M2, K3, 25.5.2018. (8,9)

(Гр. 1)

$\boxed{1.}$ $f(x,y) = \sin x + \cos y$, $A(0, \pi/2)$

[5]

$$T(x,y) = f(0,\pi/2) + (f'_x(0,\pi/2)(x-0) + f'_y(0,\pi/2)(y-\pi/2)$$
$$+ \frac{1}{2}(f''_{xx}(0,\pi/2)(x-0)^2 + f''_{yy}(0,\pi/2)(y-\pi/2)^2$$
$$+ f''_{xy}(0,\pi/2)(x-0)(y-\pi/2)) \quad 2$$

$f(0,\pi/2) = 0$

$f'_x = \cos x \overset{(0,\pi/2)}{=} 1$, $f'_y = -\sin y \overset{(0,\pi/2)}{=} -1$ 1

$f''_{xx} = -\sin x \overset{(0,\pi/2)}{=} 0$, $f''_{xy} = 0$; $f''_{yy} = -\cos y \overset{(0,\pi/2)}{=} 0$ 1

$T(x,y) = x - y + \frac{\pi}{2}$ 1

$\boxed{1.}$ (Гр. 2) $f(x,y) = \cos x + \sin y$, $A(0, \pi/2)$.

$f(0,\pi/2) = 2$

$f'_x = -\sin x \overset{(0,\pi/2)}{=} 0$, $f'_y = \cos y \overset{(0,\pi/2)}{=} 0$

$f''_{xx} = -\cos x \overset{(0,\pi/2)}{=} -1$, $f''_{xy} = 0$, $f''_{yy} = -\sin y \overset{(0,\pi/2)}{=} -1$

$$T(x,y) = 2 + \frac{1}{2}\left(-x^2 - y^2 + \pi y - \frac{\pi^2}{4}\right) = -\frac{1}{2}x^2 - \frac{1}{2}y^2 + \frac{\pi}{2}y - \frac{\pi^2}{8} + 2$$

$\boxed{2}$ (Т.1) $y' = \frac{y}{x} - \frac{y^3}{x^3}$ О.Р. = ?

[5]

– хомогена

$\frac{y}{x} = u$ ① ; $y = xu$, $y' = u + xu'$ ①

$(u = u(x))$

$\cancel{u} + xu' = \cancel{u} - u^3$ ①

$x\frac{du}{dx} = -u^3$

$\frac{du}{u^3} \overset{①}{=} -\frac{dx}{x}$ – раздвојене пром.

$/\int$

$-\frac{1}{2u^2} = \underbrace{-\ln|x| + C}_{= \ln(c/x)}$

$-\frac{x^2}{2y^2} = \ln(c/x)$ О.Р. ①

$\boxed{2}$ (Т.2) $y' = \frac{y^3}{x^3} + \frac{y}{x}$ О.Р. = ?

– хомогена

$\frac{y}{x} = u$; $y = xu$, $y' = u + xu'$

$(u = u(x))$

$\cancel{u} + xu' = u^3 + \cancel{u}$

$x\frac{du}{dx} = u^3$

$\frac{du}{u^3} = \frac{dx}{x}$ $/\int$

$-\frac{1}{2u^2} = \ln|x| + C$

$-\frac{x^2}{2y^2} = \ln(cx)$ О.Р.

$\boxed{3.}$ (гр.1) $z = e^{x-y}(x^2-2y^2)$. Екстреми?

$\boxed{10}$

$$\frac{\partial z}{\partial x} = e^{x-y}(x^2-2y^2) + e^{x-y}(2x) = e^{x-y}(x^2+2x-2y^2) = 0$$

~2 + 0.5

$$\frac{\partial z}{\partial y} = -e^{x-y}(x^2-2y^2) + e^{x-y}(-4y) = -e^{x-y}(x^2-2y^2+4y) = 0$$

$$x^2+2x-2y^2=0$$

$$x^2-2y^2+4y=0 \quad / \cdot(-1) \quad (+)$$

$$2x-4y=0 \Rightarrow x=2y$$

$$4y^2+4y-2y^2=0$$

$$y^2+2y=0$$

$$y(y+2)=0$$

$$y_1=0, \quad y_2=-2 \quad 2$$

$$x_1=0, \quad x_2=-4$$

$(0,0)$; $(-4,-2)$ – стационарне тачке

$$\frac{\partial^2 z}{\partial x^2} = e^{x-y}(x^2+2x-2y^2) + e^{x-y}(2x+2) = e^{x-y}(x^2+4x-2y^2+2)$$

$$1.5 \quad \frac{\partial^2 z}{\partial y^2} = e^{x-y}(x^2-2y^2+4y) - e^{x-y}(-4y+4) = e^{x-y}(x^2-2y^2+8y-4)$$

$$\frac{\partial^2 z}{\partial x \partial y} = -e^{x-y}(x^2+2x-2y^2) + e^{x-y}(-4y) = -e^{x-y}(x^2+2x-2y^2+4y)$$

