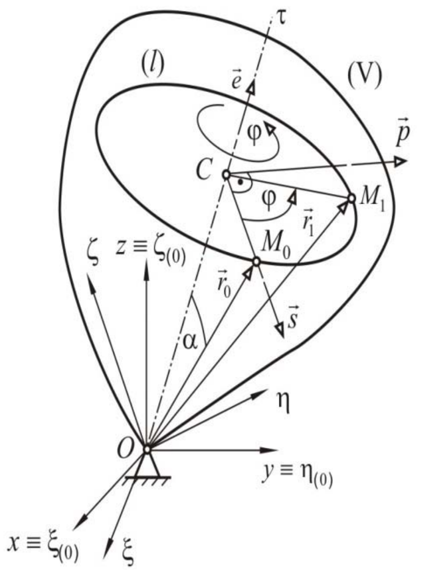
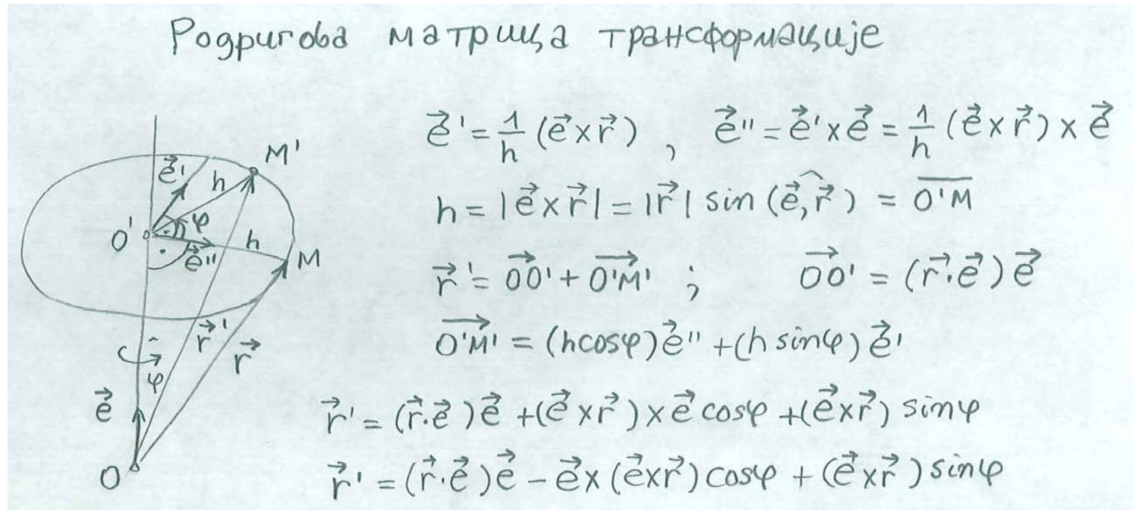
Родригов образац, Родригова матрица трансформације





$$\{\vec{r}_0\} = \begin{Bmatrix} x_0 \\ y_0 \\ z_0 \end{Bmatrix}, \quad \{\vec{r}_1\} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$\{\vec{r}_1\} = [A^r]\{\vec{r}_0\}$$

$$\vec{r}_1 = \vec{r}_0 - |\vec{e}\times\vec{r}_0|(1-\cos\varphi)\vec{s} + |\vec{e}\times\vec{r}_0|(\sin\varphi)\vec{p}$$

$$\vec{r}_1 = \vec{r}_0 + (1-\cos\varphi)\vec{e}\times(\vec{e}\times\vec{r}_0) + (\sin\varphi)\vec{e}\times\vec{r}_0$$

$$[A^r] = [I] + (1-\cos\varphi)[e^d]^2 + \sin\varphi[e^d]$$

$$\overline{CM}_0 = \overline{CM}_1 = |\vec{e}\times\vec{r}_0| = h$$

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = [A^r]\begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix}$$

- A)

$$[e^d][A^r] = [e^d]cos\varphi + [e^d]^2 sin\varphi,$$

$$\frac{\partial[A^r]}{\partial\varphi} = [e^d]cos\varphi + [e^d]^2 sin\varphi,$$

Ојлеров образац за брзину преко Родриговог приступа

$$\vec{e}\times(\vec{e}\times\vec{r}) = \vec{e}(\vec{e}\cdot\vec{r}) - \vec{r}(\vec{e}\cdot\vec{e}) \qquad \left(\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})\right)$$

$$\vec{r}\,' = \vec{r} + \vec{e}\times(\vec{e}\times\vec{r})(1-\cos\varphi) + (\vec{e}\times\vec{r})\sin\varphi, \qquad \left(\vec{e}\times([e^d]\{\vec{r}\}) = [e^d]^2\{\vec{r}\}\right)$$

$$\{\vec{r}\,'\} = \{\vec{r}\} + [e^d]^2(1-\cos\varphi)\{\vec{r}\} + [e^d]\sin\varphi\{\vec{r}\}$$

$$\{\vec{r}\,'\} = [A_r]\{\vec{r}\} \rightarrow [A_r] = [I] + [e^d]^2(1-\cos\varphi) + [e^d]\sin\varphi$$

$$\Delta\vec{r} = \vec{r}\,' - \vec{r} = \vec{e}\times(\vec{e}\times\vec{r})(1-\cos\Delta\varphi) + (\vec{e}\times\vec{r})\sin\Delta\varphi$$

$$\frac{d\vec{r}}{d\varphi} = \lim_{\Delta\varphi\to 0}\frac{\Delta\vec{r}}{\Delta\varphi} = \vec{e}\times(\vec{e}\times\vec{r})\lim_{\Delta\varphi\to 0}\frac{1-\cos\Delta\varphi}{\Delta\varphi} + (\vec{e}\times\vec{r})\lim_{\Delta\varphi\to 0}\frac{\sin\Delta\varphi}{\Delta\varphi} = \vec{e}\times\vec{r}$$

