M2; K2; 5,6; 28.5.2018.

$\boxed{1}$ (7.1) $z = xy + xe^{y/x}$

$\boxed{5}$

$$\frac{\partial z}{\partial x} = y + e^{y/x} + xe^{y/x} \cdot y\left(-\frac{1}{x^2}\right) = \overbrace{y + e^{y/x} - \frac{y}{x}e^{y/x}}^{= y + \left(1 - \frac{y}{x}\right)e^{y/x}} \quad 0.5$$

$$\frac{\partial z}{\partial y} = x + xe^{y/x} \cdot \frac{1}{x} = x + e^{y/x} \quad 0.5$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \quad 0.5$$

$$\frac{\partial^2 z}{\partial x^2} = \cancel{e^{y/x} \cdot y\left(-\frac{1}{x^2}\right)} - y\left(-\frac{1}{x^2}\right)e^{y/x} + \left(1 - \frac{y}{x}\right)e^{y/x} \cdot y \cdot \left(-\frac{1}{x^2}\right) \quad 1$$

$$= \frac{y}{x^2}e^{y/x} - \frac{y}{x^2}\left(1 - \frac{y}{x}\right)e^{y/x} = \frac{y^2}{x^3}e^{y/x}$$

$$\frac{\partial^2 z}{\partial x \partial y} = 1 + e^{y/x} \cdot y\left(-\frac{1}{x^2}\right) = 1 - \frac{y}{x^2}e^{y/x} \quad 1$$

$$\frac{\partial^2 z}{\partial y^2} = e^{y/x} \cdot \frac{1}{x} = \frac{1}{x}e^{y/x} \quad 1$$

$$d^2 z = \frac{\partial^2 z}{\partial x^2}dx^2 + 2\frac{\partial^2 z}{\partial x \partial y}dx\,dy + \frac{\partial^2 z}{\partial y^2}dy^2 \quad 0.5$$

$\boxed{1.}$ (1.2) $z = xy + ye^{x/y}$
$\boxed{5}$

$$\frac{\partial z}{\partial x} = y + ye^{x/y} \cdot \frac{1}{y} = y + e^{x/y} \quad 0.5$$

$$\frac{\partial z}{\partial y} = x + e^{x/y} + ye^{x/y} \cdot x\left(-\frac{1}{y^2}\right) = x + \left(1 - \frac{x}{y}\right)e^{x/y} \quad 0.5$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \quad 0.5$$

$$\frac{\partial^2 z}{\partial x^2} = e^{x/y} \cdot \frac{1}{y} = \frac{1}{y}e^{x/y} \quad 1$$

$$\frac{\partial^2 z}{\partial x \partial y} = 1 + e^{x/y} \cdot x\left(-\frac{1}{y^2}\right) = 1 - \frac{x}{y^2}e^{x/y} \quad 1$$

$$\frac{\partial^2 z}{\partial y^2} = -x\left(-\frac{1}{y^2}\right)e^{x/y} + \left(1 - \frac{x}{y}\right) \cdot e^{x/y} \cdot x\left(-\frac{1}{y^2}\right)$$

$$= \cancel{\frac{x}{y^2}e^{x/y}} - \frac{x}{y^2}\left(\cancel{1} - \frac{x}{y}\right)e^{x/y} = \frac{x^2}{y^3}e^{x/y} \quad 1$$

$$dz^2 = \frac{\partial^2 z}{\partial x^2}dx^2 + \frac{\partial^2 z}{\partial x \partial y}dxdy + \frac{\partial^2 z}{\partial y^2}dy^2 \quad 0.5$$

$\boxed{2}$ (7.1) $z = \frac{8}{x} + \frac{x}{y} + y \quad (x \neq 0,\ y \neq 0)$

[10]

$$\frac{\partial z}{\partial x} = -\frac{8}{x^2} + \frac{1}{y} = 0 \qquad \frac{-8y + x^2}{x^2 y} = 0 \qquad -8y + x^2 = 0$$

