

Mašinski Fakultet Univerziteta u Beogradu



## **Mašinski elementi II**

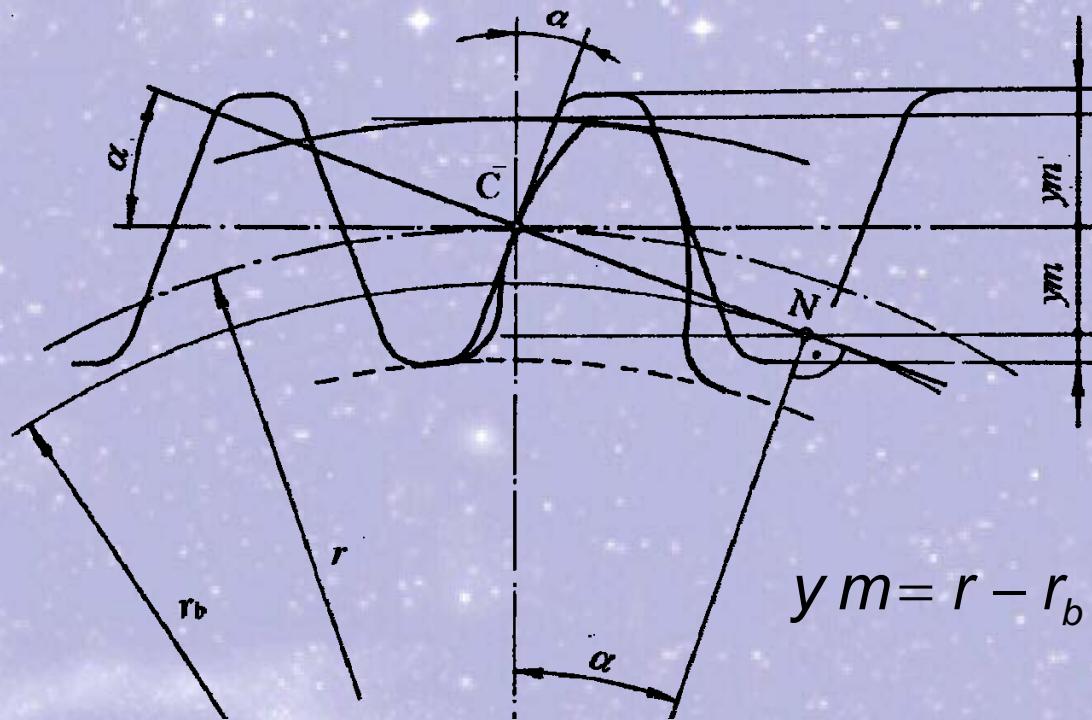
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# **Geometrija i kinematika zupčanih parova sa pravim zupcima deo 2.**

Petak 11. 03. 2011.

