Генералисане силе у динамици крутих сила

Генералисана сила од сила пригушивача (вискозно трење)

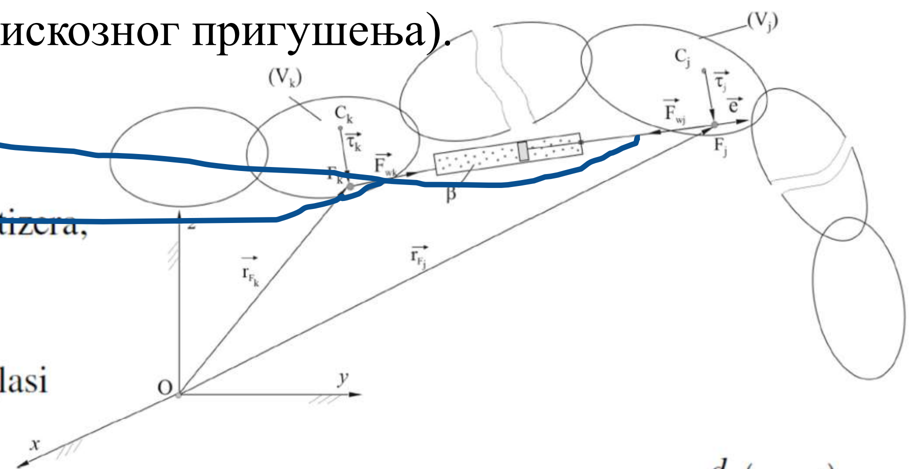
$$\vec{F}_{wj} = -\vec{F}_{wk} = -\beta \vec{v}_{rk},$$

(β – коефицијент вискозног пригушења).

$\vec{F}_{wj}$ sila viskoznog trenja koja deluje na klip,

$\vec{F}_{wk}$ sila viskoznog trenja koja deluje na cilindar amortizera,

$\vec{v}_{rk}$ relativna brzina klipa u odnosu na amortizer, izraz za virtualni rad sila viskoznog trenja glasi



$$\delta A^{w} = \vec{F}_{wk} \cdot \delta \vec{r}_{F_k} + \vec{F}_{wj} \cdot \delta \vec{r}_{F_j} = \vec{F}_{wj} \cdot \delta\left(\overrightarrow{F_k F_j}\right).$$

$$\overrightarrow{F_k F_j} = \overline{F_k F_j}\,\vec{e}, \quad |\vec{e}| = 1 \qquad \vec{v}_{rk} = \vec{e}\,\frac{d}{dt}\left(\overline{F_k F_j}\right),$$

$$\delta\left(\overrightarrow{F_k F_j}\right) = \vec{e}\,\delta\left(\overline{F_k F_j}\right) + \overline{F_k F_j}\,\delta\vec{e} \qquad \vec{e} \cdot \delta\vec{e} = 0.$$

$$\boxed{\delta A^{w} = -\beta \frac{d}{dt}\left(\overline{F_k F_j}\right)\delta\left(\overline{F_k F_j}\right),}$$

Пошто је

$$\overline{F_k F_j} = \sqrt{\overrightarrow{F_k F_j} \cdot \overrightarrow{F_k F_j}},$$

$$\frac{\partial\left(\overline{F_k F_j}\right)}{\partial q^{\alpha}} = \frac{\overrightarrow{F_k F_j}}{\overline{F_k F_j}} \cdot \frac{\partial\left(\overrightarrow{F_k F_j}\right)}{\partial q^{\alpha}}.$$

$$\frac{d}{dt}\left(\overline{F_k F_j}\right) = \sum_{\beta=1}^{n} \frac{\partial\left(\overline{F_k F_j}\right)}{\partial q^{\beta}}\dot{q}^{\beta},$$

$$\frac{d}{dt}\left(\overline{F_k F_j}\right) = \sum_{\beta=1}^{n} \frac{\overrightarrow{F_k F_j}}{\overline{F_k F_j}} \cdot \frac{\partial\left(\overrightarrow{F_k F_j}\right)}{\partial q^{\beta}}\dot{q}^{\beta}$$