~2,5

$$\frac{\partial z}{\partial y} = -\frac{x}{y^2} + 1 = 0 \qquad \frac{-x + y^2}{y^2} = 0 \qquad -x + y^2 = 0$$

$$\cancel{y^2 = 8y = 0}$$

$$x = y^2$$

$$-8y + y^4 = 0$$

$$y(y^3 - 8) = 0$$

$$y(y-2)(y^2 + 2y + 4) = 0$$

$$\cancel{y_1 = 0} \qquad y_2 = 2$$

$$\cancel{x_1 = 0} \qquad x_2 = 4$$

Стационарна тачка: $\cancel{(0,0)}$, $(4,2)$ ~3

$$\frac{\partial^2 z}{\partial x^2} = -8(-2)x^{-3} = \frac{16}{x^3}$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{y^2} \qquad \sim 1,5$$

$$\frac{\partial^2 z}{\partial y^2} = -x(-2)y^{-3} = \frac{2x}{y^3}$$

~~(0,0) A =~~

$(4,2)$: $A = \frac{\partial^2 z}{\partial x^2}(4,2) = \frac{1}{4}$; $B = \frac{\partial^2 z}{\partial x \partial y}(4,2) = -\frac{1}{4}$

$C = \frac{\partial^2 z}{\partial y^2}(4,2) = \cancel{\frac{1}{2}}\ 1$

$\Delta = AC - B^2 =$ ~~$\frac{1}{16} - 1 < 0$ није екстрем~~ ~3

$\frac{1}{4} - \frac{1}{16} = \frac{3}{16} > 0$ јесте екстрем

$A > 0$ $(C > 0)$ минимум; $z_{min} = \frac{8}{4} + \frac{4}{2} + 2 = 6$

$\boxed{2}$ (4.2) $z = \frac{8}{y} + \frac{y}{x} + x \quad (x \neq 0,\ y \neq 0)$

$\boxed{10}$

$$\left.\begin{aligned} \frac{\partial z}{\partial x} &= -\frac{y}{x^2} + 1 = 0 \\ \frac{\partial z}{\partial y} &= -\frac{8}{y^2} + \frac{1}{x} = 0 \end{aligned}\right| \quad \left.\begin{aligned} \frac{-y + x^2}{x^2} &= 0 \\ \frac{-8x + y^2}{xy^2} &= 0 \end{aligned}\right| \quad \begin{aligned} &-y + x^2 = 0 \Rightarrow y = x^2 \\ &-8x + y^2 = 0 \end{aligned}$$

~2,5

$$-8x + x^4 = 0$$

$$x(x-2)(x^2+2x+4) = 0$$

$\cancel{x_1 = 0}$ $\quad x_2 = 2$

$y_2 = 4$ ~3

Стаціонарна точка: $(2,4)$

~1,5

$$\frac{\partial^2 z}{\partial x^2} = -y\,\frac{-2}{x^3} = \frac{2y}{x^3} \overset{(2,4)}{=} 1 = A$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{x^2} \overset{(2,4)}{=} -\frac{1}{4} = B$$

$$\frac{\partial^2 z}{\partial y^2} = -8 \cdot \frac{-2}{y^3} = \frac{16}{y^3} \overset{(2,4)}{=} \frac{1}{4} = C$$

$\Delta = AC - B^2 = \frac{1}{4} - \frac{1}{16} = \frac{3}{16} > 0$ Точка екстр.

$A > 0$ $(C > 0)$ Мінімум

$z_{min} = 6$.

$(z(2,4))$

~3

3 (7.1) $(x^2-2xy-y^2)dy+(y^2-2xy-x^2)dx=0$, $y(1)=-2$

[10]

$$/ : x^2dx$$

$$\left(1-2\frac{y}{x}-\left(\frac{y}{x}\right)^2\right)y'+\left(\left(\frac{y}{x}\right)^2-2\frac{y}{x}-1\right)=0$$

$$y'=\frac{\left(\frac{y}{x}\right)^2-2\frac{y}{x}-1}{\left(\frac{y}{x}\right)^2+2\frac{y}{x}-1}$$ – хомогена (2)

$\frac{y}{x}=z$ (0.5), $y=xz$, $y'=z+xz'$ (0.5)

$$z+xz'=\frac{z^2-2z-1}{z^2+2z-1}$$

$$x\frac{dz}{dx}=\frac{z^2-2z-1}{z^2+2z-1}-z=\frac{z^2-2z-1-z^3-2z^2+z}{z^2+2z-1}$$

$$=-\frac{z^3+z^2+z+1}{z^2+2z-1}$$

$$\frac{z^2+2z-1}{z^3+z^2+z+1}dz=-\frac{dx}{x}$$ – раздвојене променљиве (2)

$$/\int$$

(2) $$I=\int\frac{z^2+2z-1}{\underbrace{z^3+z^2+z+1}_{z^2(z+1)+(z+1)}}dz=\int\frac{z^2+2z-1}{(z^2+1)(z+1)}dz$$

