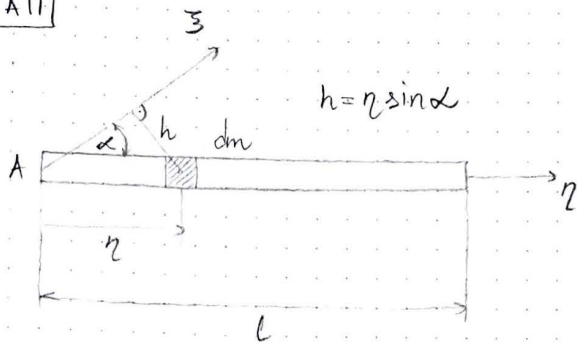


ШТАП



$\xi \Rightarrow$  произволна оса под углом  $\alpha$

ШТАП  $\Rightarrow$  линејски елемент

линејска густина  $g = \frac{m}{l} = \text{const.}$

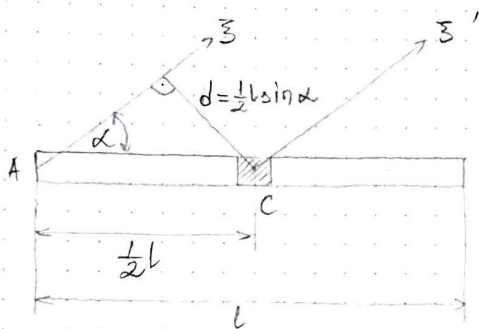
$$g = \frac{dm}{d\eta} \Rightarrow dm = g d\eta = \frac{m}{l} d\eta$$

$$J_{A\xi} = \int_m h^2 dm = \int_0^l \eta^2 \sin^2 \alpha \cdot \frac{m}{l} d\eta$$

$$J_{A\xi} = \frac{m}{l} \sin^2 \alpha \int_0^l \eta^2 d\eta = \frac{m}{l} \sin^2 \alpha \cdot \frac{l^3}{3}$$

$$J_{A\xi} = \frac{1}{3} m l^2 \sin^2 \alpha$$

$$J_{A\eta} = 0 \quad (h=0)$$



$$J_{A\xi} = J_{C\xi'} + m d^2 \Rightarrow \text{ШТАПЕРОВА ТЕОРЕМА}$$

$$J_{C\xi'} = J_{A\xi} - m d^2, \quad d = \frac{1}{2} l \sin \alpha$$

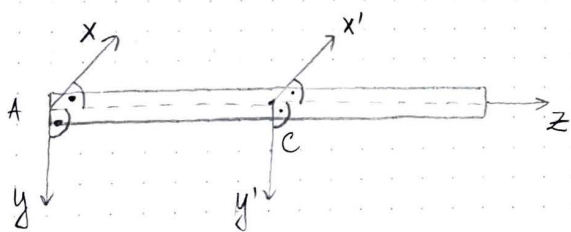
$$= \frac{1}{3} m l^2 \sin^2 \alpha - \frac{1}{4} m l^2 \sin^2 \alpha$$

$d$  - нормално растојанство од  $\xi$

$$J_{C\xi'} = \frac{1}{12} m l^2 \sin^2 \alpha$$

$$J_{C\eta'} = 0$$

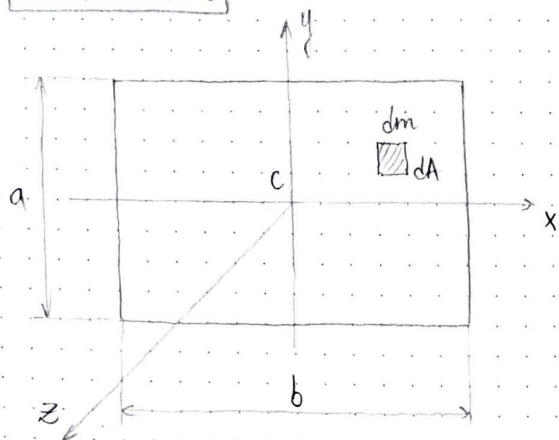
$$\alpha = 90^\circ = \frac{\pi}{2}$$



$$J_{Ax} = J_{Ay} = \frac{1}{3} m l^2 \quad [\text{kgm}^2]$$

$$J_{Cx} = J_{Cy} = \frac{1}{12} m l^2 \quad [\text{kgm}^2]$$

# ПРЯМОУГОЛЬНИК



$$S = \frac{m}{A} \Rightarrow \text{поверхностная плотность}, \quad A = ab, \quad S = \frac{m}{ab}$$

