M2, kol. 1 ; 17/18 ; 5,6 ; 16.4.2018.

1. [5] (Z.1) $\int_1^3\left(\frac{\ln^2 x}{x}+xe^x\right)dx=\underbrace{\int_1^3\frac{\ln^2 x}{x}dx}_{I_1}+\int_1^3 xe^x dx$

$I_1=\left|\begin{matrix}\ln x=t\\ \frac{dx}{x}\overset{(1)}{=}dt\end{matrix}\right|=\int_0^{\ln 3 \ (1)} t^2dt=\frac{t^3}{3}\Big|_0^{\ln 3}=\frac{\ln^3 3}{3}$ (0.5)

$I_2=\left|\begin{matrix}u=x, & dv=e^xdx\\ du\overset{(1)}{=}dx, & v=e^x\end{matrix}\right|=[xe^x]\Big|_1^3-\underbrace{\int_1^3 e^xdx}_{[e^x]|_1^3}\overset{(1)}{}=3e^3-\cancel{e}-e^3+\cancel{e}=2e^3$ (0.5)

$I=\frac{\ln^3 3}{3}+2e^3.$

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1. [5] (Z.2) $\int_1^2\left(xe^x-\frac{\ln^3 x}{x}\right)dx=\underbrace{\int_1^2 xe^xdx}_{I_1}-\underbrace{\int_1^2\frac{\ln^3 x}{x}dx}_{I_2}$

$I_1=\left|\begin{matrix}u=x, & dv=e^xdx\\ du\overset{(1)}{=}dx, & v=e^x\end{matrix}\right|=[xe^x]\Big|_1^2-\underbrace{\int_1^2 e^xdx}_{[e^x]|_1^2}\overset{(1)}{}=2e^2-\cancel{e}-e^2+\cancel{e}$
$=e^2$ (0.5)

$I_2=\left|\begin{matrix}\ln x=t\\ \frac{dx}{x}\overset{(1)}{=}dt\end{matrix}\right|=\int_0^{\ln 2 \ (1)} t^3dt=\left[\frac{t^4}{4}\right]\Big|_0^{\ln 2}=\frac{\ln^4 2}{4}$ (0.5)

$I=e^2-\frac{\ln^4 2}{4}$

[2] (Zd. 1) [10]

$$I=\int \frac{(2\cos^2x-3\cos x-5)\sin x}{(2-\cos x)(\cos^2x+1)}dx = \left| \begin{array}{l}\cos x = t \\ -\sin x\, dx = dt\end{array}\right| \ (1) =$$

$$=\int \frac{2t^2-3t-5}{(t-2)(t^2+1)}dt$$

$$\frac{2t^2-3t-5}{(t-2)(t^2+1)} = \frac{A}{t-2} + \frac{Bt+C}{t^2+1} \quad \Big/ (t-2)(t^2+1) \ (1)$$

$(1+1)$

$$2t^2-3t-5 = A(t^2+1)+(Bt+C)(t-2)$$

$$= At^2+A+Bt^2-2Bt+Ct-2C$$

$$t^2:\ 2 = A+B \Rightarrow A = 2-B$$

(1) $t:\ -3 = -2B+C$

$$1:\ -5 = \underbrace{A-2C}_{2-B-2C}$$

$$\left.\begin{array}{l} 2B-C=3 \ /\cdot 2 \\ B+2C=7 \end{array}\right\} (+$$

(1)

$$5B = 13 \Rightarrow B = \frac{13}{5}$$

$$C = 2B-3 = \frac{26}{5}-\frac{15}{5} = \frac{11}{5}$$

$$A = 2-B = \frac{10}{5}-\frac{13}{5} = -\frac{3}{5}$$

(1)

$$I = \int\left(\frac{-3/5}{t-2} + \frac{\overbrace{13/5\ t+11/5}^{\frac{13}{5}\cdot\frac{1}{2}\cdot 2t+\frac{11}{5}}}{t^2+1}\right)dt =$$

$$= -\frac{3}{5}\int\frac{dt}{t-2} + \frac{13}{10}\int\frac{\overbrace{2t\,dt}^{=d(t^2+1)}}{t^2+1} + \frac{11}{5}\int\frac{dt}{t^2+1}$$

