Машински факултет
УНИВЕРЗИТЕТА У БЕОГРАДУ

Директан задатак кинематике у механици робота –“ниво координата“

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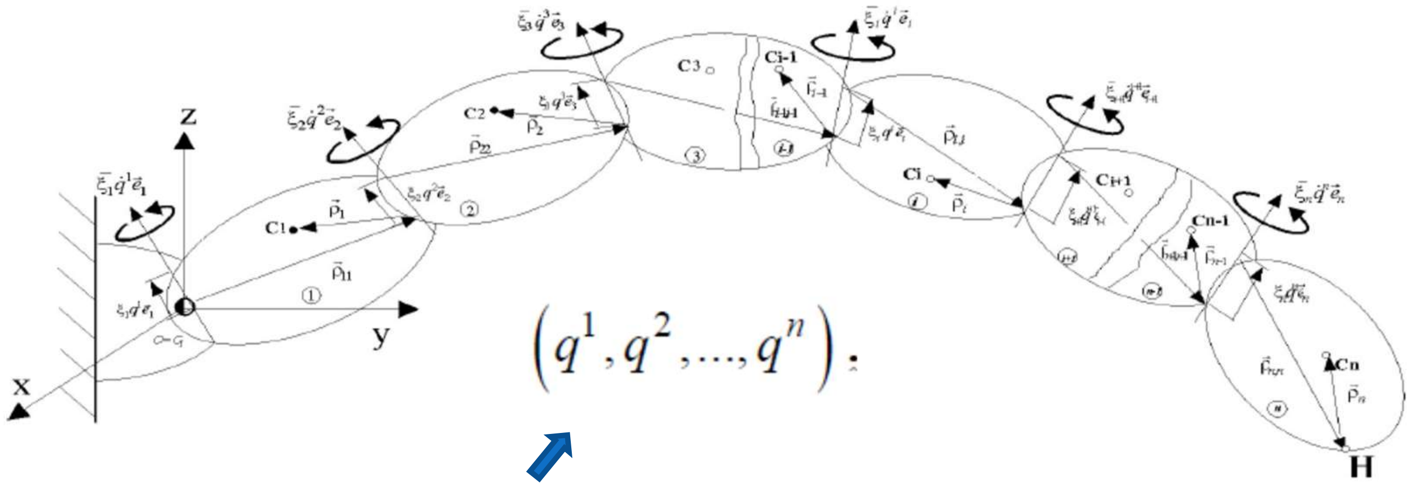
- Спољашње и унутрашње координате роботског система



$$\left(q^1, q^2, \ldots, q^n\right).$$

- спољашње координате роботског система

$$\bar{q}^1, \bar{q}^2, \ldots, \bar{q}^m \qquad m < 6 .$$

- унутрашње координате роботског система

$$H(x_H, y_H, z_H)$$

$$\psi, \theta, \varphi$$

$$H = H\left(\bar{q}^1, \bar{q}^2, \bar{q}^3\right), \quad V_n = V_n\left(\bar{q}^4, \bar{q}^5, \bar{q}^6\right),$$

$$x_H = x_H\left(\bar{q}^1, \bar{q}^2, \bar{q}^3\right), \qquad \psi = \psi\left(\bar{q}^4, \bar{q}^5, \bar{q}^6\right),$$

$$y_H = y_H\left(\bar{q}^1, \bar{q}^2, \bar{q}^3\right), \qquad \theta = \theta\left(\bar{q}^4, \bar{q}^5, \bar{q}^6\right),$$

$$z_H = z_H\left(\bar{q}^1, \bar{q}^2, \bar{q}^3\right). \qquad \varphi = \varphi\left(\bar{q}^4, \bar{q}^5, \bar{q}^6\right).$$

- **Директан кинематички задатак- решавање**

$$\overline{q}^{a} = \overline{q}^{a}\left(q^{1}, q^{2}, \ldots, q^{n}\right), \quad a = 1, 2, \ldots, m,\ m \le 6,$$

