4.2 Brzina centra inercije krutog tela (V_i)

Vektor položaja centra inercije C_i krutog tela (V_i) u odnosu na nepokretnu tačku O (vidi sl.4.1) odredjen je relacijom:

$$\overrightarrow{OC_i} = \vec{r}_i = \sum_{k=1}^{i}\left(\vec{\rho}_k + \xi_k \vec{e}_k q^k\right) + \vec{\tau}_i \,. \tag{4.22}$$

Prethodna relacija ukazuje na činjenicu da važi:

$$\vec{r}_i = \vec{r}_i\left(q^1, q^2, ..., q^i\right), \tag{4.23}$$

odakle je

$$\vec{v}_{Ci} = \vec{v}_i = \sum_{\alpha=1}^{i} \vec{T}_{\alpha(i)} \dot{q}^{\alpha} \tag{4.24}$$

gde je $\vec{T}_{\alpha(i)} = \partial \vec{r}_i / \partial q^{\alpha}$ pošto je prema (4.11)

$$\begin{aligned} &\frac{\partial \vec{\rho}_k}{\partial q^{\alpha}} = \bar{\xi}_{\alpha} \vec{e}_{\alpha} \times \vec{\rho}_k \quad \forall \alpha \le k, \\ &\frac{\partial \vec{\rho}_k}{\partial q^{\alpha}} = 0 \qquad\qquad \forall \alpha > k. \\ &\frac{\partial \vec{e}_k}{\partial q^{\alpha}} = \bar{\xi}_{\alpha} \vec{e}_{\alpha} \times \vec{e}_k \quad \forall \alpha \le k, \\ &\frac{\partial \vec{e}_k}{\partial q^{\alpha}} = 0 \qquad\qquad \forall \alpha > k. \end{aligned} , \tag{4.25}$$

sledi (vidi (4.22)):

$$\begin{aligned} &\vec{T}_{\alpha(i)} = \bar{\xi}_{\alpha} \vec{e}_{\alpha} \times \left[\sum_{k=\alpha}^{i}\left(\vec{\rho}_k + \xi_k \vec{e}_k q^k\right) + \vec{\tau}_i\right] + \xi_{\alpha} \vec{e}_{\alpha} \quad \forall \alpha \le i \\ &\vec{T}_{\alpha(i)} = 0 \qquad\qquad\qquad\qquad\qquad \forall \alpha > i \end{aligned} , \tag{4.26}$$

i

$$\vec{v}_i = \sum_{\alpha=1}^{n} \vec{T}_{\alpha(i)} \dot{q}^{\alpha} \,, \tag{4.27}$$

Poslednji izraz u matričnoj formi* ima oblik:

---

* Gornji indeks (i) označava da su koordinate odgovarajućeg vektora date u odnosu na lokalni koordinatni sistem $C_i \xi_i \eta_i \zeta_i$ vezan za (Vi). Specijalno, kada je $i = 0$ u pitanju su koordinate date u odnosu na nepokretni koordinatni sistem *Oxyz*. Kada je u pitanju

$$\left\{\vec{v}_i^{(0)}\right\} = \sum_{\alpha=1}^{n} \left\{\vec{T}_{\alpha(i)}^{(0)}\right\} \dot{q}^{\alpha}, \tag{4.28}$$

gde je (vidi (4.25)):

$$\left\{\vec{T}_{\alpha(i)}^{(0)}\right\} = \bar{\xi}_{\alpha}\left[A_{0,\alpha}\right]\left[e_{\alpha}^{d}\right]\left\{\sum_{k=\alpha}^{i}\left[A_{\alpha,k}\right]\left(\left\{\vec{\rho}_k\right\} + \xi_k q^k \left\{\vec{e}_k\right\}\right) + \left[A_{\alpha,i}\right]\left\{\vec{\tau}_i\right\}\right\} +$$
$$+ \xi_{\alpha}\left[A_{0,\alpha}\right]\left\{\vec{e}_{\alpha}\right\} \qquad \forall \alpha \le i, \tag{4.29}$$
$$\left\{\vec{T}_{\alpha(i)}^{(0)}\right\} = 0 \qquad \forall \alpha > i$$