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}}{d\varphi}\frac{d\varphi}{dt} = (\vec{e}\times\vec{r})\dot{\varphi} = (\dot{\varphi}\vec{e})\times\vec{r} = \vec{\omega}\times\vec{r}$$

Последњи израз представља добро познати Ојлеров образац за брзину (из Механике 2)

Veze između ugaone brzine, ugaonog ubrzanja i matrice transformacije u slučaju sfernog kretanja tela

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = [A] \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} = \frac{d[A]}{dt} \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix}$$

$$\xi, \eta, \zeta = const$$

$$\vec{v} = \vec{\omega} \times \vec{OM}$$

$$\{\vec{v}\} = \begin{Bmatrix} v_x \\ v_y \\ v_z \end{Bmatrix} = \frac{d[A]}{dt} \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix}$$

$$\{\vec{v}\} = [\omega^d] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$\{\vec{v}\} = [\omega^d][A] \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix}$$

$$\left[\frac{d[A]}{dt} - [\omega^d][A] \right] \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix} = 0$$

$$[\omega^d] = \frac{d[A]}{dt}[A]^T$$

$$\begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} = \begin{bmatrix} \dot{\alpha}_{11} & \dot{\alpha}_{12} & \dot{\alpha}_{13} \\ \dot{\alpha}_{21} & \dot{\alpha}_{22} & \dot{\alpha}_{23} \\ \dot{\alpha}_{31} & \dot{\alpha}_{32} & \dot{\alpha}_{33} \end{bmatrix} \begin{bmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} \\ \alpha_{12} & \alpha_{22} & \alpha_{32} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} \end{bmatrix}$$

$$\omega_x = \dot{\alpha}_{31}\alpha_{21} + \dot{\alpha}_{32}\alpha_{22} + \dot{\alpha}_{33}\alpha_{23},$$
$$\omega_y = \dot{\alpha}_{11}\alpha_{31} + \dot{\alpha}_{12}\alpha_{32} + \dot{\alpha}_{13}\alpha_{33},$$
$$\omega_z = \dot{\alpha}_{21}\alpha_{11} + \dot{\alpha}_{22}\alpha_{12} + \dot{\alpha}_{23}\alpha_{13}.$$

$$\begin{Bmatrix}\omega_\xi\\ \omega_\eta\\ \omega_\zeta\end{Bmatrix}=[A]^T\begin{Bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{Bmatrix}$$

$$\begin{aligned}\omega_\xi &= \alpha_{13}\dot{\alpha}_{12}+\alpha_{23}\dot{\alpha}_{22}+\alpha_{33}\dot{\alpha}_{32},\\ \omega_\eta &= \alpha_{11}\dot{\alpha}_{13}+\alpha_{21}\dot{\alpha}_{23}+\alpha_{31}\dot{\alpha}_{33},\\ \omega_\zeta &= \alpha_{12}\dot{\alpha}_{11}+\alpha_{22}\dot{\alpha}_{21}+\alpha_{32}\dot{\alpha}_{31}.\end{aligned}$$

- Vektor ugaonog ubrzanja krutog dela $\vec{\varepsilon}=\dfrac{d\vec{\omega}}{dt}$

$$\{\vec{\varepsilon}\}=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{Bmatrix}=\begin{Bmatrix}\dot{\omega}_x\\ \dot{\omega}_y\\ \dot{\omega}_z\end{Bmatrix}$$

$$[\varepsilon^d]=[\dot{\omega}^d]$$

$$[\varepsilon^d]=\frac{d^2[A]}{dt^2}[A]^T+\frac{d[A]}{dt}\frac{d[A]^T}{dt}$$

$$\begin{bmatrix}0 & -\varepsilon_z & \varepsilon_y\\ \varepsilon_z & 0 & -\varepsilon_x\\ -\varepsilon_y & \varepsilon_x & 0\end{bmatrix}=\begin{bmatrix}\ddot{\alpha}_{11} & \ddot{\alpha}_{12} & \ddot{\alpha}_{13}\\ \ddot{\alpha}_{21} & \ddot{\alpha}_{22} & \ddot{\alpha}_{23}\\ \ddot{\alpha}_{31} & \ddot{\alpha}_{32} & \ddot{\alpha}_{33}\end{bmatrix}\begin{bmatrix}\alpha_{11} & \alpha_{21} & \alpha_{31}\\ \alpha_{12} & \alpha_{22} & \alpha_{32}\\ \alpha_{13} & \alpha_{23} & \alpha_{33}\end{bmatrix}+$$
$$+\begin{bmatrix}\dot{\alpha}_{11} & \dot{\alpha}_{12} & \dot{\alpha}_{13}\\ \dot{\alpha}_{21} & \dot{\alpha}_{22} & \dot{\alpha}_{23}\\ \dot{\alpha}_{31} & \dot{\alpha}_{32} & \dot{\alpha}_{33}\end{bmatrix}\begin{bmatrix}\dot{\alpha}_{11} & \dot{\alpha}_{21} & \dot{\alpha}_{31}\\ \dot{\alpha}_{12} & \dot{\alpha}_{22} & \dot{\alpha}_{32}\\ \dot{\alpha}_{13} & \dot{\alpha}_{23} & \dot{\alpha}_{33}\end{bmatrix}.$$