$\underline{\underline{(0,0)}}$ 2

$$A = \frac{\partial^2 z}{\partial x^2}(0,0) = 2, \quad C = \frac{\partial^2 z}{\partial y^2}(0,0) = -4, \quad B = \frac{\partial^2 z}{\partial x \partial y}(0,0) = 0$$

$\Delta = AC - B^2 = -8 < 0$ није екстрем ($(0,0)$ је седласта тачка)

$\underline{(-4,-2)}$ 2

$$A = \frac{\partial^2 z}{\partial x^2}(-4,-2) = \frac{\cancel{16}}{e^2}, \quad C = \frac{\partial^2 z}{\partial y^2}(-4,-2) = -\frac{12}{e^2}, \quad B = +\frac{8}{e^2}$$

$$\Delta = AC - B^2 = + \frac{\overset{6}{\cancel{16}} \cdot 12}{e^4} - \frac{64}{e^4} \overset{=8/e^4}{>} 0 \quad \text{јесте екстрем}$$

$A < 0$ ($C < 0$) $\underline{\text{максимум}}$ $\quad (z_{max} = z(-4,-2) = 8/e^2)$

[3] (Г.2) $z = e^{y-x}(y^2-2x^2)$

[10]

$$\frac{\partial z}{\partial x} = -e^{y-x}(y^2-2x^2) + e^{y-x}(-4x) = -e^{y-x}(-2x^2+4x+y^2) = 0$$

~2 + 0.5

$$\frac{\partial z}{\partial y} = e^{y-x}(y^2-2x^2) + e^{y-x}(2y) = e^{y-x}(-2x^2+y^2+2y) = 0$$

$$2x^2-4x-y^2=0$$

$$2x^2-y^2-2y=0 \quad |\cdot(-1) \quad (+)$$

$$-4x+2y=0 \Rightarrow y=2x$$

$$2x^2-4x-4x^2=0$$

$$x^2+2x=0$$

$$x(x+2)=0$$

$$x_1=0, \quad x_2=-2$$

$$y_1=0, \quad y_2=-4$$

2

$(0,0)$, $(-2,-4)$ – стационарне тачке

$$\frac{\partial^2 z}{\partial x^2} = e^{y-x}(-2x^2+4x+y^2) - e^{y-x}(-4x+4) = e^{y-x}(-2x^2+8x+y^2-4)$$

1.5

$$\frac{\partial^2 z}{\partial y^2} = e^{y-x}(-2x^2+y^2+2y) + e^{y-x}(2y+2) = e^{y-x}(-2x^2+y^2+4y+2)$$

$$\frac{\partial^2 z}{\partial x \partial y} = -e^{y-x}(-2x^2+y^2+2y) + e^{y-x}(-4x) = -e^{y-x}(-2x^2+4x+y^2+2y)$$

2 $\underline{(0,0)}$ $A=-4$, $C=2$, $B=0$

$\Delta = AC-B^2 = -8 < 0$ нема екстрема

2 $\underline{(-2,-4)}$ $A=-12/e^2$, $C=-6/e^2$, $B=+8/e^2$

$\Delta = AC-B^2 = 72/e^2 - 64/e^2 = 8/e^2 > 0$ има екстрем

$A<0$ $(C<0)$ максимум $z_{max} = z(-2,-4) = 8/e^2$

$y(1)=2$

4 (тк-1) $xdy-(4y+x^2\sqrt{y})dx=0 \quad |:xdx$

[10] $y'-\frac{4}{x}y=xy^{1/2}$ – Бернулли (1)

$|:y^{1/2}$

$\underline{y^{-1/2}y'}-\frac{4}{x}y^{1/2}=x$

замена: $y^{1/2}=z$ (1)

$\frac{1}{2}\underline{y^{-1/2}y'}=z'$ (1)

$2z'-\frac{4}{x}z=x \quad |:2$

$z'-\frac{2}{x}z=\frac{x}{2}$ – линейное (1)

$z=e^{-\int-\frac{2}{x}dx}\left(C+\int\frac{x}{2}e^{\int-\frac{2}{x}dx}dx\right)$ (1)

$=e^{2\ln|x|}\left(C+\frac{1}{2}\int xe^{-2\ln|x|}dx\right)$

$=x^2\left(C+\frac{1}{2}\int x\cdot\frac{1}{x^2}dx\right)=x^2\left(C+\frac{1}{2}\cdot\ln|x|\right)$ (2)

$y=z^2=x^4\left(C+\frac{1}{2}\ln|x|\right)^2$ о.р. (1)

$y(1)=2: \quad 2=1(C+0)^2 \Rightarrow C=\sqrt{2}$ (1)

$y=x^4\left(\sqrt{2}+\frac{1}{2}\ln|x|\right)^2$ ч.р. (1)

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4 (тк-2) $(x^2y^{1/2}+4y)dx-xdy=0, \quad y(1)=3$

$|:xdx$

$xy^{1/2}+\frac{4}{x}y-y'=0$

$y'-\frac{4}{x}y=xy^{1/2}$ (тк 1 ...)

...

$y(1)=3: \quad 3=1(C+0)^2 \Rightarrow C=\sqrt{3}$

$y=x^4\left(\sqrt{3}+\frac{1}{2}\ln|x|\right)^2.$