Варијација је сада $$\delta\left(\overline{F_kF_j}\right)=\sum_{\alpha=1}^{n}\frac{\overrightarrow{F_kF_j}}{\overline{\overline{F_kF_j}}}\cdot\frac{\partial\left(\overrightarrow{F_kF_j}\right)}{\partial q^{\alpha}}\delta q^{\alpha}$$

$$\delta A^{w}=-\beta\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\frac{\overrightarrow{F_kF_j}}{\overline{\overline{F_kF_j}}}\cdot\frac{\partial\left(\overrightarrow{F_kF_j}\right)}{\partial q^{\beta}}\dot{q}^{\beta}\frac{\overrightarrow{F_kF_j}}{\overline{\overline{F_kF_j}}}\cdot\frac{\partial\left(\overrightarrow{F_kF_j}\right)}{\partial q^{\alpha}}\delta q^{\alpha}$$

$$\delta A^{w}=\sum_{\alpha=1}^{n}Q_{\alpha}^{w}\delta q^{\alpha}$$

Како је и упоређивањем са претходним изразом , добија се

$$\boxed{Q_{\alpha}{}^{W}=-\beta\frac{\overrightarrow{F_KF_j}}{\overline{\overline{F_KF_j}}}\frac{\partial\overrightarrow{F_KF}_j}{\partial q^{\alpha}}\sum_{\beta=1}^{n}\frac{\overrightarrow{F_KF_j}}{\overline{\overline{F_KF_j}}}\frac{\partial\overrightarrow{F_KF}_j}{\partial q^{\beta}}\dot{q}^{\beta}}$$

Ако се уочени вектор , $\overrightarrow{F_kF_j}$ прикаже преко одговарајућих вектора,види слику и одреди парцијални извод по датој генералиснаој координати, добија се

$$Q_{\alpha}^{w}=-\beta\sum_{\beta=1}^{n}\frac{\vec{r}_{C_j}-\vec{r}_{C_k}+\vec{\tau}_j-\vec{\tau}_k}{\left|\vec{r}_{C_j}-\vec{r}_{C_k}+\vec{\tau}_j-\vec{\tau}_k\right|^{2}}\cdot\left(\vec{T}_{\beta(j)}-\vec{T}_{\beta(k)}+\vec{\Omega}_{\beta(j)}\times\vec{\tau}_j-\vec{\Omega}_{\beta(k)}\times\vec{\tau}_k\right)*$$

$$*\left(\vec{r}_{C_j}-\vec{r}_{C_k}+\vec{\tau}_j-\vec{\tau}_k\right)\cdot\left(\vec{T}_{\alpha(j)}-\vec{T}_{\alpha(k)}+\vec{\Omega}_{\alpha(j)}\times\vec{\tau}_j-\vec{\Omega}_{\alpha(k)}\times\vec{\tau}_k\right)\dot{q}^{\beta}\ ,$$

Други начин одређивања :

Уводи се дисипативна Рејлијева функција

$$\Phi = \frac{1}{2}\beta v_r^2$$

Из аналитичке механике, (механике 3) је познато да се генералисана сила може одредити:

$$Q_\alpha^\beta = -\frac{\partial \Phi}{\partial \dot{q}^\alpha} = -\frac{\cancel{2}}{\cancel{2}}\beta v_r \cdot \frac{\partial v_r}{\partial \dot{q}^\alpha} = -\beta v_r \cdot \frac{\partial v_r}{\partial \dot{q}^\alpha}$$

$$l = \overline{F_k F_j}\left(q^1, q^2, \ldots, q^n\right) \qquad v_r = \frac{dl}{dt} = \sum_{\beta=1}^{n} \frac{\partial l}{\partial q^\beta}\dot{q}^\beta$$

$$\frac{\partial v_r}{\partial \dot{q}^\alpha} = \frac{\partial}{\partial \dot{q}^\alpha}\left(\sum_{\alpha=1}^{n}\frac{\partial l}{\partial q^\alpha}\dot{q}^\alpha\right) = \frac{\partial l}{\partial q^\alpha}$$

$$\boxed{Q_\alpha^\beta = -\beta \frac{\partial l}{\partial q^\alpha}\sum_{\beta=1}^{n}\frac{\partial l}{\partial q^\beta}\dot{q}^\beta}$$

$$\frac{\partial l}{\partial q^\alpha} = \frac{\partial \overline{F_k F_j}\left(q^1, q^2, \ldots, q^n\right)}{\partial q^\alpha} = \frac{\partial}{\partial q^\alpha}\left(\sqrt{\overrightarrow{F_k F_j} \cdot \overrightarrow{F_k F_j}}\right) = \frac{\overrightarrow{F_k F_j}}{\overline{F_k F_j}}\frac{\partial}{\partial q^\alpha}\left(\overrightarrow{F_k F_j}\right)$$