$$dm = S dA, \quad dA = dx dy$$

$$dm = \frac{m}{ab} dx dy$$

$$\begin{aligned} Y_{cx} &= \int_m (y^2 + z^2) dm = \int_m y^2 dm = \frac{m}{ab} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} y^2 dy dx = \\ &= \frac{m}{ab} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left( \frac{y^3}{3} \Big|_{-\frac{a}{2}}^{\frac{a}{2}} \right) dx = \frac{m}{3ab} \left( \frac{1}{8} a^3 + \frac{1}{8} a^3 \right) \cdot x \Big|_{-\frac{b}{2}}^{\frac{b}{2}} = \\ &= \frac{m}{3ab} \cdot \frac{2}{48} a^3 \cdot b \end{aligned}$$

$$Y_{cx} = \frac{1}{12} ma^2$$

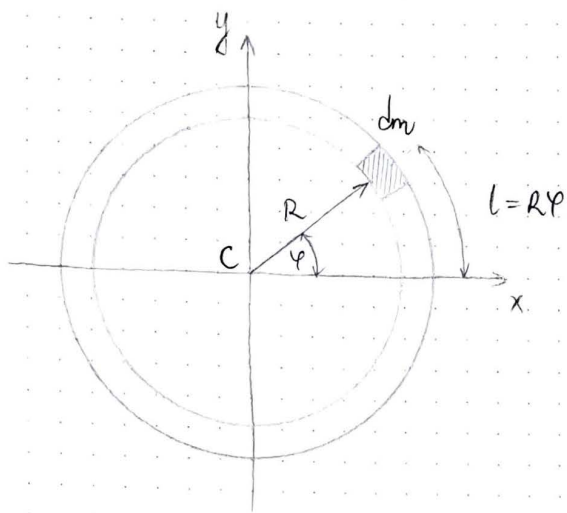
$$Y_{cy} = \frac{1}{12} mb^2$$

$$\begin{aligned} Y_{cz} &= \int_m (x^2 + y^2) dm = \frac{m}{ab} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 dx dy + \frac{m}{ab} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} y^2 dx dy = \\ &= \frac{m}{3ab} \cdot \frac{1}{4} b^3 \cdot a + \frac{m}{3ab} \cdot \frac{1}{4} a^3 \cdot b \end{aligned}$$

$$Y_{cz} = \frac{1}{12} m(a^2 + b^2)$$

$$\star a = b \Rightarrow \underline{\underline{Y_{cz} = \frac{1}{6} ma^2}}$$

# ТАНАК ПРСТЕН



$$S = \frac{dm}{dl} \Rightarrow \text{массовая плотность}$$

$$l = R\varphi \Rightarrow dl = R d\varphi$$

$$S = \frac{dm}{R d\varphi} = \frac{m}{2R\pi} \Rightarrow dm = \frac{m}{2\pi} d\varphi$$

$$x = R \cos \varphi$$

$$y = R \sin \varphi$$

$$J_{Cx} = \int_m (y^2 + z^2) dm = \frac{m}{2\pi} \int_0^{2\pi} R^2 \sin^2 \varphi d\varphi$$

$$\sin^2 \varphi = \frac{1 - \cos 2\varphi}{2}$$

$$J_{Cx} = \frac{mR^2}{4\pi} \int_0^{2\pi} d\varphi - \frac{mR^2}{4\pi} \int_0^{2\pi} \cos 2\varphi d\varphi$$

$$J_{Cx} = \frac{mR^2}{4\pi} \cdot 2\pi - \frac{mR^2}{4\pi} (\sin 2\varphi - 0)$$