(0.5) (1) (0.5)

$$= -\frac{3}{5}\ln|t-2| + \frac{13}{10}\ln|t^2+1| + \frac{11}{5}\operatorname{arctg} t + C$$

$$= \frac{1}{10}\left(13\ln(\cos^2x+1) - 6\ln|\cos x-2| + 22\operatorname{arctg}(\cos x)\right) + C$$

[2] (Zd. 2)

$$\int \frac{(2\sin^2x-3\sin x-5)\cos x}{(2-\sin x)(\sin^2x+1)}dx = \left| \begin{array}{l}\sin x = t \\ \cos x\, dx = dt\end{array}\right| \ (1)$$

$$= -\int\frac{(2t^2-3t-5)dt}{(t-2)(t^2+1)} = \dots (\text{Zd } 1)$$

$\boxed{3.}$ (Гр.1) [10]

$$\int \frac{5-x}{\sqrt[3]{2+x}}\,dx = \left| \begin{array}{l} 2+x=t^3 \\ x=t^3-2 \\ dx=3t^2dt \end{array} \right| = \int \frac{5-t^3+2}{\cancel{\sqrt[3]{t^3}}}\, 3t^{\cancel{2}}dt =$$

$$= 3\int (3t - t^4)\,dt = 3\cdot\left(3\frac{t^2}{2} - \frac{t^5}{5}\right) + C = \frac{9}{2}(2+x)^{2/3} - \frac{3}{5}(2+x)^{5/3} + C$$

$\boxed{3.}$ (Гр.2) [10]

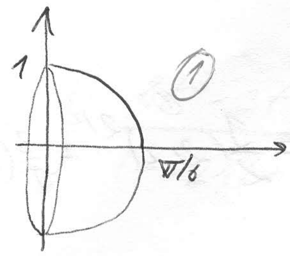
$$\int \frac{5+x}{\sqrt[3]{2-x}}\,dx = \left| \begin{array}{l} 2-x=t^3 \\ x=2-t^3 \\ dx=-3t^2dt \end{array} \right| = \int \frac{5+2-t^3}{\cancel{\sqrt[3]{t^3}}}\,(-3t^{\cancel{2}}dt) =$$

$$= 3\int (t^4 - 7t)\,dt = 3\left(\frac{t^5}{5} - 7\frac{t^2}{2}\right) + C =$$

$$= \frac{3}{5}(2-x)^{5/3} - \frac{7}{2}(2-x)^{2/3} + C$$

$\boxed{4}$ (Z.1) $y=\cos 3x$, $x\in[0,\pi/6]$, око x-осе; $P_x=?$

$[10]$



$+$ ф-ла

$$P_x=2\pi\int_0^{\pi/6}|\cos 3x|\sqrt{1+\underbrace{((\cos 3x)')^2}_{(-3\sin 3x)^2=9\sin^2 3x}}\,dx$$

$$=\left\{\begin{matrix} x\Big|_0^{\pi/6}, & 3x\Big|_0^{\pi/2} \\ |\cos 3x|=\cos 3x & \end{matrix}\right\}=$$

$$=2\pi\int_0^{\pi/6}\cos 3x\sqrt{1+(3\sin 3x)^2}\,dx=\left\{\begin{matrix} 3\sin 3x=\operatorname{sh}t \\ 9\cos 3x\,dx=\operatorname{ch}t\,dt\end{matrix}\right\}=$$

$$=2\pi\int_0^{\operatorname{arsh}3}\underbrace{\sqrt{\underbrace{1+\operatorname{sh}^2 t}_{\operatorname{ch}^2 t}}}_{|\operatorname{ch}t|=\operatorname{ch}t}\,\frac{\operatorname{ch}t\,dt}{9}=\frac{2\pi}{9}\int_0^{\ln(3+\sqrt{10})}\operatorname{ch}^2 t\,dt=$$