Matrični zapis izraza (4.27) može da se dovede i na oblik:

$$\left\{\vec{v}_i^{(0)}\right\} = [E]\{\dot{q}\}, \tag{4.30}$$

gde je

$$[E] \in R^{3\times n} \Rightarrow [E] = \left[\left\{\vec{T}_{1(i)}^{(0)}\right\} \vdots \left\{\vec{T}_{2(i)}^{(0)}\right\} \vdots \ldots \vdots \left\{\vec{T}_{n(i)}^{(0)}\right\}\right], \tag{4.31}$$

$$\{\dot{q}\} \in R^{n\times 1} \Rightarrow \{\dot{q}\}^T = \left(\dot{q}^1, \dot{q}^2, \ldots, \dot{q}^n\right). \tag{4.32}$$

4.2.3 Brzina vrha hvataljke H robotskog sistema

Primer 3. *Izvesti izraz za brzinu vrha hvataljke H robotskog sistema, slika 4.2.*

Položaj vrha hvataljke je odredjen sa:

$$\vec{r}_H = \sum_{\alpha=1}^{n}\left(\vec{\rho}_{\alpha\alpha} + \bar{\xi}_{\alpha}\vec{e}_{\alpha} q^{\alpha}\right), \quad \Rightarrow \vec{r}_H = \vec{r}_H\left(q^1, q^2, \ldots, q^n\right), \tag{4.25}$$

tako da je, primenom iste procedure kao i u prethodnom primeru dobija

$$\vec{v}_H = \frac{d\vec{r}_H}{dt} = \sum_{\beta=1}^{n}\frac{\partial \vec{r}_H}{\partial q^{\beta}}\dot{q}^{\beta} = \sum_{\beta=1}^{n}\vec{\tau}_{\beta(n)}\dot{q}^{\beta}, \tag{4.26}$$

---

vektor vezan za telo (Vi) (na primer $\vec{\rho}_i = \overrightarrow{A_i B_i}$, $A_i, B_i \in (V_i)$) tada u matričnom zapisu može da se izostavi gornji indeks (i) ako se radi o koordinatama toga vektora, datim u odnosu na $C_i\xi_i\eta_i\zeta_i$ (tj. o koordinatama datim u odnosu na lokalni koordinatni sistem za koje je vezan pomenuti vektor). Ova primedba se odnosi i na dualne objekte tih vektora.

gde je $\vec{\tau}_{\beta(n)} = \dfrac{\partial \vec{r}_H}{\partial q^\beta}$ i on se razlikuje od kvazibaznog vektora $\vec{T}_{\beta(n)}$ po odsustvu vektora $\vec{\rho}_n$ i njegovih izvoda što se vidi iz sledećih izraza:

$$\vec{T}_{\beta(n)} = \bar{\xi}_\beta \vec{e}_\beta \times \left(\sum_{\alpha=\beta}^{n} \left(\vec{\rho}_{\alpha\alpha} + \xi_\alpha \vec{e}_\alpha q^\alpha \right) + \vec{\rho}_n \right) + \xi_\beta \vec{e}_\beta \,, \tag{4.27}$$

$$\vec{\tau}_{\beta(n)} = \bar{\xi}_\beta \vec{e}_\beta \times \left(\sum_{\alpha=\beta}^{n} \left(\vec{\rho}_{\alpha\alpha} + \xi_\alpha \vec{e}_\alpha q^\alpha \right) \right) + \xi_\beta \vec{e}_\beta \,. \tag{4.28}$$