# Granični broj zubaca



$$r_b = r \cos \alpha$$

$$y_m = r - r_b \cos \alpha = \frac{mz}{2} - \frac{mz}{2} \cos^2 \alpha$$

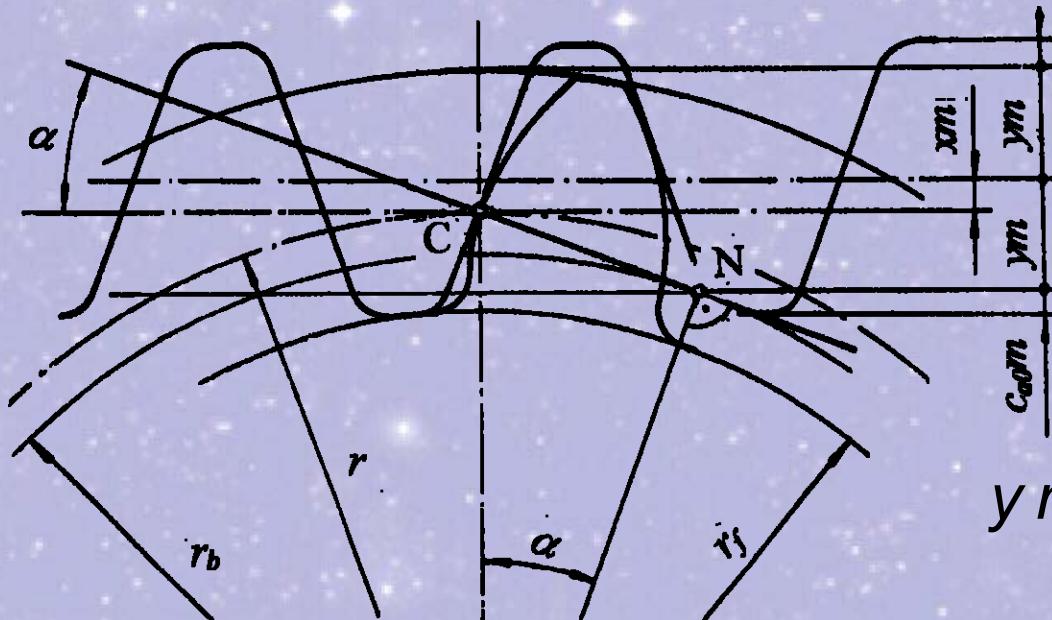
Za koeficijent visine zupca  $y=1$ , sledi:

$$m = \frac{mz}{2} (1 - \cos^2 \alpha)$$

$$\text{Granični broj zubaca } Z_g = \frac{2y}{\sin^2 \alpha} = \frac{2}{\sin^2 \alpha} = 17,1 \quad \text{za } \alpha = \alpha_n = 20^\circ$$



# Koeficijent pomeranja profila



Pomeranje profila  
x·m

$$y_m = r - r_b \cos \alpha + x_{\min} m$$

$$x_{\min} m = y_m - (r - r_b \cos \alpha) = y_m - \left( \frac{mz}{2} - \frac{mz}{2} \cos^2 \alpha \right)$$

Minimalni koeficijent  
pomeranja profila

$$x_{\min} = y - \frac{z}{2} \sin^2 \alpha$$

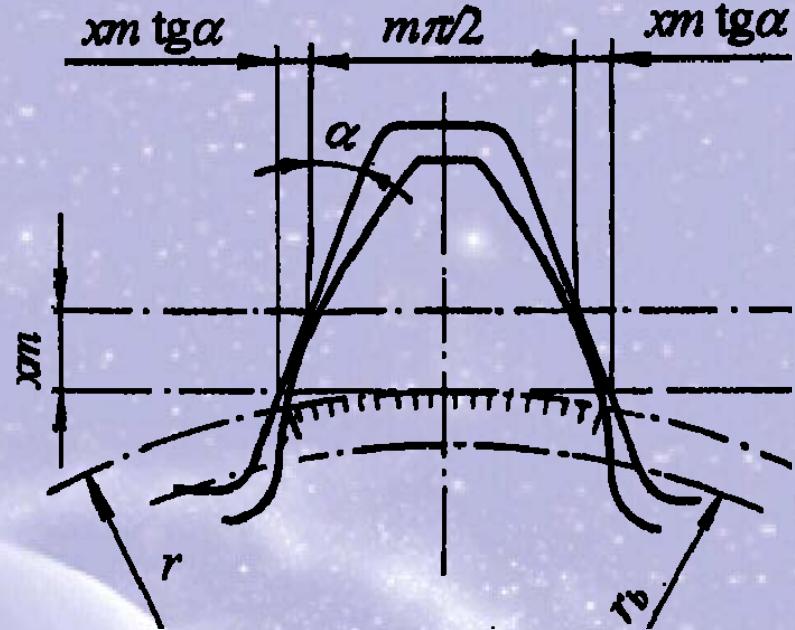
za  $z=17,1$   $x_{\min}=0$

za  $z<17,1$   $x_{\min}>0$

# Mere profila zupca



## Lučna debeljina zupca



Lučna debeljina zupca na bilo  
kojoj kružnici:

Lučna debeljina zupca na  
podeonoj kružnici:

$$s = m \left( \frac{\pi}{2} + 2x \operatorname{tg} \alpha \right)$$

bez pomeranja profila,  $xm=0$

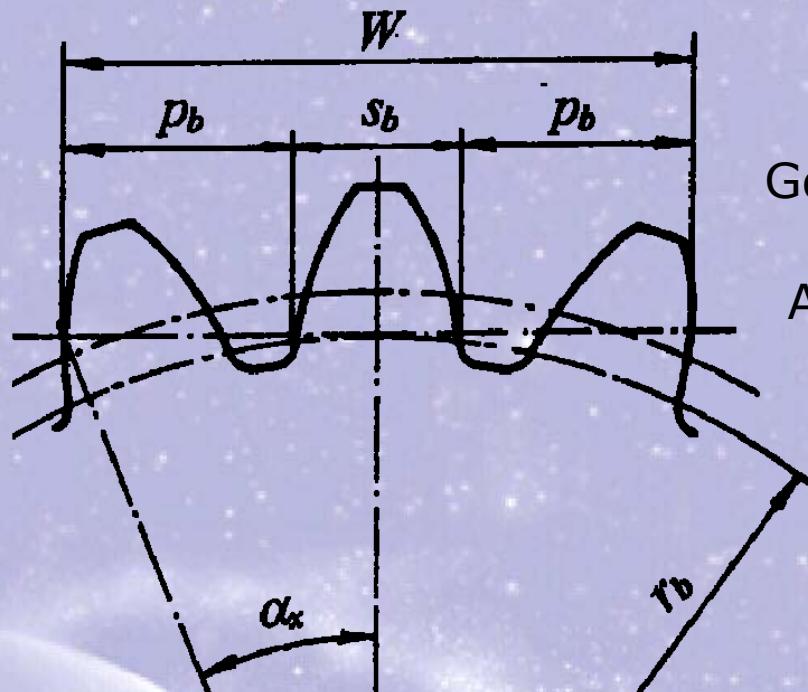
$$s = m \frac{\pi}{2} = \frac{p}{2}$$

$$s_y = d_y \left( \frac{s}{d} + \operatorname{inv} \alpha - \operatorname{inv} \alpha_y \right)$$

# Mere profila zupca



## Mera preko zubaca



$$W = (z_w - 1)p_b + s_b$$

Gde je  $z_w$  merni broj zubaca (na slici  $z_w=3$ )

Ako se zameni:

$$p_b = p \cos \alpha = m\pi \cos \alpha$$

$$s_b = d_b \left( \frac{s}{d} + \operatorname{inv} \alpha \right) \quad \text{gde su:}$$