- **Инверзан кинематички задатак- решавање**

$$q^{i}(t) = f^{i}(\overline{q}^{1}, \overline{q}^{2}, \overline{q}^{3}, \overline{q}^{4}, \overline{q}^{5}, \overline{q}^{6}), \quad i = 1, 2, , \ldots n$$

- **Директан кинематички задатак- случај позиционирања**

Базни случај -позиционирање

$$\vec{r}_H = \overline{OH} \;\rightarrow \vec{r}_H = \sum_{k=1}^{n}\left(\vec{\rho}_{kk} + \xi_k \vec{e}_k q^k\right)$$

$$\left\{\vec{r}_H^{(0)}\right\} = \sum_{k=1}^{n}\left[A_{o,k}\right]\left(\left\{\vec{\rho}_{kk}^{(k)}\right\} + \xi_k \left\{\vec{e}_k^{(k)}\right\} q^k\right)$$

$$\left\{\vec{\rho}_{kk}^{(k)}\right\} = \left(\rho_{kk1} \quad \rho_{kk2} \quad \rho_{kk3}\right)^T, \; \left\{\vec{e}_k^{(k)}\right\} = \left(e_{k1} \quad e_{k2} \quad e_{k3}\right)^T$$

$$\left\{\vec{r}_H^{(0)}\right\} = \left(x_H \quad y_H \quad z_H\right)^T$$

$$\left[A_{0,k}\right] = \begin{bmatrix} \alpha_{11}^k & \alpha_{12}^k & \alpha_{13}^k \\ \alpha_{21}^k & \alpha_{22}^k & \alpha_{23}^k \\ \alpha_{31}^k & \alpha_{32}^k & \alpha_{33}^k \end{bmatrix}$$

$$x_H = \sum_{k=1}^{n}\left[\alpha_{11}^k\left(\rho_{kk1} + \xi_k e_{k1} q^k\right) + \alpha_{12}^k\left(\rho_{kk2} + \xi_k e_{k2} q^k\right) + \alpha_{13}^k\left(\rho_{kk3} + \xi_k e_{k3} q^k\right)\right]$$

$$y_H = \sum_{k=1}^{n}\left[\alpha_{21}^k\left(\rho_{kk1} + \xi_k e_{k1} q^k\right) + \alpha_{22}^k\left(\rho_{kk2} + \xi_k e_{k2} q^k\right) + \alpha_{23}^k\left(\rho_{kk3} + \xi_k e_{k3} q^k\right)\right]$$

$$z_H = \sum_{k=1}^{n}\left[\alpha_{31}^k\left(\rho_{kk1} + \xi_k e_{k1} q^k\right) + \alpha_{32}^k\left(\rho_{kk2} + \xi_k e_{k2} q^k\right) + \alpha_{33}^k\left(\rho_{kk3} + \xi_k e_{k3} q^k\right)\right]$$

$$\alpha_{ij}^k = \alpha_{ij}\left(q^1, q^2, \ldots, q^n\right), \quad i, j = 1, 2, 3; \quad k = 1, 2, \ldots, n$$

$$x_H = x_H\left(q^1, q^2, \ldots, q^n\right),$$
$$y_H = y_H\left(q^1, q^2, \ldots, q^n\right),$$
$$z_H = z_H\left(q^1, q^2, \ldots, q^n\right)$$