4.3 Ubrzanje centra inercije krutog tela (V_i)

Ako se izraz (4.27) diferencira po vremenu dobijamo:

$$\vec{a}_{Ci} = \vec{a}_i = \sum_{\alpha=1}^{i} \vec{T}_{\alpha(i)} \ddot{q}^\alpha + \sum_{\alpha=1}^{i} \frac{d\vec{T}_{\alpha(i)}}{dt} \dot{q}^\alpha \,, \tag{4.33}$$

Kako je $\vec{T}_{\alpha(i)} = \vec{T}_{\alpha(i)}\left(q^1, q^2, ..., q^i\right)$ sledi:

$$\frac{d\vec{T}_{\alpha(i)}}{dt} = \sum_{\beta=1}^{i} \frac{\partial \vec{T}_{\alpha(i)}}{\partial q^\beta} \dot{q}^\beta \,. \tag{4.34}$$

Kako je $\vec{T}_{\alpha(i)} = \partial \vec{r}_i / \partial q^\alpha$ (uz ispunjenje poznatih uslova o neprekidnosti i diferenijabilnosti funkcije $\vec{r}_i = \vec{r}_i(t),\ i = 1,2,...,n$) očigledno sledi:

$$\frac{\partial \vec{T}_{\alpha(i)}}{\partial q^\beta} = \frac{\partial^2 \vec{r}_i}{\partial q^\alpha \partial q^\beta} \Rightarrow \frac{\partial \vec{T}_{\alpha(i)}}{\partial q^\beta} = \frac{\partial \vec{T}_{\beta(i)}}{\partial q^\alpha} \,, \tag{4.35}$$

tako da je izraz (4.33) moguće dovesti na oblik:

$$\vec{a}_i = \sum_{\alpha=1}^{i} \vec{T}_{\alpha(i)} \ddot{q}^\alpha + \sum_{\alpha=1}^{i} \sum_{\beta=1}^{i} \vec{\Gamma}_{\alpha\beta(i)} \dot{q}^\alpha \dot{q}^\beta \,, \tag{4.36}$$

gde je

$$\vec{\Gamma}_{\alpha\beta(i)} = \frac{\partial \vec{T}_{\alpha(i)}}{\partial q^\beta} \,, \tag{4.37}$$

i

$$\vec{\Gamma}_{\alpha\beta(i)} = \vec{\Gamma}_{\beta\alpha(i)} \,, \tag{4.38}$$

U slučaju $\alpha \le \beta$ sledi (vidi (4.37) i (4.26)):

$$\vec{\Gamma}_{\alpha\beta(i)} = \bar{\xi}_\alpha \vec{e}_\alpha \times \frac{\partial}{\partial q^\beta}\left[\sum_{k=\alpha}^{i}\left(\vec{\rho}_k + \xi_k \vec{e}_k q^k\right) + \vec{\tau}_i\right], \tag{4.39}$$

očigledno je da važi:

$$\frac{\partial}{\partial q^\beta}\left[\sum_{k=\alpha}^{i}\left(\vec{\rho}_k + \xi_k \vec{e}_k q^k\right) + \vec{\tau}_i\right] = \frac{\partial}{\partial q^\beta}\left[\sum_{k=1}^{i}\left(\vec{\rho}_k + \xi_k \vec{e}_k q^k\right) + \vec{\tau}_i\right], \tag{4.40}$$

ili (vidi (4.22) i imajući u vidu $\vec{T}_{\alpha(i)} = \partial \vec{r}_i / \partial q^\alpha$)

$$\frac{\partial}{\partial q^\beta}\left[\sum_{k=\alpha}^{i}\left(\vec{\rho}_k + \xi_k \vec{e}_k q^k\right) + \vec{\tau}_i\right] = \vec{T}_{\beta(i)} \,. \tag{4.41}$$

Poslednji izraz daje relaciji (4.39) oblik:

$$\vec{\Gamma}_{\alpha\beta(i)} = \bar{\xi}_\alpha \vec{e}_\alpha \times \vec{T}_{\beta(i)} \quad \forall \ \alpha \le \beta \,. \tag{4.42}$$

