

## Promena pritiska u dvofaznom toku

Bilans količine kretanja prema modelu dva fluida

- bilans količine kretanja za tečnu fazu

$$\frac{\partial(\alpha_1 \rho_1 u_1)}{\partial t} + \frac{\partial(\alpha_1 \rho_1 u_1^2)}{\partial z} = -\alpha_1 \frac{\partial p}{\partial z} + a_{21} \tau_{21} - a_{1z} \tau_{1z} - \alpha_1 \rho_1 g \sin \theta + \Gamma_{21} u_2 - \Gamma_{12} u_1 \quad (1)$$

- bilans količine kretanja za gasnu fazu

$$\frac{\partial(\alpha_2 \rho_2 u_2)}{\partial t} + \frac{\partial(\alpha_2 \rho_2 u_2^2)}{\partial z} = -\alpha_2 \frac{\partial p}{\partial z} - a_{21} \tau_{21} - a_{2z} \tau_{2z} - \alpha_2 \rho_2 g \sin \theta - \Gamma_{21} u_2 + \Gamma_{12} u_1 \quad (2)$$

Sabiranjem jednačina (1) i (2) za stacionarne uslove  $\left(\frac{\partial}{\partial t} = 0\right)$  i korišćenjem relacije

$$\alpha_1 + \alpha_2 = 1$$

$$\frac{dp}{dz} = -(a_{1z} \tau_{1z} + a_{2z} \tau_{2z}) - (\alpha_1 \rho_1 + \alpha_2 \rho_2) g \sin \theta - \frac{d}{dz} (\alpha_1 \rho_1 u_1^2 + \alpha_2 \rho_2 u_2^2)$$

$$\rho = \alpha_1 \rho_1 + \alpha_2 \rho_2$$

$$u_1 = \frac{(1-\chi)G}{\alpha_1 \rho_1} \Rightarrow \alpha_1 \rho_1 u_1^2 = \frac{(1-\chi)^2 G^2}{\alpha_1 \rho_1}$$

$$u_2 = \frac{\chi G}{\alpha_2 \rho_2} \Rightarrow \alpha_2 \rho_2 u_2^2 = \frac{\chi^2 G^2}{\alpha_2 \rho_2}$$

$$\frac{dp}{dz} = -(a_{1z} \tau_{1z} + a_{2z} \tau_{2z}) - \rho g \sin \theta - G^2 \frac{d}{dz} \left( \frac{(1-\chi)^2}{\alpha_1 \rho_1} + \frac{\chi^2}{\alpha_2 \rho_2} \right)$$

$$\frac{dp}{dz} = \left( \frac{dp}{dz} \right)_{tr} + \left( \frac{dp}{dz} \right)_g + \left( \frac{dp}{dz} \right)_{ak}$$

Ukupna promena pritiska usled:

- gravitacije,
- trenja na zidovima,
- ubrzanja (akceleracije).

Međufazno trenje ne utiče direktno na promenu pritiska, ali utiče posredno jer međufazno trenje utiče na strukturu dvofaznog toka i  $\alpha_k$ , a samim tim in a ostale članove u gornjoj jednačini.

Za homogeno strujanje  $u_1 = u_2$  i  $\chi = x$ , i može se napisati

$$a_{1z} \tau_{1z} + a_{2z} \tau_{2z} = (a\tau)_{tr}$$

$$\text{Pošto je po definiciji } x = \frac{\rho_2 V_2}{\rho_1 V_1 + \rho_2 V_2} = \frac{\alpha_2 \rho_2}{\alpha_1 \rho_1 + \alpha_2 \rho_2}$$

$$1 - x = \frac{\rho_1 V_1}{\rho_1 V_1 + \rho_2 V_2} = \frac{\alpha_1 \rho_1}{\alpha_1 \rho_1 + \alpha_2 \rho_2}$$

jednačina za jediničnu promenu pritiska se može napisati u sledećem obliku

$$\frac{dp}{dz} = -(\alpha\tau)_{tr} - \rho g \sin \theta - G^2 \frac{d}{dz} \left( \frac{1}{\rho} \right)$$

što predstavlja promenu pritiska kako za homogeno dvofazno strujanje, tako i za jednofazno strujanje.

Množitelj pada pritiska usled trenja u dvofaznom toku,  $\Phi^2$

$$\left( -\frac{dp}{dz} \right)_{tr,DT} = (\alpha\tau)_{tr} = \frac{4}{D} f \frac{\rho u^2}{2} = \frac{4}{D} f \frac{G^2}{2\rho} = \left( \frac{4}{D} f_{10} \frac{G^2}{2\rho_1} \right) \left( \frac{f}{f_{10}} \frac{\rho_1}{\rho} \right) = \left( -\frac{dp}{dz} \right)_{tr,10} \Phi_{10}^2$$

$$\Phi_{20}^2, \Phi_1^2, \Phi_2^2 \quad f \approx f_{10}, \quad \Phi_{10}^2 = \frac{\rho_1}{\rho} = \frac{v}{v_1} = \left[ 1 + x \left( \frac{v_2}{v_1} - 1 \right) \right]$$

$$\text{npr. } \left( -\frac{dp}{dz} \right)_{tr,DT} = \left( -\frac{dp}{dz} \right)_{tr,1} \Phi_1^2 = \frac{4}{D} f_1 \frac{(1-x)^2 G^2}{2\rho_1} \Phi_1^2$$

