

1. Solutions of the problems send to email:

numericke.metode.metode@gmail.com

till 23:59 on 19.01.2015. sharp. Solutions of the problems which arrive with some delay are not going to be considered, regardless of the excuse.

2. In the subject field of the email put

KMA.NM.999/GG

where:

- KMA-denotes Mathematics department
- NM-denotes Numerical methods
- 999/GG-student number including leading zero

For example, if your student number is 23 and you have been enrolled to the faculty in 2011, your subject field should be:

KMA.NM.023/11

Similarly, if your student number is 124 and you have been enrolled in 2011, your subject should be

KMA.NM.124/11

3. Solutions of the problems, program in Matlab, figures as illustrations in JPEG format, text in Word or scanned text on the paper sheet, send as attachment of your email.

4. Every problem put in the different zip archive with names

zadatak01.zip, zadatak02.zip, zadatak03.zip, zadatak04.zip, zadatak05.zip

and put archives in your attachment.

5. Last received email is the only one to be considered, so, in your last email put all the solutions you want to hand in.
6. Every cheating, like copying solutions of your colleague as your own, is going to be sanctioned, in another words, it is better for you to hand in one solution instead of three copied solutions.
7. Solution of every problem brings 25% to your score. Maximal possible score is 125%.

III test in Numerical methods

- 1 a) Construct Lagrange interpolation polynomial for 30 equidistant nodes on the interval $[1, 10]$, for the function $f(x) = (x - 1)^2(x - 5)^2(x - 9)^2 + 1$. Show on the graph the number of significant digits with which interpolation polynomial approximates function f on the interval $[1, 10]$.
 - b) Using graph of the Lebesgue function, for the given distribution of interpolation nodes, give the theoretical bound for the number of significant digits on the interval $[0, 10]$, Compare the results obtained with the theoretical method and with the experiment. Explain the reason why results are in the good agreement.
 - c) Using graph of the Lebesgue function give the estimation of the minimal number of significant digits obtained by interpolation of the function $f(x) = (x - 1)^2(x - 5)^2(x - 10)^2 + 1$ on the interval $[1, 10]$, if the input data have at least 7 significant digits.
2. Suppose you are given measurement equipment which has upper bound for the relative error 10^{-m} , $m \in \mathbb{N}$, and that you are sampling constant function $f(x) = .1$, on the interval $[0, 5]$ with the sampling step $h = .001$.
 - a) For $m = 5$, determine Newton interpolation polynomial of the highest possible degree n , constructed in the interpolation nodes $x_k = 0 + kh$, $k = 0, \dots, n$, such that the number of significant digits at the point x_n with which Newton interpolation polynomial approximates function f is at least two.
 - b) Plot the graph of the dependence of the highest degree of the Newton interpolation polynomial n , for achieving two significant digits at point x_n , with respect to the number of significant digits of the input data m . Explain why this graph is linear.
3. a) Plot the dependence of the number of significant digits with respect to the degree of exactness $n = 2k$, $k = 1, \dots, 30$, for determining derivative of the function $f(x) = \log x$, at the point $x = 5$, for one-sided and two-sided methods for differentiation, with $h = .1$.
 - b) Compare obtained results with theoretical results. Explain the reason of the discrepancy. Explain why formula for two-sided method has the number of significant digits which, except for the small degrees of exactness, does not depend on the degree of exactness n .

4. Using composite trapezoidal rule for the computation of the integral

$$\int_{-1}^1 \left(1 + \frac{\sin(1000\pi \log(2+x)/\log(3))}{2+x} \right) dx = 2,$$

plot the dependence of the number of significant digits as a function of the logarithm of the number of the subintervals $N = 2^k$, $k = 1, \dots, 20$. Compare obtained results with the theoretical error estimation. Show that the increment in the significant digits, when number of subintervals increases two times, for the theoretical estimate and experimental results are the same. Explain why theoretical estimate gives the smaller number of the significant digits than the number of significant digits obtained using experiment with the same number of subintervals.

5. a) Find the exact solution of the Cauchy problem

$$y' = 2xy, \quad y(0) = 1.$$

b) Write the functions in `Matlab` which computes the solution of the Cauchy problem, using Euler method

$$y_{n+1} = y_n + hf_k, \quad y_0 = 1,$$

and Adams-Bashforth method

$$y_{n+3} = y_{n+2} + \frac{h}{12}(23f_{n+2} - 15f_{n+1} + 5f_n), \quad y_0 = 1, \quad y_1 \approx 1 + h62 + \frac{h^4}{2}, \quad y_2 \approx 1 + 4h^2 + 8h^4.$$

c) Plot the graph of the dependence of the number of significant digits of the approximation of the solution $y(1)$ with respect to the logarithm of the step $h = 2^{-k}$, $k = 1, \dots, 15$. Give the estimate of the order of the Adams-Bashforth method using graph.

d) Plot the graph of the dependence of the number of significant digits of the solution of the Cauchy problem, when the computation is performed using Adams-Bashforth method, on the interval $[0, 1]$ for $h = 2^{-10}$, Explain the shape of the graph.

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