U slučaju $\alpha > \beta$ sledi(vidi (4.37), (4.11))

$$\begin{aligned}\vec{\Gamma}_{\alpha\beta(i)} = \bar{\xi}_\alpha \bar{\xi}_\beta \left(\vec{e}_\beta \times \vec{e}_\alpha\right) \times \left[\sum_{k=\alpha}^{i}\left(\vec{\rho}_k + \xi_k \vec{e}_k q^k\right) + \vec{\tau}_i\right] + \\ + \bar{\xi}_\alpha \bar{\xi}_\beta \left(\vec{e}_\beta \times \vec{e}_\alpha\right) + \bar{\xi}_\alpha \vec{e}_\alpha \times \left\{\bar{\xi}_\beta \vec{e}_\beta \times \left[\sum_{k=\alpha}^{i}\left(\vec{\rho}_k + \xi_k \vec{e}_k q^k\right) + \vec{\tau}_i\right]\right\}\end{aligned}, \tag{4.43}$$

što se nakon razvijanja dvostrukih vektorskih proizvoda može dovesti na oblik:

$$\vec{\Gamma}_{\alpha\beta(i)} = \bar{\xi}_\beta \vec{e}_\beta \times \left\{\bar{\xi}_\alpha \vec{e}_\alpha \times \left[\sum_{k=\alpha}^{i}\left(\vec{\rho}_k + \xi_k \vec{e}_k q^k\right) + \vec{\tau}_i\right] + \xi_\alpha \vec{e}_\alpha\right\}, \tag{4.44}$$

ili, prema (4.26):

$$\vec{\Gamma}_{\alpha\beta(i)} = \bar{\xi}_\beta \vec{e}_\beta \times \vec{T}_{\alpha(i)} \quad \forall \ \alpha > \beta \,. \tag{4.45}$$

Prema (4.38) sledi da se pri sračunavanju vektora (4.37) može da koristi ili izraz (4.42) ili (4.45) a potom uzme u obzir činjenica da važi (4.38) (na primer, ako se odredjuje $\vec{\Gamma}_{53(11)}$ može se iskoristiti izraz (4.42) prema kome se odredjuje $\vec{\Gamma}_{53(11)}$ i potom uzme u obzir činjenica da važi: $\vec{\Gamma}_{53(11)} = \vec{\Gamma}_{35(11)}$). Izraz (4.36) može da se napiše i u obliku:

$$\vec{a}_i = \sum_{\alpha=1}^{n} \vec{T}_{\alpha(i)} \ddot{q}^\alpha + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \vec{\Gamma}_{\alpha\beta(i)} \dot{q}^\alpha \dot{q}^\beta , \tag{4.46}$$

Pošto važi (vidi (4.26))

$$\vec{T}_{\alpha(i)} = 0 \quad \forall \alpha > i , \tag{4.47}$$

i* (vidi (4.35)) i (4.37))):

$$\vec{\Gamma}_{\alpha\beta(i)} = 0 \quad \forall \ \sup(\alpha,\beta) > i \,, \tag{4.48}$$

Očigledno je da izrazi (4.42)) i (4.45) mogu da se napišu u jedinstvenoj formi

$$\vec{\Gamma}_{\alpha\beta(i)} = \bar{\xi}_a \vec{e}_a \times \vec{T}_{b(i)} \quad , \tag{4.49}$$

gde je**

$$a = \inf(\alpha,\beta), \quad b = \sup(\alpha,\beta)\,, \tag{4.50}$$

U matričnoj formi izraz (4.46) ima oblik:

$$\left\{\vec{a}_i^{(0)}\right\} = \sum_{\alpha=1}^{n} \left\{\vec{T}_{\alpha(i)}^{(0)}\right\} \ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \left\{\vec{\Gamma}_{\alpha\beta(i)}^{(0)}\right\} \dot{q}^{\alpha}\dot{q}^{\beta}, \tag{4.51}$$