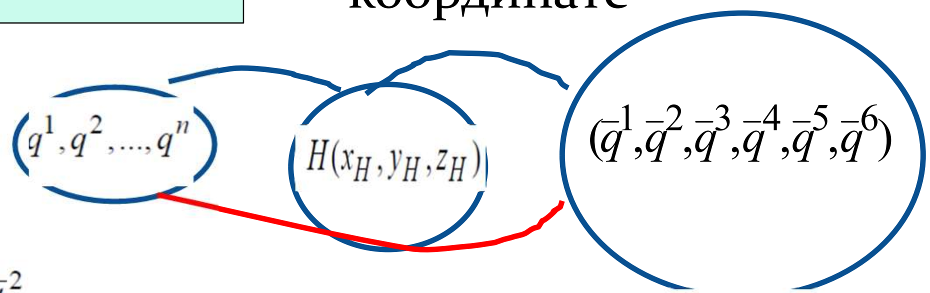
Општи случај -позиционирање

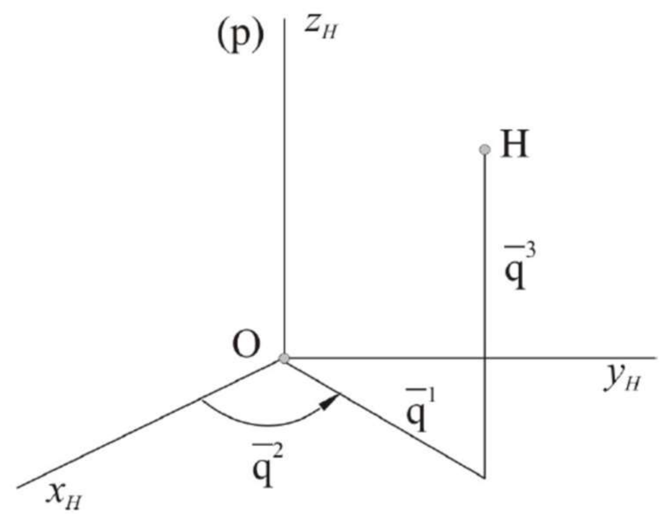
$$x_H = x_H\left(\bar{q}^1, \bar{q}^2, \bar{q}^3\right),$$
$$y_H = y_H\left(\bar{q}^1, \bar{q}^2, \bar{q}^3\right),$$
$$z_H = z_H\left(\bar{q}^1, \bar{q}^2, \bar{q}^3\right),$$

$$x_H = \bar{q}^1 \cos \bar{q}^2,$$
$$y_H = \bar{q}^1 \sin \bar{q}^2,$$
$$z_H = \bar{q}^3.$$

$$\bar{q}^1 = \sqrt{x_H^2 + y_H^2},$$
$$\bar{q}^2 = arctg\left(\frac{y_H}{x_H}\right) + k\pi,\ k = 0, \pm 1, \pm 2, \ldots,$$
$$\bar{q}^3 = z_H.$$

- Поларно-цилиндричне координате





$$\bar{q}^1 = \sqrt{x_H^2(q^1, q^2, ..., q^n) + y_H^2(q^1, q^2, ..., q^n)},$$

$$\bar{q}^2 = arctg\left(\frac{y_H(q^1, q^2, ..., q^n)}{x_H(q^1, q^2, ..., q^n)}\right) + k\pi, \quad k = 0, \pm 1, \pm 2, ...$$

$$\bar{q}^3 = z_H\left(q^1, q^2, ..., q^n\right).$$

$$\Rightarrow$$

$$\bar{q}^1 = \bar{q}^1\left(q^1, q^2, ..., q^n\right),$$

$$\bar{q}^2 = \bar{q}^2\left(q^1, q^2, ..., q^n\right),$$

$$\bar{q}^3 = \bar{q}^3\left(q^1, q^2, ..., q^n\right).$$

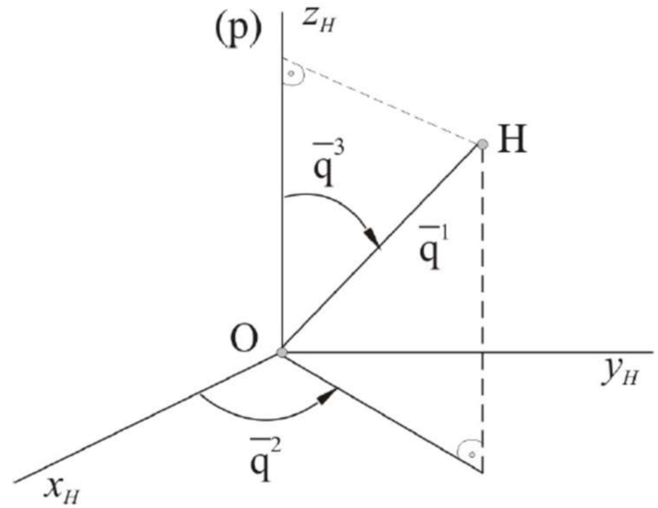
Сингуларна права z-оса $x_H = 0,\ y_H = 0,$