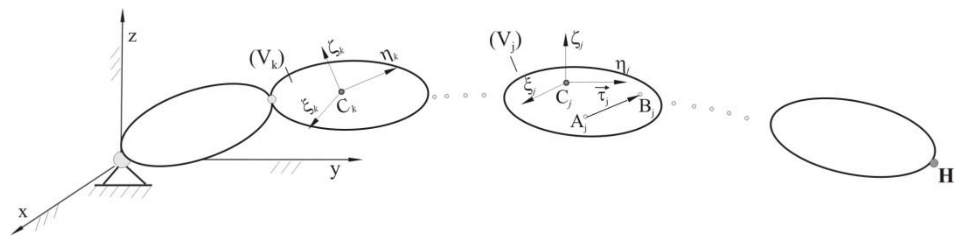
$$\begin{aligned}\varepsilon_x &= \ddot{\alpha}_{31}\alpha_{21}+\ddot{\alpha}_{32}\alpha_{22}+\ddot{\alpha}_{33}\alpha_{23}+\dot{\alpha}_{31}\dot{\alpha}_{21}+\dot{\alpha}_{32}\dot{\alpha}_{22}+\dot{\alpha}_{33}\dot{\alpha}_{23},\\ \varepsilon_y &= \ddot{\alpha}_{11}\alpha_{31}+\ddot{\alpha}_{12}\alpha_{32}+\ddot{\alpha}_{13}\alpha_{33}+\dot{\alpha}_{11}\dot{\alpha}_{31}+\dot{\alpha}_{12}\dot{\alpha}_{32}+\dot{\alpha}_{13}\dot{\alpha}_{33},\\ \varepsilon_z &= \ddot{\alpha}_{21}\alpha_{11}+\ddot{\alpha}_{22}\alpha_{12}+\ddot{\alpha}_{23}\alpha_{13}+\dot{\alpha}_{21}\dot{\alpha}_{11}+\dot{\alpha}_{22}\dot{\alpha}_{12}+\dot{\alpha}_{23}\dot{\alpha}_{13}.\end{aligned}$$

$$\begin{Bmatrix}\varepsilon_\xi\\ \varepsilon_\eta\\ \varepsilon_\zeta\end{Bmatrix}=[A]^T\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{Bmatrix}$$

$$\begin{aligned}\varepsilon_\xi &= \dot{\alpha}_{13}\dot{\alpha}_{12}+\dot{\alpha}_{23}\dot{\alpha}_{22}+\dot{\alpha}_{33}\dot{\alpha}_{32}+\alpha_{13}\ddot{\alpha}_{12}+\alpha_{23}\ddot{\alpha}_{22}+\alpha_{33}\ddot{\alpha}_{32},\\ \varepsilon_\eta &= \dot{\alpha}_{11}\dot{\alpha}_{13}+\dot{\alpha}_{21}\dot{\alpha}_{23}+\dot{\alpha}_{31}\dot{\alpha}_{33}+\alpha_{11}\ddot{\alpha}_{13}+\alpha_{21}\ddot{\alpha}_{23}+\alpha_{31}\ddot{\alpha}_{33},\\ \varepsilon_\zeta &= \dot{\alpha}_{12}\dot{\alpha}_{11}+\dot{\alpha}_{22}\dot{\alpha}_{21}+\dot{\alpha}_{32}\dot{\alpha}_{31}+\alpha_{12}\ddot{\alpha}_{11}+\alpha_{22}\ddot{\alpha}_{21}+\alpha_{32}\ddot{\alpha}_{31}.\end{aligned}$$

Сложене матрице трансформације

- Ортогоналне матрице
- Родригова матрица
- Сложене матрице трансформације



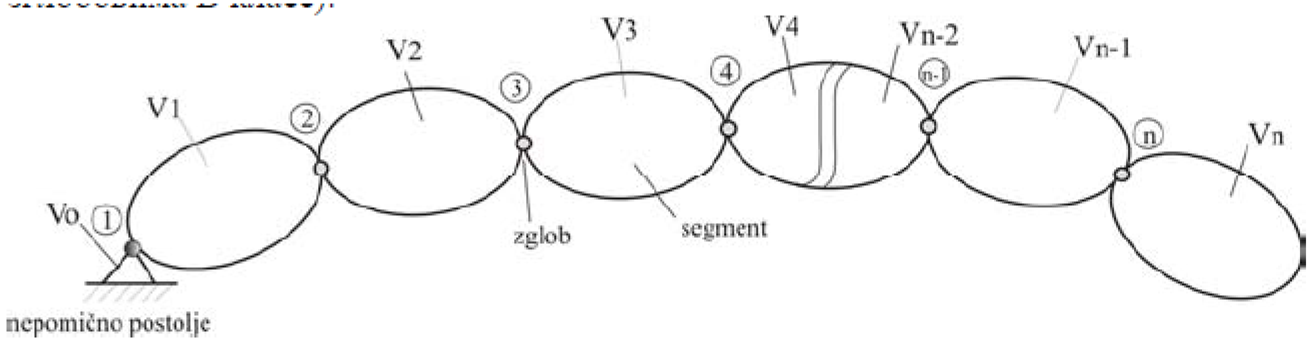
$$\left[A_{k,j}\right]=\left[A_{k+1}^r\right]\left[A_{k+2}^r\right]\ldots\left[A_{j-1}^r\right]\left[A_j^r\right]=\prod_{s=k+1}^{s=j}\left[A_s^r\right]$$