gde je (uzimamo da je $\vec{\Gamma}_{\alpha\beta(i)}$ sračunat prema (4.42) a u slučaju $\alpha > \beta$ iskorišćena je osobina simetrije po indeksima α i β):

$$\left\{\vec{\Gamma}_{\alpha\beta(i)}^{(0)}\right\} = \bar{\xi}_{\alpha} \left[A_{0,\alpha}\right]\left[e_{\alpha}^{d}\right]\left[A_{\alpha,\beta}\right]\left\{\vec{T}_{\beta(i)}^{(\beta)}\right\}, \tag{4.52}$$

gde je

$$\left\{\vec{T}_{\beta(i)}^{(\beta)}\right\} = \bar{\xi}_{\beta}\left[e_{\beta}^{d}\right]\left\{\sum_{k=\alpha}^{i}\left[A_{\beta,k}\right]\left(\left\{\vec{\rho}_k\right\} + \xi_k q^k \left\{\vec{e}_k\right\}\right) + \left[A_{\beta,i}\right]\vec{\tau}_i\right\} + \xi_{\beta}\left\{\vec{e}_{\beta}\right\}, \tag{4.53}$$

4.4 Ugaona brzina krutog tela (V_i)

Ugaona brzina tela (V_1) koje je posredstvom cilindričnog (prizmatičnog) zgloba (1) (vidi sl.*) vezano za postolje, ima ugaonu brzinu:

$$\vec{\omega}_1 = \bar{\xi}_1 \vec{e}_1 \dot{q}_1 \,. \tag{4.54}$$

Kruto telo (V_2) relativno se kreće u odnosu na telo (V_1) (ili se obrće oko ose zgloba (2) vezane za telo (V_1) ili se translatorno pravolinijski kreće duž ose zgloba (2) vezane za telo (V_1)). Relativna ugaona brzina tela (V_2) u odnosu na (V_1) iznosi:

$$\vec{\omega}_{2r} = \bar{\xi}_2 \vec{e}_2 \dot{q}^2 \,, \tag{4.55}$$

Prema izrazu za apsolutnu brzinu tela (V_2):

---

* $\sup(\alpha,\beta) = \alpha \ \ \forall \alpha \geq \beta; \ \ \sup(\alpha,\beta) = \beta \ \ \forall \beta > \alpha$

** $\inf(\alpha,\beta) = \alpha \ \ \forall \alpha \leq \beta; \ \ \inf(\alpha,\beta) = \beta \ \ \forall \alpha > \beta$

$$\vec{\omega}_2 = \vec{\omega}_{2p} + \vec{\omega}_{2r} .$$

(4.56)

gde je $\vec{\omega}_{2p}$ -prenosna ugaona brzina tela (V_2), uzimajući u obzir da je:

$$\vec{\omega}_{2p} = \vec{\omega}_1 , \qquad (4.57)$$

sledi

$$\vec{\omega}_2 = \bar{\xi}_1 \vec{e}_1 \dot{q}^1 + \bar{\xi}_2 \vec{e}_2 \dot{q}^2 .$$

(4.58)

Polazeći od izraza za apsolutnu ugaonu brzinu tela (V_3):

$$\vec{\omega}_3 = \vec{\omega}_{3p} + \vec{\omega}_{3r} , \qquad (4.59)$$

Koristeći očigledne relacije:

$$\vec{\omega}_{3p} = \vec{\omega}_2 ,\; \vec{\omega}_{3r} = \bar{\xi}_3 \vec{e}_3 \dot{q}^3 ,$$

(4.60)

dobija se:

$$\vec{\omega}_3 = \sum_{k=1}^{3} \bar{\xi}_k \vec{e}_k \dot{q}^k . \qquad (4.61)$$

