 0.8$

$\chi = 1\% = 0.01$

$S = ?$

$$\chi = \frac{\dot{m}_2}{\dot{m}_1+\dot{m}_2} = \frac{\rho_2\alpha_2 u_2 A}{\rho_1\alpha_1 u_1 A + \rho_2\alpha_2 u_2 A} = \frac{\rho_2\alpha_2 u_2}{\rho_1\alpha_1 u_1 + \rho_2\alpha_2 u_2} = \frac{1}{1+\frac{1}{S}\cdot\frac{1-\alpha_2}{\alpha_2}\cdot\frac{\rho_1}{\rho_2}} = \frac{1}{1+\frac{1}{S}\cdot\frac{1-\alpha_2}{\alpha_2}\cdot\frac{v''}{v'}} \rightarrow S = \left(\frac{\frac{1}{\chi}-1}{\frac{1-\alpha_2}{\alpha_2}\cdot\frac{v''}{v'}}\right)^{-1}$$

$$p = 0.1MPa\quad \frac{v''}{v'} = \frac{1.694}{0.0010432} = 1623.85\quad S = 4.1$$

$$p = 10MPa\quad \frac{v''}{v'} = \frac{0.01803}{0.0014521} = 12.42\quad S = 0.031$$

5. Pokazati da su maseni udeo pare (statički kvalitet) i energetski udeo pare (TD kvalitet) jednaki kada se parna i tečna faza nalaze u termičkoj ravnoteži.

$x = \frac{m_2}{m} = \frac{m_2}{m_1 + m_2}$ maseni udeo pare (statički kvalitet)

$x_T = \frac{h - h'}{h'' - h'}$ energetski udeo pare (termodinamički kvalitet)

Termička ravnoteža: $T_1=T_2=Tsat$, $h_1=h'$, $h_2=h''$

$$H = m_1 h_1 + m_2 h_2 \left[J \right]$$
$$H = m_1 h' + m_2 h''$$
$$mh = m_1 h' + m_2 h'' / : m$$
$$h = \frac{m_1}{m} h' + \frac{m_2}{m} h''$$
$$\frac{m_2}{m} = x, \frac{m_1}{m} = 1 - x$$
$$h = (1 - x) h' + x h'' \rightarrow x = \frac{h - h'}{h'' - h'} = x_T$$

6. Odrediti prividnu brzinu dvofazne mešavine azota (idealnog gasa) i vode na 50^0C koja se nalazi na pritisku od 1,5 bara. faktor klizanja odrediti na osnovu korelacije Zivi-a $S = (\rho_1 / \rho_2)^{\frac{1}{3}}$. Zapreminski udeo gasne faze iznosi 0,3, a brzina 2 m/s.

$$u_2 = 2 \frac{m}{s}$$
$$\alpha_2 = 0.3$$
$$p = 1.5\ bar, t = 50^0 C \rightarrow v = 0.001012 \frac{m^3}{kg} \rightarrow \rho_{H_2O} = \rho_1 = \frac{1}{v} = 988.14 \frac{kg}{m^3}$$

Azot (idealan gas) – važi j-na stanja idealnog gasa $p = \rho RT$. Iz TD Tab – R_{N2}=296,8 J/(kgK)

$$p = 1.5 \cdot 10^5\ Pa, T = 50 + 273 = 323\ K$$
$$\rho_{N_2} = \rho_2 = \frac{p}{RT} = 1.565 \frac{kg}{m^3}$$
$$S = \left(\frac{\rho_1}{\rho_2} \right)^{\frac{1}{3}} = 8.579$$
$$j = j_1 + j_2 = \alpha_1 u_1 + \alpha_2 u_2 = u_2 \left(\frac{u_1}{u_2} \alpha_1 + \alpha_2 \right) = u_2 \left(\frac{1}{S} \alpha_1 + \alpha_2 \right) = 2 \cdot \left(\frac{1}{8.579} \cdot 0.7 + 0.3 \right) = 0.763 \frac{m}{s}$$