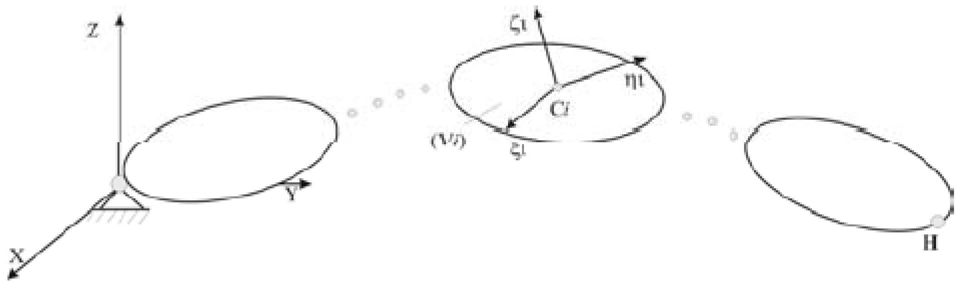
$$\det\left[A\right]=\left(\vec{\lambda}\times\vec{\mu}\right)\cdot\vec{\upsilon}=1,\quad \left[A\right]^{-1}=\left[A\right]^T.$$

$$\left[A_r\right]=I+\left(1-\cos\varphi\right)\left[e^d\right]^2+\sin\varphi\cdot\left[e^d\right]$$

$$\left\{\vec{r}_1\right\}=\left[A_r\right]\left\{\vec{r}\right\}$$

$$\left[A_{k,j}^r\right]=\left[A_{k+1}^r\right]\left[A_{k+2}^r\right]\ldots\left[A_{j-1}^r\right]\left[A_j^r\right].$$


V1
V2
V3
V4
Vn-2
Vn-1
Vn
Vo
zglob
segment
nepomično postolje


Z
X
Y
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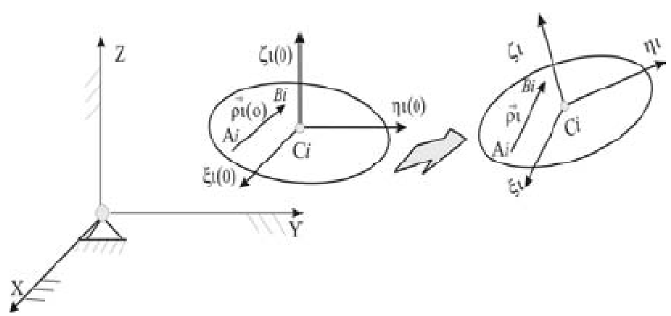
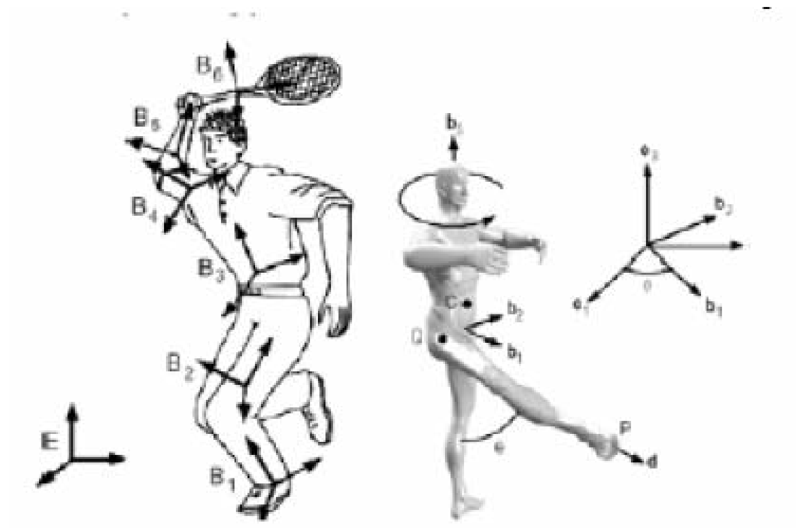

Z
Y
X
Ci
Ai
Bi



Uvodimo sledeće matrice:

$$\{\xi\} \in R^{n\times 1}, \{\bar{\xi}\} \in R^{n\times 1},$$

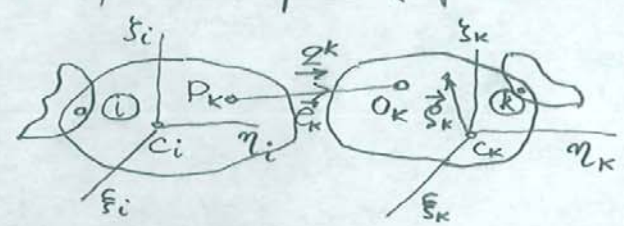
U slučaju da zglob (i) dozvoljava pravolinijsku translaciju tela (V_i) u odnosu na telo (V_{i-1}) važiće ((V_0) -nepomično postolje)

$$\xi_i = 1, \quad \bar{\xi}_i = 0,$$

U slučaju da zglob (i) dozvoljava rotaciju tela (V_i) u odnosu na osu vezanu za telo (V_{i-1}) važiće $\xi_i = 0, \quad \bar{\xi}_i = 1,$