Uopštavanjem prethodnih rezultata dolazi se do sledećeg izraza za ugaonu brzinu $(\vec{\omega}_i)$ krutog tela (V_i):

$$\vec{\omega}_i = \sum_{k=1}^{i} \bar{\xi}_k \vec{e}_k \dot{q}^k \qquad (4.62)$$

ili

$$\vec{\omega}_i = \sum_{k=1}^{n} \vec{\Omega}_{k(i)} \, \dot{q}^k , \qquad (4.63)$$

u matričnoj formi relacija (4.62) glasi:

$$\left\{ \vec{\omega}_i^{(0)} \right\} = \sum_{k=1}^{i} \bar{\xi}_k \left[A_{0,k} \right] \left\{ \vec{e}_k \right\} \dot{q}^k , \qquad (4.64)$$

ili

$$\left\{ \vec{\omega}_i^{(0)} \right\} = [F]\{\dot{q}\} , \qquad (4.65)$$

gde je

$$[F] \in R^{3\times n} \Rightarrow [F] = \left[\bar{\xi}_1 \left\{ \vec{e}_1^{(0)} \right\} \vdots \bar{\xi}_2 \left\{ \vec{e}_2^{(0)} \right\} \vdots \ldots \vdots \bar{\xi}_i \left\{ \vec{e}_i^{(0)} \right\} \right] , \qquad (4.66)$$

pri čemu je potrebno uzeti u obzir da važi

$$\left\{\vec{e}_k^{(0)}\right\} = \left[A_{0,k}\right]\left\{\vec{e}_k\right\}, \quad k = 1,2,...,i\,. \tag{4.67}$$

4.5 Ugaono ubrzanje krutog tela (V_i)

Diferenciranjem relacije (4.63) po vremenu dobija se izraz za ubrzanje tela (V_i) u obliku:

$$\vec{\varepsilon}_i = \sum_{\beta=1}^{n} \vec{\Omega}_{\beta(i)}\, \ddot{q}^{\beta} + \sum_{\beta=1}^{n}\sum_{\alpha=1}^{n} \vec{\Lambda}_{\beta\alpha(i)} \dot{q}^{\beta} \dot{q}^{\alpha}\ , \tag{4.68}$$

gde je

$$\vec{\Lambda}_{\beta\alpha(i)} = \frac{\partial \vec{\Omega}_{\beta(i)}}{\partial q^{\alpha}}, \tag{4.69}$$

odnosno sledi:

$$\vec{\Lambda}_{\beta\alpha(i)} = \bar{\xi}_{\alpha} \vec{e}_{\alpha} \times \vec{\Omega}_{\beta(i)} \Rightarrow \vec{\Lambda}_{\beta\alpha(i)} = \vec{\Omega}_{\alpha(\beta)} \times \vec{\Omega}_{\beta(i)}, \tag{4.70}$$

odakle zaključujemo da važi

$$\vec{\Lambda}_{\beta\alpha(i)} = 0 \quad \forall\ \alpha > \beta\,, \tag{4.71}$$

i, takodje,

$$\vec{\Lambda}_{\beta\alpha(i)} = 0 \quad \forall\ \beta > i\,. \tag{4.72}$$

Koristeći (4.70) i (4.71) izraz (4.68) možemo da dovedemo na oblik:

$$\vec{\varepsilon}_i = \sum_{\beta=1}^{i} \vec{\Omega}_{\beta(i)}\, \ddot{q}^{\beta} + \sum_{\beta=1}^{i}\sum_{\alpha=1}^{\beta} \vec{\Omega}_{\alpha(\beta)} \times \vec{\Omega}_{\beta(i)} \dot{q}^{\beta} \dot{q}^{\alpha}\ , \tag{4.73}$$