Martineli parametar

$$X^2 = \frac{\left( \frac{dp}{dz} \right)_{tr,1}}{\left( \frac{dp}{dz} \right)_{tr,2}}$$

$$\Phi_1^2 = 1 + \frac{C}{X} + \frac{1}{X^2}$$

$$\Phi_2^2 = 1 + CX + X^2$$

novi član koji je karakteristika dvofaznog toka

$$\left( -\frac{dp}{dz} \right)_{tr,DT} = \left( -\frac{dp}{dz} \right)_{tr,1} + C \left[ \left( -\frac{dp}{dz} \right)_{tr,1} \left( -\frac{dp}{dz} \right)_{tr,2} \right]^{1/2} + \left( -\frac{dp}{dz} \right)_{tr,2} \quad (3)$$

$$\left( -\frac{dp}{dz} \right)_{tr,DT} = f \frac{G^2}{2\rho} \frac{4}{D}$$

$$\left( -\frac{dp}{dz} \right)_{tr,1} = f_1 \frac{(1-x)^2 G^2}{2\rho_1} \frac{4}{D}$$

$$\left( -\frac{dp}{dz} \right)_{tr,2} = f_2 \frac{x^2 G^2}{2\rho_2} \frac{4}{D}$$

Ako je  $f \approx f_1 \approx f_2$  tada se iz jednačine (3) dobija

$$\frac{1}{\rho} = \frac{(1-x)^2}{\rho_1} + C \left[ \frac{(1-x)^2}{\rho_1} \frac{x^2}{\rho_2} \right]^{1/2} + \frac{x^2}{\rho_2} \quad (4)$$

$$\frac{1}{\rho} = \frac{1-x}{\rho_1} + \frac{x}{\rho_2}$$

u gornju jednačinu (4) dobija se

$$\frac{1-x}{\rho_1} + \frac{x}{\rho_2} = \frac{(1-x)^2}{\rho_1} + \frac{x^2}{\rho_2} + C \frac{x(1-x)}{(\rho_1 \rho_2)^{1/2}}$$

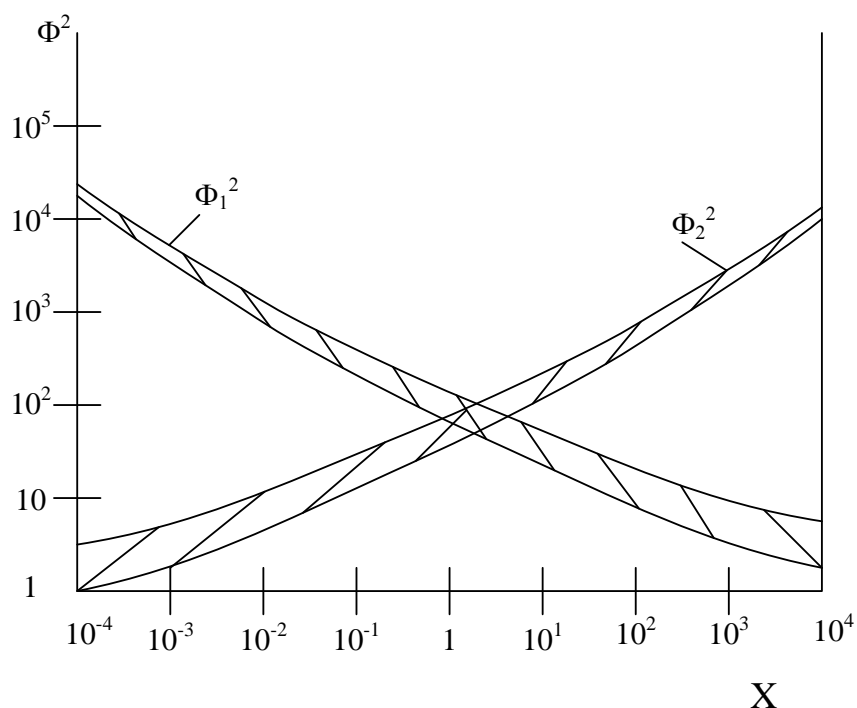
$$\frac{1-x}{\rho_1}(1-1+x) + \frac{x}{\rho_2}(1-x) = C \frac{x(1-x)}{(\rho_1 \rho_2)^{1/2}}$$

$$x(1-x) \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right) = C \frac{x(1-x)}{(\rho_1 \rho_2)^{1/2}}$$

$$C = \left( \frac{\rho_2}{\rho_1} \right)^{1/2} + \left( \frac{\rho_1}{\rho_2} \right)^{1/2}$$

Za strujanje vode i vazduha na atmosferskom pritisku  $C=28,9$ .

#### Eksperimentalna zavisnost Lockhart – Martinelli-a



Dobro se može opisati sa sledećim vrednostima koef. C

tečnost	gas	C
turbulentno	turbulentno	20
laminarno	turbulentno	12
turbulentno	laminarno	10
laminarno	laminarno	5

Za laminarno strujanje ili turbulentno u glatkoj cevi

$$X^2 = \frac{f_1 \frac{(1-x)^2 G^2}{2\rho_1} \frac{4}{D}}{f_2 \frac{x^2 G^2}{2\rho_2} \frac{4}{D}} = \frac{\frac{k}{\text{Re}_1^n} \frac{(1-x)^2}{\rho_1}}{\frac{k}{\text{Re}_2^n} \frac{x^2}{\rho_2}} = \frac{\rho_2}{\rho_1} \frac{x^n \mu_1^n}{(1-x^n) \mu_2^n} \frac{(1-x)^2}{x^2} = \frac{\rho_2}{\rho_1} \left( \frac{\mu_1}{\mu_2} \right)^n \left( \frac{1-x}{x} \right)^{2-n}$$

$$\text{Re}_1 = \frac{\alpha_1 \rho_1 u_1 D}{\mu_1} = \frac{(1-x)GD}{\mu_1}, \text{Re}_2 = \frac{xGD}{\mu_2}$$

za laminarno  $n=1$ , za turbulentno  $n=0,25$ .