Zglob (i) za koji je $\xi_i = 1$ naziva se *prizmatičnim* a zglob (i) za koji je $\bar{\xi}_i = 1$ - *cilindrični.*

** трансnaторни (призматични) зглоб

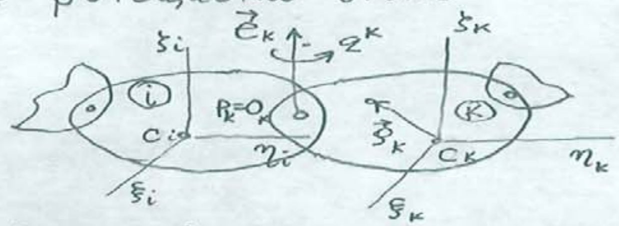


$$\{\vec{\xi}_k^{(i)}\} = \{\vec{\xi}_k^{(k)}\}$$

$$[A_{rk}] = [I]$$

$$\{\vec{e}_k^{(i)}\} = \{\vec{e}_k^{(k)}\}, \quad (\xi_k = 1, \ \bar{\xi}_k = 0)$$

* ротациони зглоб



$$\{\vec{e}_k^{(i)}\} = \{\vec{e}_k^{(k)}\}, \quad (\xi_k = 0, \ \bar{\xi}_k = 1)$$

референтна конфигурација :

$$\{\vec{\xi}_k^{(i)}\} = \{\vec{\xi}_k^{(k)}\}$$

ротација за угао q^k око $\vec{e}_k$:

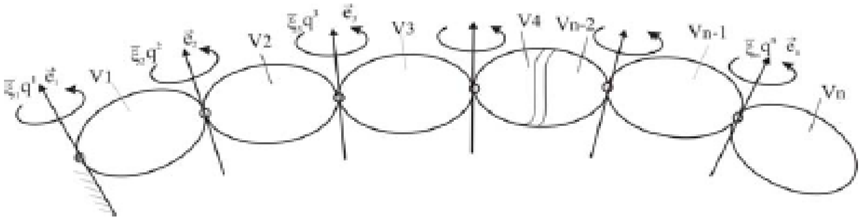
$$\{\vec{\xi}_k^{(i)}\} = [A_{rk}]\{\vec{\xi}_k^{(k)}\}$$

$$[A_{rk}] = [I] + [e^d]^2(1-\cos q^k) + [e^d]\sin q^k$$

*** општи случај

$$[A_{rk}] = [I] + \bar{\xi}_k\left([e^d]^2(1-\cos q^k) + [e^d]\sin q^k\right)$$

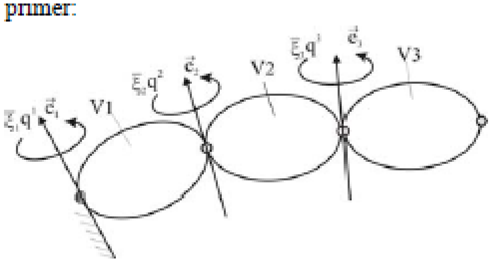
3.1 Složene matrice transformacija



q^i, i=1,2,...,n- relativne koordinate osim prve

primer:

primer:



zaustavlja se 1 i 2 segment a treći rotira oko sada nepokretne ose 3 za ugao q^3 i tako redom. Sistem je u početnom trenutku nalazio u referentnom položaju.

$(V_3(0)) \xrightarrow{\bar{\xi}_3 q^3} (V_3(I))$,

$(V_3(I)) \xrightarrow{\bar{\xi}_2 q^2} (V_3(II))$, $(V_3(0))$ -referentni položaj, (V_3) -proizvoljni položaj,

$(V_3(II)) \xrightarrow{\bar{\xi}_1 q^1} (V_3)$,

Interesuju nas projekcije uočenog vektora na ose Ox,Oz,Oy (koje odgovaraju referentnoj konfiguraciji i to posle rotacije)

$\{\vec{\rho}_3\} = \begin{Bmatrix} \xi_3(0) \\ \eta_3(0) \\ \varsigma_3(0) \end{Bmatrix}$ -referentna konfiguracija

prva rotacija $\bar{\xi}_3 q^3$

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \left[A_{R_3}\right] \begin{Bmatrix} \xi_3(0) \\ \eta_3(0) \\ \varsigma_3(0) \end{Bmatrix}$$

druga rotacija $\bar{\xi}_2 q^2$

$$\begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} = \left[A_{R_2}\right] \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \left[A_{R_2}\right]\left[A_{R_3}\right] \begin{Bmatrix} \xi_3(0) \\ \eta_3(0) \\ \varsigma_3(0) \end{Bmatrix}$$