На сличан начин добија се:

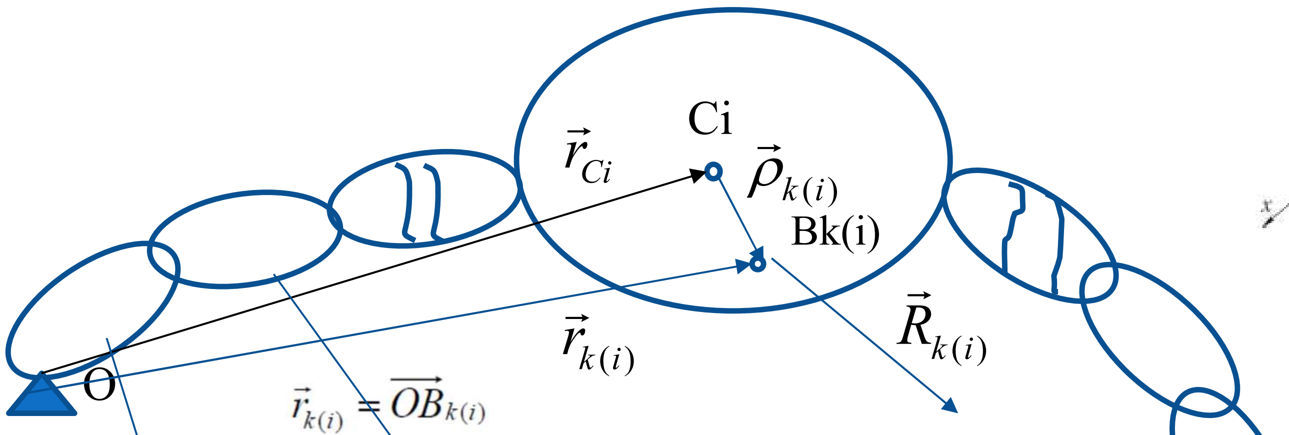
$$l = \overline{F_k F_j}\left(q^1, q^2, \ldots, q^n\right) \rightarrow \frac{\partial l}{\partial q^\beta} = \frac{\overrightarrow{F_k F}_j}{\overline{\overline{F_k F_j}}} \frac{\partial}{\partial q^\beta}\left(\overrightarrow{F_k F}_j\right)$$

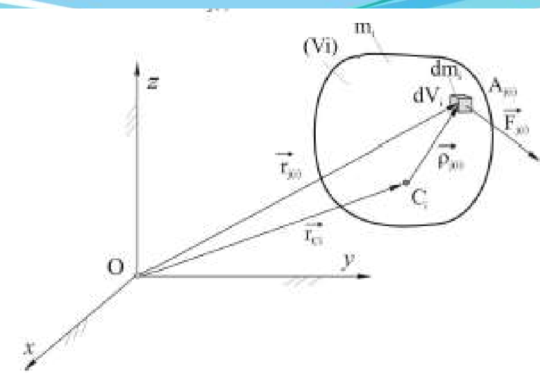
Коначно:

$$Q_\alpha{}^W = -\beta \frac{\overrightarrow{F_K F_j}}{\overline{\overline{F_K F_j}}} \frac{\partial \overrightarrow{F_K F}_j}{\partial q^\alpha} \sum_{\beta=1}^{n} \frac{\overrightarrow{F_K F_j}}{\overline{\overline{F_K F_j}}} \frac{\partial \overrightarrow{F_K F}_j}{\partial q^\beta} \dot{q}^\beta$$

$$\frac{\partial l}{\partial q^\alpha}$$

- Принцип идеалности веза роботског система





Систем реакције веза делује на дати РС:

$$\left(\left(\vec R_{1(1)},\vec R_{2(1)},\dots,\vec R_{m_1(1)}\right),\left(\vec R_{1(1)},\vec R_{2(1)},\dots,\vec R_{m_2(1)}\right),\dots,\left(\vec R_{1(1)},\vec R_{2(1)},\dots,\vec R_{m_n(1)}\right)\right),$$