ili

$$\vec{\varepsilon}_i = \sum_{\beta=1}^{i} \vec{\Omega}_{\beta(i)}\, \ddot{q}^{\beta} + \sum_{\alpha=1}^{i}\sum_{\beta=1}^{i} \vec{\Omega}_{\alpha(\beta)} \times \vec{\Omega}_{\beta(i)} \dot{q}^{\alpha} \dot{q}^{\beta}\ , \tag{4.74}$$

odakle sledi:

$$\vec{\varepsilon}_i = \sum_{\beta=1}^{i} \vec{\Omega}_{\beta(i)}\, \ddot{q}^{\beta} + \sum_{\alpha=1}^{i}\sum_{\beta=\alpha}^{i} \vec{\Omega}_{\alpha(\beta)} \times \vec{\Omega}_{\beta(i)} \dot{q}^{\alpha} \dot{q}^{\beta}\ , \tag{4.75}$$

ili

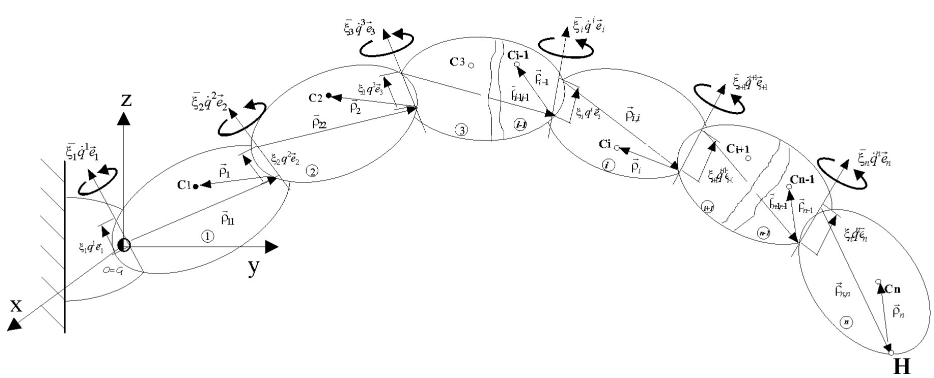
$$\vec{\varepsilon}_i = \sum_{\beta=1}^{i} \bar{\xi}_{\beta} \vec{e}_{\beta}\, \ddot{q}^{\beta} + \sum_{\alpha=1}^{i}\sum_{\beta=\alpha}^{i} \bar{\xi}_{\alpha} \bar{\xi}_{\beta} \vec{e}_{\alpha} \times \vec{e}_{\beta} \dot{q}^{\alpha} \dot{q}^{\beta}, \tag{4.76}$$

Poslednja relacija u matričnoj formi glasi:

$$\left\{\vec{\varepsilon}_i^{(0)}\right\} = \sum_{\beta=1}^{i} \bar{\xi}_\beta \left[A_{0,\beta}\right]\left\{\vec{e}_\beta\right\}\ddot{q}^\beta + \\ + \sum_{\alpha=1}^{i}\sum_{\beta=\alpha}^{i} \bar{\xi}_\alpha \bar{\xi}_\beta \left[A_{0,\alpha}\right]\left[e_\alpha^d\right]\left[A_{\alpha,\beta}\right]\left\{\vec{e}_\beta\right\}\dot{q}^\alpha \dot{q}^\beta, \tag{4.77}$$

Primer za vezbanje. *Izvesti izraz za ugaonu brzinu centra inercije i-tog robotskog segmenta* (V_i) *a zatim za robotski sistem sa pet stepeni slobode odrediti izraz za ugaonu brzinu poslednjeg –petog segmenta.*

U odnosu na nepokretno postolje imamo apsolutno obrtanje samo u slučaju prvog segmenta, gde je q^1 apsolutni ugao obrtanja.Prema tome sa $\vec{\omega}_1$ je označena apsolutna ugaona brzina obrtanja prvog segmenta (4.1).