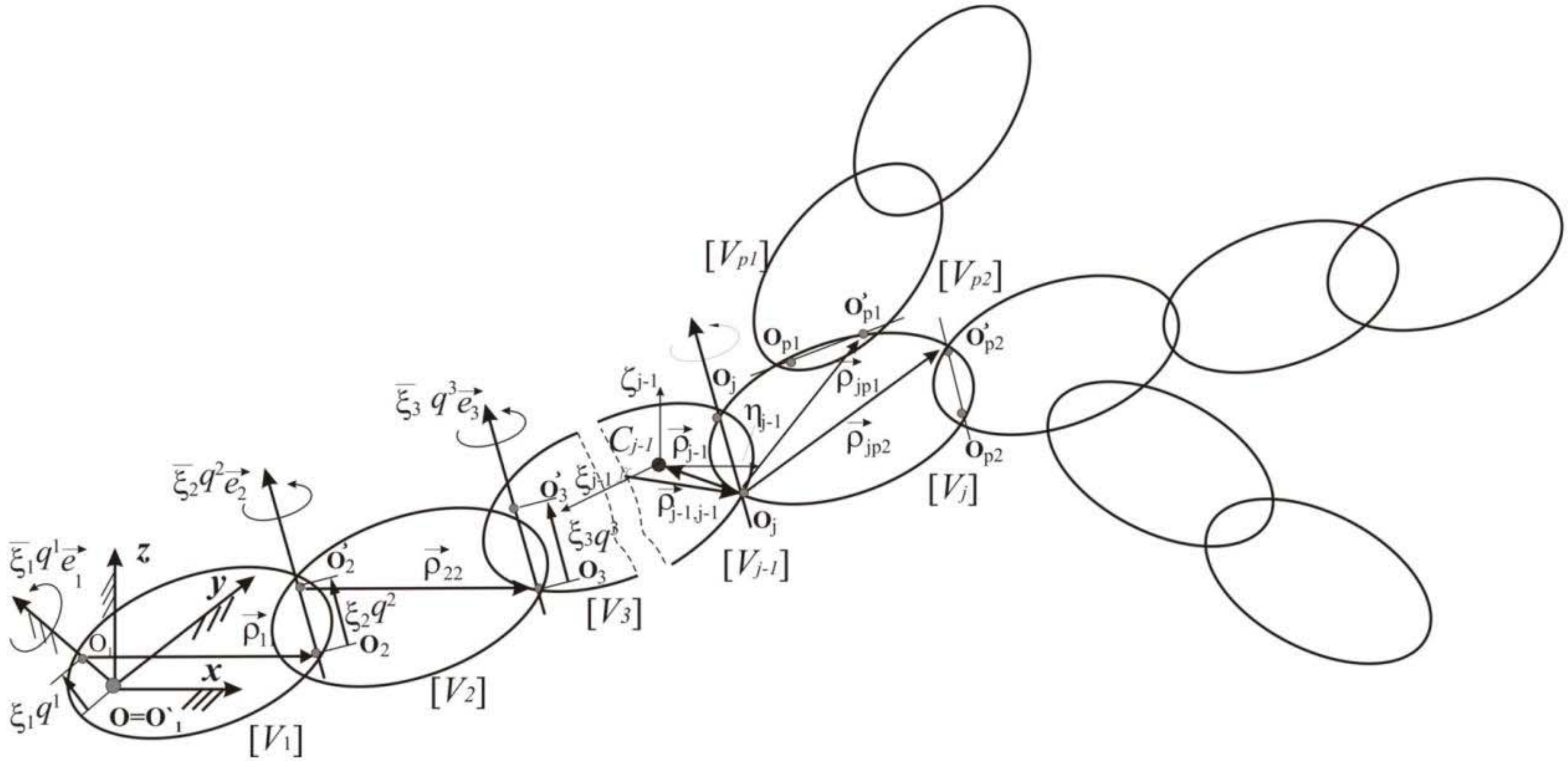
treća rotacija $\bar{\xi}_1 q^1$

$$\begin{Bmatrix} x''' = x_3 \\ y''' = y_3 \\ z''' = z_3 \end{Bmatrix} = \left[A_{R_1}\right] \begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} = \left[A_{R_1}\right]\left[A_{R_2}\right]\left[A_{R_3}\right] \begin{Bmatrix} \xi_3(0) \\ \eta_3(0) \\ \varsigma_3(0) \end{Bmatrix} = \left[A_{0,3}\right] \begin{Bmatrix} \xi_3 \\ \eta_3 \\ \varsigma_3 \end{Bmatrix}$$

$\left[A_{0,3}\right]$ složena matrica transformacije

$$\left[A_{0,3}\right] = \left[A_{R_1}\right]\left[A_{R_2}\right]\left[A_{R_3}\right]$$