$$J_{Cx} = \frac{1}{2} m R^2$$

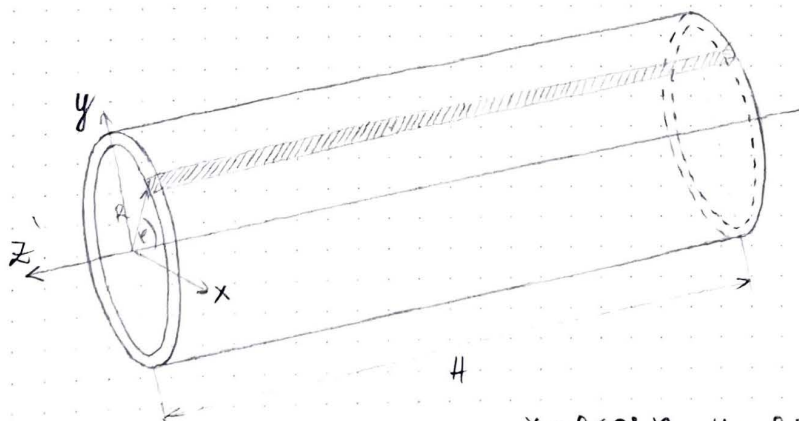
$$J_{Cy} = \frac{1}{2} m R^2$$

$$J_{Cz} = \int_m (x^2 + y^2) dm = \int_m (R^2 \cos^2 \varphi + R^2 \sin^2 \varphi) dm$$

$$= \int_m R^2 dm = R^2 \int_m dm$$

$$J_{Cz} = m R^2$$

# ЦИЛИНДР



$$dA = H \cdot R d\varphi$$

$$A = H \cdot 2R\pi$$

$$S = \frac{m}{A} = \frac{dm}{dA} \Rightarrow \text{поверхностная плотность}$$

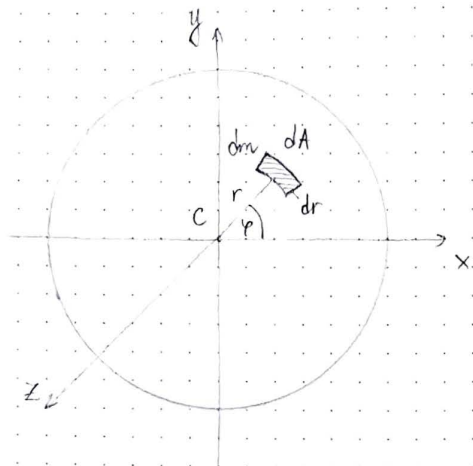
$$dm = \frac{m}{A} dA = \frac{m}{2R\pi H} \cdot H R d\varphi = \frac{m}{2\pi} d\varphi$$

$$x = R \cos \varphi, y = R \sin \varphi$$

$$J_{Cz} = \int_m (x^2 + y^2) dm = \int_0^{2\pi} (R^2 \cos^2 \varphi + R^2 \sin^2 \varphi) \frac{m}{2\pi} d\varphi = \frac{mR^2}{2\pi} \int_0^{2\pi} d\varphi$$

$$J_{Cz} = m R^2$$

# ТАНАК КРУЖНИИ ДИСК



$$S = \frac{dm}{dA} = \frac{m}{A}, \quad A = R^2\pi, \quad dA = dr \cdot r d\varphi$$

$$dm = \frac{m}{R^2\pi} \cdot r dr d\varphi$$

$$x = r \cos \varphi, \quad y = r \sin \varphi$$

$$\begin{aligned} J_{Cx} &= \int_m (y^2 + z^2) dm = \int_0^R \int_0^{2\pi} r^2 \sin^2 \varphi \cdot \frac{m}{R^2\pi} r dr d\varphi \\ &= \frac{m}{R^2\pi} \int_0^{2\pi} \left( \int_0^R r^3 dr \right) \sin^2 \varphi d\varphi, \quad \sin^2 \varphi = \frac{1 - \cos 2\varphi}{2} \\ &= \frac{m}{R^2\pi} \cdot \frac{R^4}{4} \cdot \frac{1}{2} \left( \int_0^{2\pi} d\varphi - \int_0^{2\pi} \cos 2\varphi d\varphi \right) \\ &= \frac{mR^2}{48\pi} \cdot 2\pi \end{aligned}$$

$$\boxed{J_{Cx} = \frac{1}{4} m R^2}$$

$$\boxed{J_{Cy} = \frac{1}{4} m R^2}$$

$$\begin{aligned} J_{Cz} &= \int_m (x^2 + y^2) dm = \int_m (r^2 \cos^2 \varphi + r^2 \sin^2 \varphi) dm \\ &= \int_m r^2 dm = \frac{m}{R^2\pi} \int_0^R \int_0^{2\pi} r^2 \cdot r dr d\varphi \\ &= \frac{m}{R^2\pi} \cdot \frac{R^4}{24} \cdot 2\pi \end{aligned}$$

$$\boxed{J_{Cz} = \frac{1}{2} m R^2}$$