Slika 4.4a

$$\vec{\omega}_1 = \bar{\xi}_1 \vec{e}_1 \dot{q}^1 \tag{4.1}$$

Parametar ξ koji određuje da li je uočeni zglob translatoran (T) ili rotacioni (R) je sada prema definiciji:

$$\xi_i = \begin{cases} 0, & rotacioni\, zglob \\ 1, & translatorni\, zglob \end{cases} \quad i = 1,2,3...,n \tag{4.2}$$

odnosno:

$$\bar{\xi}_i = 1 - \xi_i,\ i = 1,2,3,...n \tag{4.3}$$

Telo (V_1) vrši prenosno kretanje koje je ili translatorno ili rotaciono a telo (V_2) relativno u odnosu na (V_1) tj.:

$$\vec{\omega}_2 = \vec{\omega}_{2p} + \vec{\omega}_{2r}, \quad \vec{\omega}_{2p} = \vec{\omega}_1, \quad \vec{\omega}_{2r} = \overline{\xi}_2 \dot{q}^2 \vec{e}_2, \tag{4.4}$$

$$\vec{\omega}_2 = \overline{\xi}_1 \vec{e}_1 \dot{q}^1 + \overline{\xi}_2 \dot{q}^2 \vec{e}_2 \tag{4.5}$$

Na sličan način kao u slučaju drugog segmenta može se uočiti da i za treći segment važi:

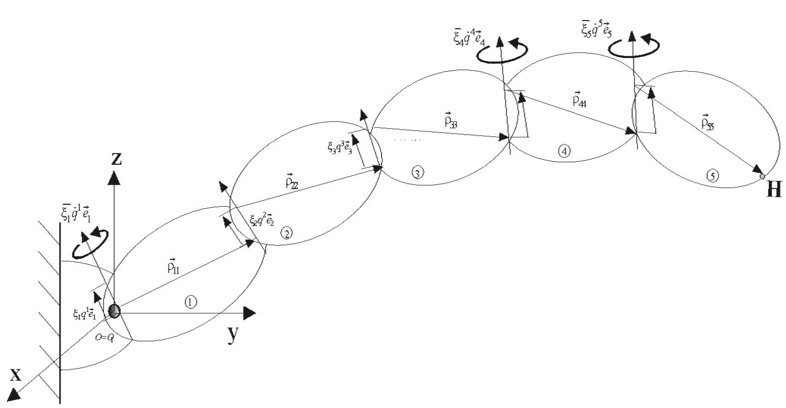
$$\vec{\omega}_3 = \vec{\omega}_{3p} + \vec{\omega}_{3r}, \quad \vec{\omega}_{3p} = \vec{\omega}_2, \quad \vec{\omega}_{3r} = \overline{\xi}_3 \dot{q}^3 \vec{e}_3,$$

(4.6)

odnosno imajući u vidu relaciju (4.5) proizilazi da je:

$$\vec{\omega}_3 = \overline{\xi}_1 \vec{e}_1 \dot{q}^1 + \overline{\xi}_2 \dot{q}^2 \vec{e}_2 + \overline{\xi}_3 \dot{q}^3 \vec{e}_3 \tag{4.7}$$

$$\vec{\omega}_i = \sum_{\alpha=1}^{i} \overline{\xi}_\alpha \vec{e}_\alpha \dot{q}^\alpha \tag{4.8}$$



Slika 4.4b

U konkretnom slučaju za dati robotski sistem sa zglobovima RTTRR tj. imajući u vidu (4.2) i (4.3) dobija se $\overline{\xi}_1 = 1$, $\overline{\xi}_2 = 0$, $\overline{\xi}_3 = 0$, $\overline{\xi}_4 = 1$, $\overline{\xi}_5 = 1$ odnosno na osnovu izraza (4.8) dobija se:

$$\vec{\omega}_5 = \dot{q}^1 \vec{e}_1 + \dot{q}^4 \vec{e}_4 + \dot{q}^5 \vec{e}_5 \tag{4.9}$$