1. Solutions of the problems send to email:

   numericke.metode.metode@gmail.com

   till 23:59 on 07.12.2014 sharp. Solutions of the problems which arrive with some delay are not going to be considered, regerdless of the excuse.

2. In the subject field of the email put

   KMA.NM.999/GG

   where:

   - KMA-denotes Mathematics department
   - NM-denotes Numerical methods
   - 999/GG-student number including leading zero

   For example, if your student number is 23 and you have been enrolled to the faculty in 2011, your subject field should be:

   KMA.NM.023/11

   Similarly, if your student number is 124 and you have been enrolled in 2011, your subject should be

   KMA.NM.124/11

3. Solutions of the problems, program in Matlab, figures as illustrations in JPEG format, text in Word or scanned text on the paper sheet, send as attachment of your email.

4. Every problem put in the different zip archive with names

   zadatak01.zip, zadatak02.zip, zadatak03.zip, zadatak04.zip, zadatak05.zip

   and put archives in your attachment.

5. Last received email is the only one to be considered, so, in your last email put all the solutions you want to hand in.

6. Every cheating, like copying solutions of your colleague as your own, is going to be sanctioned, in another words, it is better for you to hand in one solution instead of three copied solutions.

7. Solution of every problem brings 25% to your score. Maximal possible score is 125%.

University of Belgrade
Faculty of Mechanical Engineering 05.12.2014.

II test in Numerical methods

1 Determine the lower bound of the number of significant digits in the computation of the function `exp`. Write script file in `Matlab` which computes number of the significant digits in computation of the function `exp` on the interval $[-100, 100]$, with the step size .1, under the assumption that function arguments are given with the upper bound of the relative error of 10^{-5}. Plot the dependence of the number of significant digits, obtained by experiment and by the theory, with respect to the argument.

2. Prove that the alternative series

$$\sum_{k=0}^{+\infty}(-1)^k\frac{20^k}{k!}$$

is convergent. Find the sum of the series. Write the function in `Matlab` which computes the sum of the series (sum of the series can not be `NaN`) and determine the relative error and the number of significant digits of the result. Using the model of the format `double` with the 16 decimal digits in mantissa explain your results.

3. Using `Matlab` create upper triangular matrix

$$A_k = \begin{bmatrix} 10^{-k} & 0 & 0 & \dots & 0 \\ 1 & 10^{-k} & 0 & \dots & 0 \\ 1 & 1 & 10^{-k} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 10^{-k} \end{bmatrix}.$$

of order 10. Compute the vector $b_k = A_k x$, where x is the vector with all coordinates equal to one.

For $k = 1, \dots, 15$, using function `linsolve` solve the system $A_k x_k^G = b_k$. Plot dependence of the number of the significant digits in the solution x_k^G and dependence of the condition number of the matrix A_k with respect to k. Give the interpretation of your figure.

4. Consider the iterative method

$$x^{k+1} = x^k + \frac{20}{7}(b - Ax^k), \quad k \in \mathbb{N}_0, \quad A = \begin{bmatrix} \frac{23}{180} & \frac{17}{180} & \frac{11}{180} \\ -\frac{3}{20} & \frac{7}{20} & \frac{3}{20} \\ -\frac{11}{45} & \frac{4}{45} & \frac{47}{90} \end{bmatrix}, \quad b = \begin{bmatrix} \frac{17}{60} \\ \frac{7}{20} \\ \frac{11}{30} \end{bmatrix}.$$

Prove that the series x^k, $k \in \mathbb{N}_0$ converge to the soluton of the system of linear equations $Ax = b$. Determine the increment of the number of sigificant digits per iteration. Determine approximately the number of iterations needed to achieve machine precision. Implement method in the script file in `Matlab` and plot the dependence of the number of significnat digits with respect to the number of iterations. Compare theoretical and experimental results.

5. Write the function `fS` in `Matlab` which implements Steffensen method

$$x_{k+1} = x_k - \frac{(f(x_k))^2}{f(x_k + f(x_k)) - f(x_k)},$$

where function `fS` accepts as arguments initial approximation `x0`, maximal allowed relative error `prec`, maximal allowed absolute error `acc`, and maximal allowed number of iterations `maxIter`.

Apply the function `fS` to solve the equation $\sin x - \cos x = 0$ with initial approximation $x_0 = 1$. Try to determine the order of the convergence of iterative process by the computation of the increment of significant digits in every iteration. Give some comparision of Newton and Steffensen method.

Show analytically that Steffensen method has the order of the convergence two and determine the asymptotic error constant.

prof. Aleksandar Cvetkovic, PhD