Принцип идеалности веза је облика за наш РС:

$$\sum_{i=1}^{n}\left(\sum_{k=1}^{m_i}\vec R_{k(i)}\cdot\delta\vec r_{k(i)}\right)=0\,,$$

$$\delta A^{w}\left(\vec R_{k(i)}\right)=\vec R_{k(i)}\cdot\delta\left(\vec r_{Ci}+\vec\rho_{k(i)}\right)=\sum_{\alpha=1}^{n}\vec R_{k(i)}\cdot\vec T_{\alpha(i)}\delta q^{\alpha}+\sum_{\alpha=1}^{n}\underbrace{\vec R_{k(i)}\cdot\left(\vec\Omega_{\alpha(i)}\times\vec\rho_{k(i)}\right)}_{2x}\delta q^{\alpha}$$

Укупни виртуални рад за сегмент Vci:

$$\delta A^{w}{}_{i} = \sum_{k=1}^{mi}\left(\sum_{\alpha=1}^{n}\vec{R}_{k(i)}\cdot\vec{T}_{\alpha(i)}\delta q^{\alpha} + \sum_{\alpha=1}^{n}\vec{\Omega}_{\alpha(i)}\cdot\vec{M}_{ck(i)}\cdot\delta q^{\alpha}\right)$$

$$= \sum_{\alpha=1}^{n}\left(\left(\sum_{k=1}^{mi}\vec{R}_{k(i)}\right)\cdot\vec{T}_{\alpha(i)} + \left(\sum_{k=1}^{mi}\vec{M}_{ck(i)}\right)\cdot\vec{\Omega}_{\alpha(i)}\right)\delta q^{\alpha}$$

$$\vec{R}_{R(i)} = \sum_{k=1}^{m_i}\vec{R}_{k(i)},$$

$$\vec{M}^{r}_{C_iR(i)} = \sum_{k=1}^{m_i}\vec{M}_{C_i}\left(\vec{R}_{k(i)}\right).$$

$$\boxed{\delta A^{w}{}_{i} = \sum_{\alpha=1}^{n}\left(\vec{R}_{R(i)}\cdot\vec{T}_{\alpha(i)} + \vec{M}^{r}_{CiR(i)}\cdot\vec{\Omega}_{\alpha(i)}\right)\delta q^{\alpha}}$$

- Укупни виртуални рад за цео РС

$$\delta A_{\Sigma}^{w} = \sum_{i=1}^{n} \delta A_{i}^{w} = \sum_{i=1}^{n} \sum_{\alpha=1}^{n} \left(\vec{R}_{R(i)} \cdot \vec{T}_{\alpha(i)} + \vec{M}_{CiR(i)}^{r} \cdot \vec{\Omega}_{\alpha(i)} \right) \delta q^{\alpha} = 0$$

$$= \sum_{\alpha=1}^{n} \sum_{i=1}^{n} \left(\vec{R}_{R(i)} \cdot \vec{T}_{\alpha(i)} + \vec{M}_{CiR(i)}^{r} \cdot \vec{\Omega}_{\alpha(i)} \right) \delta q^{\alpha} = \sum_{\alpha=1}^{n} Q_{\alpha}^{r} \delta q^{\alpha} = 0$$

$$\Downarrow$$

$$\sum_{i=1}^{n} \sum_{k=1}^{m_i} \vec{R}_{(i)k} \cdot \delta \vec{r}_{(i)k} = \sum_{\alpha=1}^{n} Q_{\alpha}^{r} \cdot \delta q^{\alpha} = 0 \, ,$$

Q_{α}^{r} Генералисана сила од система реакција веза

$$Q_{\alpha}^{r} = \sum_{i=1}^{n} \left(\vec{R}_{R(i)} \cdot \vec{T}_{\alpha(i)} + \vec{M}_{C_i R(i)}^{r} \cdot \vec{\Omega}_{\alpha(i)} \right) = 0, \ \alpha = 1,2,\dots,n.$$