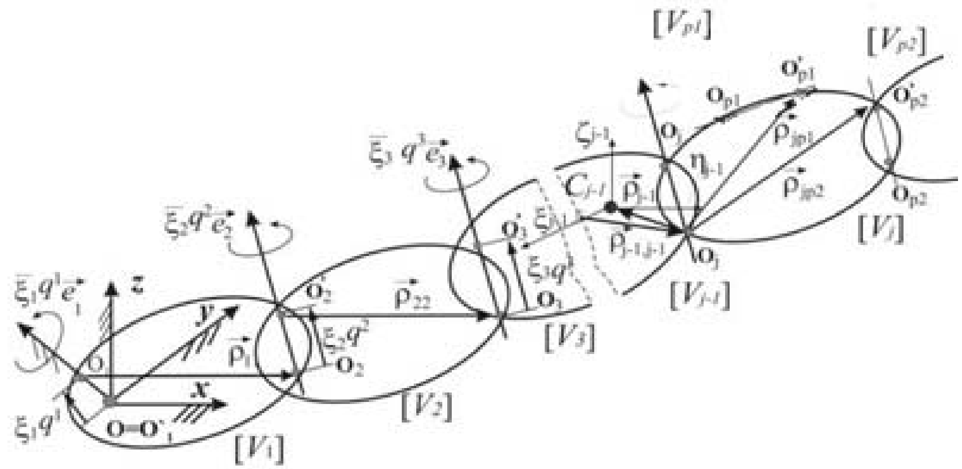
Основне геометријске карактеристике



Геометрија система крутих тела

- Врсте кинематских парова

$$\xi_i + \bar{\xi}_i = 1 \quad i = 1,2\ldots n.$$



- Кар. вектори

$$\vec{\rho}_{ii} = \overrightarrow{O_iO_{i+1}}$$

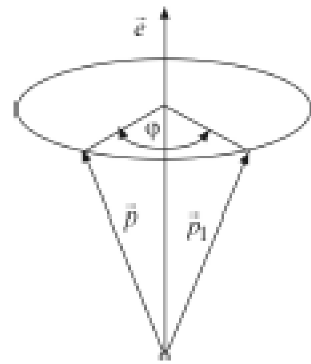
$$\vec{\rho}_i = \overrightarrow{O_{i+1}C_i} \qquad \vec{R}_i = \overrightarrow{O_iC_i} = \sum_{\alpha=1}^{i}\left(\vec{\rho}_{\alpha\alpha} + \xi_\alpha q_\alpha \vec{e}_\alpha\right) + \vec{\rho}_i$$

- Положај врха хватаљке

$$\vec{R}_H = \overrightarrow{O_iH} = \vec{R}_n - \vec{\rho}_n = \sum_{\alpha=1}^{i}\left(\vec{\rho}_{\alpha\alpha} + \xi_\alpha q_\alpha \vec{e}_\alpha\right)$$

Пример

Primer. *Poznat je vektor $\bar{p} = (1 \quad 0 \quad 2)^T$. Odrediti vektor $\bar{p}_1$ koji se dobija rotacijom vektora $\bar{p}$ oko ose*



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koja je određena jediničnim vektorom $\bar{e} = (1/\sqrt{3} \quad -1/\sqrt{3} \quad 1/\sqrt{3})^T$, za ugao $\varphi = 60^\circ$.

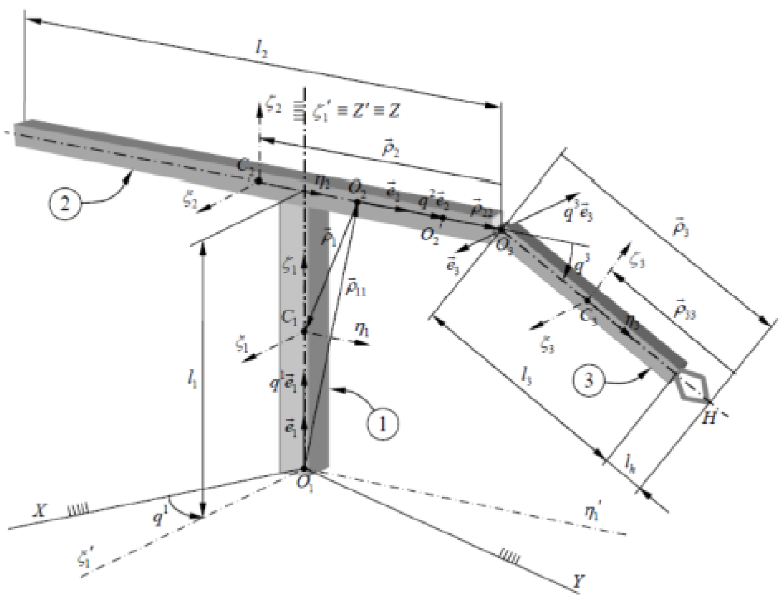
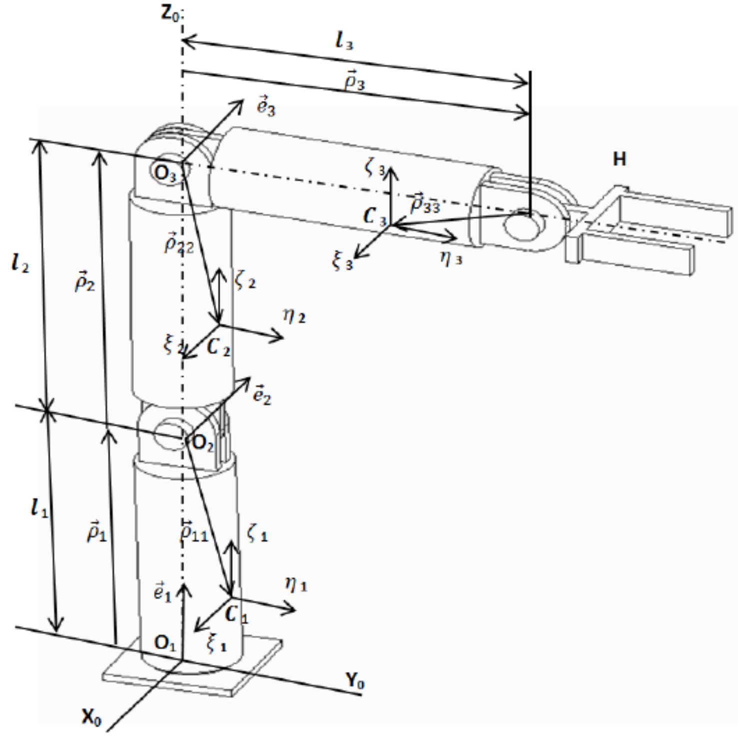
Na osnovu Rodrigovog obrasca ima se:

$$\bar{p}_1 = \bar{p} + (1-\cos\varphi)\bar{e} \times (\bar{e} \times \bar{p}) + (\sin\varphi)\bar{e} \times \bar{p} \qquad (3.7)$$

gde su:

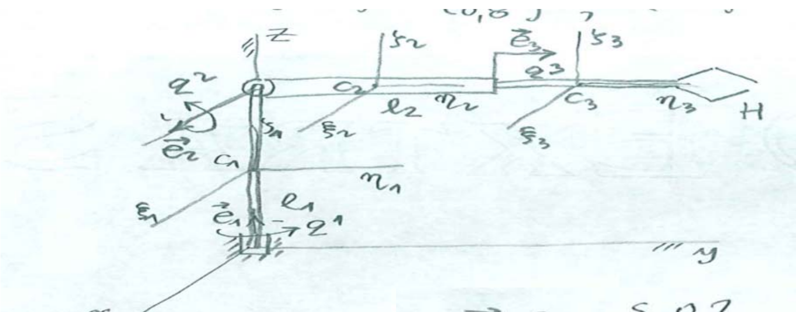
$$\bar{e} \times \bar{p} = \begin{bmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \\ 1 & 0 & 2 \end{bmatrix} = \frac{1}{\sqrt{3}}(-2\bar{i} - \bar{j} + \bar{k}), \quad \bar{e} \times (\bar{e} \times \bar{p}) = \begin{bmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \\ -2/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} = -\bar{j} - \bar{k}$$

$$\bar{p}_1 = \bar{i} + 2\bar{k} + (1 - \cos(\pi/3))(-\bar{j} - \bar{k}) + \sin(\pi/3)\left(\frac{1}{\sqrt{3}}(-2\bar{i} - \bar{j} + \bar{k})\right) = -\bar{j} + 2\bar{k} = (0 \quad -1 \quad 2)^T$$

За дати роботски систем са три степена слободе који се налази у референтном положају, решити директни кинематички задатак, тј. одредити спољашње координате врха хватаљке $\bar{q}^i$ ($i=1,2,3$ – базни случај позиционирања) ако су вредности унутрашњих координата познате: $q^1=0{,}2$ rad, $q^2=0{,}4$ rad, $q^3=0{,}6$ m. Такође је познато:

$$\{\vec{\rho}_{11}\}=\begin{Bmatrix}0\\0\\0{,}8\end{Bmatrix},\quad \{\vec{\rho}_{22}\}=\begin{Bmatrix}0\\0{,}6\\0\end{Bmatrix},\quad \{\vec{\rho}_{33}\}=\begin{Bmatrix}0\\0{,}4\\0\end{Bmatrix}.$$



$\bar{q}^1=x_H$, $\bar{q}^2=y_H$, $\bar{q}^3=z_H$ → спољашње координате које одређују позицију врха хватаљке у односу на непокретни систем $Oxyz$!

$$\{\vec{e}_1\}=\begin{Bmatrix}0\\0\\1\end{Bmatrix},\quad \{\vec{e}_2\}=\begin{Bmatrix}1\\0\\0\end{Bmatrix},\quad \{\vec{e}_3\}=\begin{Bmatrix}0\\1\\0\end{Bmatrix}$$

Положај врха хватаљке у систему $Oxyz$:

$$\begin{Bmatrix}x_H\\y_H\\z_H\end{Bmatrix}=\sum_{j=1}^{3}[A_{0,j}]\left(\{\vec{\rho}_{jj}+\xi_j q^j\vec{e}_j\}\right)=[A_{0,1}]\{\vec{\rho}_{11}\}+[A_{0,2}]\{\vec{\rho}_{22}\}+$$
$$+[A_{0,3}]\left(\{\vec{\rho}_{33}+q^3\vec{e}_3\}\right)$$

$$[A_{0,1}] = \begin{bmatrix} \cos q^1 & -\sin q^1 & 0 \\ \sin q^1 & \cos q^1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad [A_{1,2}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q^2 & -\sin q^2 \\ 0 & \sin q^2 & \cos q^2 \end{bmatrix}, \quad [A_{2,3}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A_{0,2}] = [A_{0,1}][A_{1,2}] = \begin{bmatrix} \cos q^1 & -\sin q^1 \cos q^2 & \sin q^1 \sin q^2 \\ \sin q^1 & \cos q^1 \cos q^2 & -\cos q^1 \sin q^2 \\ 0 & \sin q^2 & \cos q^2 \end{bmatrix}$$

$$[A_{0,3}] = [A_{0,2}][A_{2,3}] = [A_{0,2}][I] = [A_{0,2}]$$

$$\begin{Bmatrix} x_H \\ y_H \\ z_H \end{Bmatrix} = \begin{Bmatrix} -(1+q^3)\sin q^1 \cos q^2 \\ (1+q^3)\cos q^1 \cos q^2 \\ 0{,}8 + (1+q^3)\sin q^2 \end{Bmatrix}_{\substack{q^1=0{,}2 \\ q^2=0{,}4 \\ q^3=0{,}6}} = \begin{Bmatrix} -0{,}3 \\ 1{,}44 \\ 1{,}42 \end{Bmatrix} [m].$$