У сингуларном случају је

$$: \bar{q}^1 = 0,\quad \bar{q}^2 = \psi\left(x_H, y_H, z_H\right),\quad \bar{q}^3 = z_H.$$

$$\bar{q}^1 = 0,\quad \bar{q}^2 = \psi(q^1, q^2, \dots, q^n),\quad \bar{q}^3 = z_H(q^1, q^2, \dots, q^n).$$

Primer 2. *Odrediti Jakobijevu matricu* $[J_1]$ *koja se odnosi na pozicioniranje vrha* ***H*** *robotske hvataljke u slučaju da su spoljašnje koordinate sferne.*



$$\bar{q}^1 = \sqrt{x_H^2 + y_H^2 + z_H^2},$$

$$\bar{q}^2 = arctg\left(\frac{y_H}{x_H}\right) + k\pi,\ k = 0, \pm 1, \pm 2, \ldots$$

$$\bar{q}^3 = arctg\left(\frac{\sqrt{x_H^2 + y_H^2}}{z_H}\right) + k\pi,\ k = 0, \pm 1, \pm 2, \ldots$$

$$\bar{q}^1 = \sqrt{x_H^2(q^1, q^2, \ldots, q^n) + y_H^2(q^1, q^2, \ldots, q^n) + z_H^2(q^1, q^2, \ldots, q^n)},$$

$$\bar{q}^2 = arctg\left(\frac{y_H(q^1, q^2, \ldots, q^n)}{x_H(q^1, q^2, \ldots, q^n)}\right) + k\pi,\quad k = 0, \pm 1, \pm 2, \ldots$$

$$\bar{q}^3 = arctg\left(\frac{\sqrt{x_H^2(q^1, q^2, \ldots, q^n) + y_H^2(q^1, q^2, \ldots, q^n)}}{z_H(q^1, q^2, \ldots, q^n)}\right) + k\pi,\quad k = 0, \pm 1, \pm 2, \ldots$$

Сингуларитети:

$(x_H = 0,\ y_H = 0,\ z_H)$ $\quad \bar{q}^2$ ⇒

$$\bar{q}^1 = \left|z_H(q^1, q^2, \ldots, q^n)\right|,$$

$$\bar{q}^2 = \psi(q^1, q^2, \ldots, q^n),$$

$$\bar{q}^3 = k\pi,\ k = 0, \pm 1, \pm 2, \ldots,$$

$(x_H = 0,\ y_H = 0,\ z_H = 0)$ $\quad \bar{q}^2$ i $\bar{q}^3$ ⇒

$$\bar{q}^1 = 0,$$

$$\bar{q}^2 = \psi_1(q^1, q^2, \ldots, q^n),$$

$$\bar{q}^3 = \psi_2(q^1, q^2, \ldots, q^n).$$

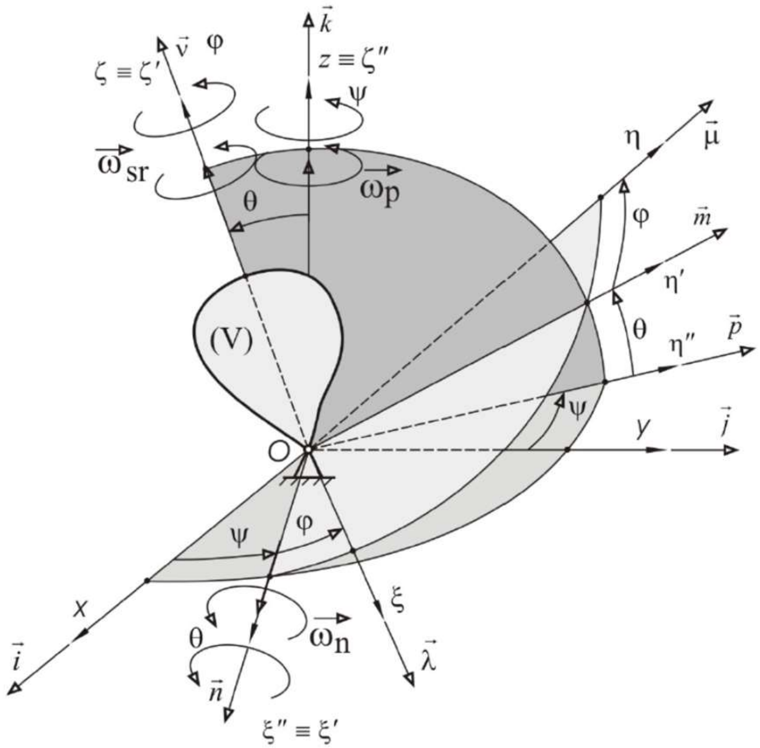
Базни случај -оријентације

$\overline{q}^4 = \psi,$

$\overline{q}^5 = \theta,$

$\overline{q}^6 = \varphi.$

$$\vec{\omega}_n = \dot{\psi}\vec{k} + \dot{\theta}\vec{n} + \dot{\varphi}\vec{\nu} \ ,$$



- Ојлерова матрица трансформације

$$[A]=\begin{bmatrix} c\psi\, c\varphi - s\psi\, c\theta\, s\varphi & -c\psi\, s\varphi - s\psi\, c\theta\, c\varphi & s\psi\, s\theta \\ s\psi\, c\varphi + c\psi\, c\theta\, s\varphi & -s\psi\, s\varphi + c\psi\, c\theta\, c\varphi & -c\psi\, s\theta \\ s\theta\, s\varphi & s\theta\, c\varphi & c\theta \end{bmatrix}.$$

$$[A]=\left[\alpha_{ij}(\psi,\theta,\varphi)\right],\ i,j=1,2,3,$$

Са друге стране имамо сложену Родригову матрицу трансформације у функцији унутрашњих координата (*)

$$[A]=[A_{0,n}]=[A_1^r][A_2^r]\dots[A_n^r].$$

$$[A]=\left[\beta_{ij}(q^1,q^2,\dots,q^n)\right],\ i,j=1,2,3.$$

$$\alpha_{ij}(\overline{q}^4,\overline{q}^5,\overline{q}^6)=\beta_{ij}(q^1,q^2,\dots,q^n)\quad i,j=1,2,3,$$

На основу израза (*)

$$\psi = arctg\left(-\frac{\beta_{13}(q^1,q^2,...,q^n)}{\beta_{23}(q^1,q^2,...,q^n)}\right) + k\pi, \quad k = 0, \pm1, \pm2, ...,$$

$$\theta = arctg\left(\frac{\pm\sqrt{\beta_{13}^2(q^1,q^2,...,q^n) + \beta_{23}^2(q^1,q^2,...,q^n)}}{\beta_{33}(q^1,q^2,...,q^n)}\right) + k_1\pi, \quad k_1 = 0, \pm1, \pm2, ...$$

$$\varphi = arctg\left(\frac{\beta_{31}(q^1,q^2,...,q^n)}{\beta_{32}(q^1,q^2,...,q^n)}\right) + k_2\pi, \quad k_2 = 0, \pm1, \pm2, ...$$

- Случај сингуларитета $\theta = k\pi, \; k = 0, \pm1, \pm2, ...$

$$[A] = \begin{bmatrix} cos\left[\psi + (-1)^k \varphi\right] & (-1)^k sin\left[\psi + (-1)^k \varphi\right] & 0 \\ sin\left[\psi + (-1)^k \varphi\right] & (-1)^k cos\left[\psi + (-1)^k \varphi\right] & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\psi = arctg\left(-\frac{\beta_{21}(q^1,q^2,...,q^n)}{\beta_{11}(q^1,q^2,...,q^n)}\right) + k_1\pi + (-1)^{k+1}\phi(q^1,q^2,...,q^n)$$

$$\theta = k\pi,$$

$$\varphi = \phi(q^1,q^2,...,q^n)$$

$$k = 0, \pm1, \pm2, ... \quad k_2 = 0, \pm1, \pm2, ...$$