$$=\frac{2\pi}{9}\int_0^{\ln(3+\sqrt{10})}\left(\frac{e^t+e^{-t}}{2}\right)^2dt=\frac{\pi}{18}\int_0^{\ln(3+\sqrt{10})}\left(e^{2t}+2+e^{-2t}\right)dt=$$

$$=\frac{\pi}{18}\left(\frac{1}{2}e^{2t}+2t-\frac{1}{2}e^{-2t}\right)\Big|_0^{\ln(3+\sqrt{10})}=$$

$$=\frac{\pi}{18}\left(\frac{1}{2}\left(e^{2\ln(3+\sqrt{10})}\cancel{-1}\right)+2\left(\ln(3+\sqrt{10})-0\right)-\frac{1}{2}\left(e^{-2\ln(3+\sqrt{10})}\cancel{-1}\right)\right)$$

$$=\frac{\pi}{36}(3+\sqrt{10})^2+\frac{\pi}{9}\ln(3+\sqrt{10})-\frac{\pi}{36}\,\frac{1}{(3+\sqrt{10})^2}$$

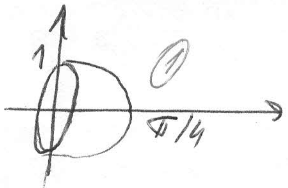
$$=\frac{\pi}{36}\left((3+\sqrt{10})^2-\frac{(3-\sqrt{10})^2}{\underbrace{((3+\sqrt{10})(3-\sqrt{10}))^2}_{=1}}\right)+\frac{\pi}{9}\ln(3+\sqrt{10})$$

$$=\frac{\pi}{36}\left(\cancel{9}+\cancel{10}+6\sqrt{10}-\cancel{9}-\cancel{10}+6\sqrt{10}\right)+\frac{\pi}{9}\ln(3+\sqrt{10})$$

$$=\frac{\pi\sqrt{10}}{3}+\frac{\pi}{9}\ln(3+\sqrt{10})=\frac{\pi}{9}\left(3\sqrt{10}+\ln(3+\sqrt{10})\right)$$

[4] (Пр.2) $y = \cos 2x$, $x \in [0, \pi/4]$, око x-осе; $P_x = ?$

[10]



$$P_x = 2\pi\int_0^{\pi/4} |\cos 2x| \sqrt{1+\left((\cos 2x)'\right)^2}\,dx$$

$(-2\sin 2x)^2 = 4\sin^2 2x$

$$= \left\{ \begin{matrix} x\big|_0^{\pi/4}, \quad 2x\big|_0^{\pi/2} \\ |\cos 2x| = \cos 2x \end{matrix} \right\} =$$

$$= 2\pi\int_0^{\pi/4}\cos 2x\sqrt{1+(2\sin 2x)^2}\,dx = \left\{ \begin{matrix} 2\sin 2x = \operatorname{sh} t \\ 4\cos 2x\,dx = \operatorname{ch} t\,dt \end{matrix} \right\} =$$

$$= 2\pi\int_0^{\operatorname{ash} 2}\sqrt{1+\operatorname{sh}^2 t}\cdot\frac{\operatorname{ch} t\,dt}{4} = \frac{\pi}{2}\int_0^{\ln(2+\sqrt5)}\operatorname{ch}^2 t\,dt \overset{(\text{Пр.}1)}{=} \ldots$$

$$\ldots = \frac{\pi}{8}\left(\frac12 e^{2t} + 2t - \frac12 e^{-2t}\right)\Big|_0^{\ln(2+\sqrt5)}$$

$$= \frac{\pi}{8}\left(\frac12\left(e^{2\ln(2+\sqrt5)} \cancel{-1}\right) + 2\left(\ln(2+\sqrt5) - 0\right) - \frac12\left(e^{-2\ln(2+\sqrt5)} \cancel{-1}\right)\right)$$

$$= \frac{\pi}{16}(2+\sqrt5)^2 + \frac{\pi}{4}\ln(2+\sqrt5) - \frac{\pi}{16}\,\frac{1}{(2+\sqrt5)^2} = \text{[illegible]}$$