$$\frac{z^2+2z-1}{(z^2+1)(z+1)}=\frac{Az+B}{z^2+1}+\frac{C}{z+1}\quad /(z^2+1)(z+1)$$

$$z^2+2z-1=(Az+B)(z+1)+C(z^2+1)$$

$$=Az^2+Az+Bz+B+Cz^2+C$$

z^2: $1=A+C \Rightarrow C=1-A$

z: $2=A+B$

1: $-1=B+C$ ✓

$$\left\{\begin{array}{l}A+B=2\\B-A=-2\end{array}\right.\ (+)\qquad 2B=0\Rightarrow \boxed{B=0}$$

$$\boxed{A=2},\ \boxed{C=-1}$$

$$I = \int \left(\frac{2z}{z^2+1} + \frac{-1}{z+1}\right) dz = \ln(z^2+1) - \ln|z+1| = \ln\frac{z^2+1}{|z+1|}$$

$$\ln\frac{z^2+1}{|z+1|} = -\ln|x| + \underbrace{C}_{\ln C} = \ln\frac{C}{|x|}$$

$$\frac{z^2+1}{z+1} = \frac{C}{x}$$

$$\frac{\left(\frac{y}{x}\right)^2+1}{\frac{y}{x}+1} = \frac{C}{x}$$

$$\frac{\frac{x^2+y^2}{\cancel{x^2}}}{\frac{x+y}{\cancel{x}}} = \frac{C}{\cancel{x}}$$

$$\frac{x^2+y^2}{x+y} = C \quad \text{о.р.} \quad 1$$

$$y(1) = -2$$

$$\frac{1^2+(-2)^2}{1+(-2)} = C \quad \Rightarrow \quad C = -5 \quad 1$$

$$\frac{x^2+y^2}{x+y} = -5 \quad \text{ч.р.} \quad 1$$

~~$\boxed{3}$ (1.2) $(x^2-2xy-y^2)dy + (x^2+2xy-y^2)dx = 0$, $y(-1)=2$~~

$\boxed{3}$ (1.2) $(y^2+2xy-x^2)\,dy + (x^2+2xy-y^2)\,dx = 0$, $y(-1) = 2$

...

$$\frac{(-1)^2+2^2}{-1+2} = C \Rightarrow C = 5; \quad \frac{x^2+y^2}{x+y} = 5$$

[4] (17.1) $y+(x-1)(y'+y^2)=0 \quad \Big| : (x-1)$ о.р.

[10]

$y'+\frac{1}{x-1}y=-y^2$ — Бернулли ①

$\Big| : y^2$

$y^{-2}y'+\frac{1}{x-1}y^{-1}=-1$

$y^{-1}=z \;;\; -y^{-2}y'=z'$

$-z'+\frac{1}{x-1}z=-1$

$z'-\frac{1}{x-1}z=1$ — линейн. ④

$$z=e^{-\int\left(-\frac{1}{x-1}\right)dx}\left(C+\int 1\cdot e^{\int\left(-\frac{1}{x-1}\right)dx}dx\right)$$

$$=e^{\ln|x-1|}\left(C+\int e^{-\ln|x-1|}dx\right)=$$

$$=(x-1)\left(C+\int\frac{dx}{x-1}\right)=(x-1)\left(C+\ln|x-1|\right) \quad ④$$

$$y=\frac{1}{(x-1)(C+\ln|x-1|)} \quad ①$$

$\boxed{4}$ (Ч.2) $y-(x-1)(y'+y^2)=0 \quad \Big| : (-(x-1))$

$\boxed{10}$

$$y' - \frac{1}{x-1}y = y^2 \quad \Big| : y^2 \quad \text{— Бернулли} \quad 1$$

$$y^{-2}y' - \frac{1}{x-1}y^{-1} = 1$$

$$y^{-1} = z, \quad -y^{-2}y' = z'$$

$$-z' - \frac{1}{x-1}z = 1$$

$$z' + \frac{1}{x-1}z = -1 \quad \text{— линейное} \quad 4$$

$$z = e^{-\int \frac{1}{x-1}dx}\left(C + \int (-1)\, e^{\int \frac{1}{x-1}dx}\right)$$

$$= e^{-\ln|x-1|}\left(C - \int e^{\ln|x-1|}dx\right)$$

$$= \frac{1}{x-1}\left(C - \int (x-1)\,dx\right) = \frac{1}{x-1}\left(C - \frac{x^2}{2} + x\right) \quad 4$$

$$y = \frac{x-1}{C - \frac{x^2}{2} + x} \quad \text{— о.р.} \quad 1$$