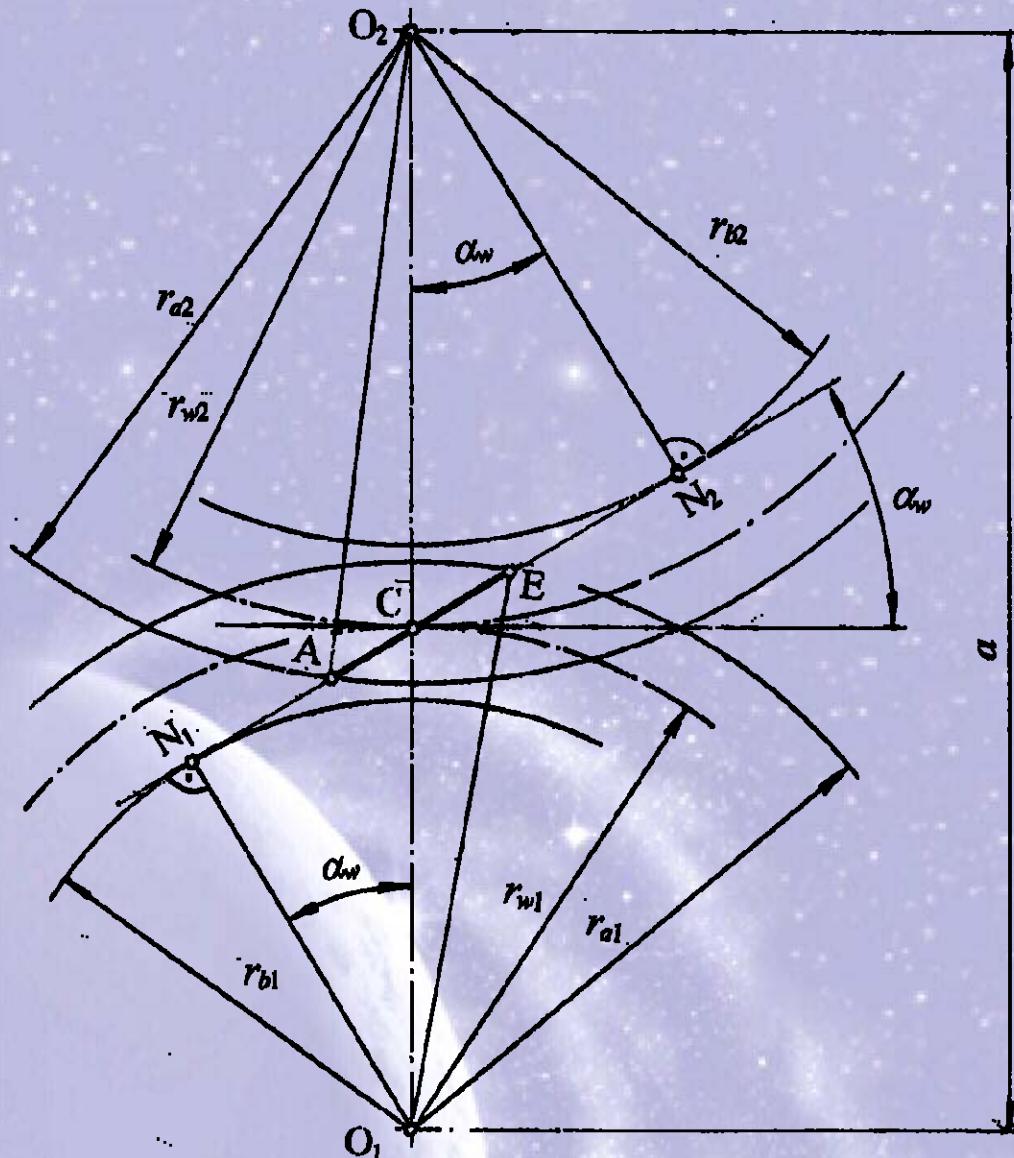
$$d = m z \qquad s = m \left( \frac{\pi}{2} + 2x \operatorname{tg} \alpha \right)$$

dobija se:  $W = m \cos \alpha [\pi(z_w - 0,5) + z \operatorname{inv} \alpha + 2x \operatorname{tg} \alpha]$

bez pomeranja profila,  $xm=0 \quad z_w = \frac{\alpha}{\pi} + 0,5 \quad$  gde je  $\alpha / \text{rad}$

# Sprezanje pravozubih zupčanika

$O_1O_2 = a$  - međuosno rastojanje



$$\begin{aligned} N_1N_2 &= N_1C + CN_2 = \\ &= r_{w1} \sin \alpha_w + r_{w2} \sin \alpha_w = \\ &= (r_{w1} + r_{w2}) \sin \alpha_w \end{aligned}$$

odakle sledi:

$$N_1N_2 = a \sin \alpha_w$$

gde su:

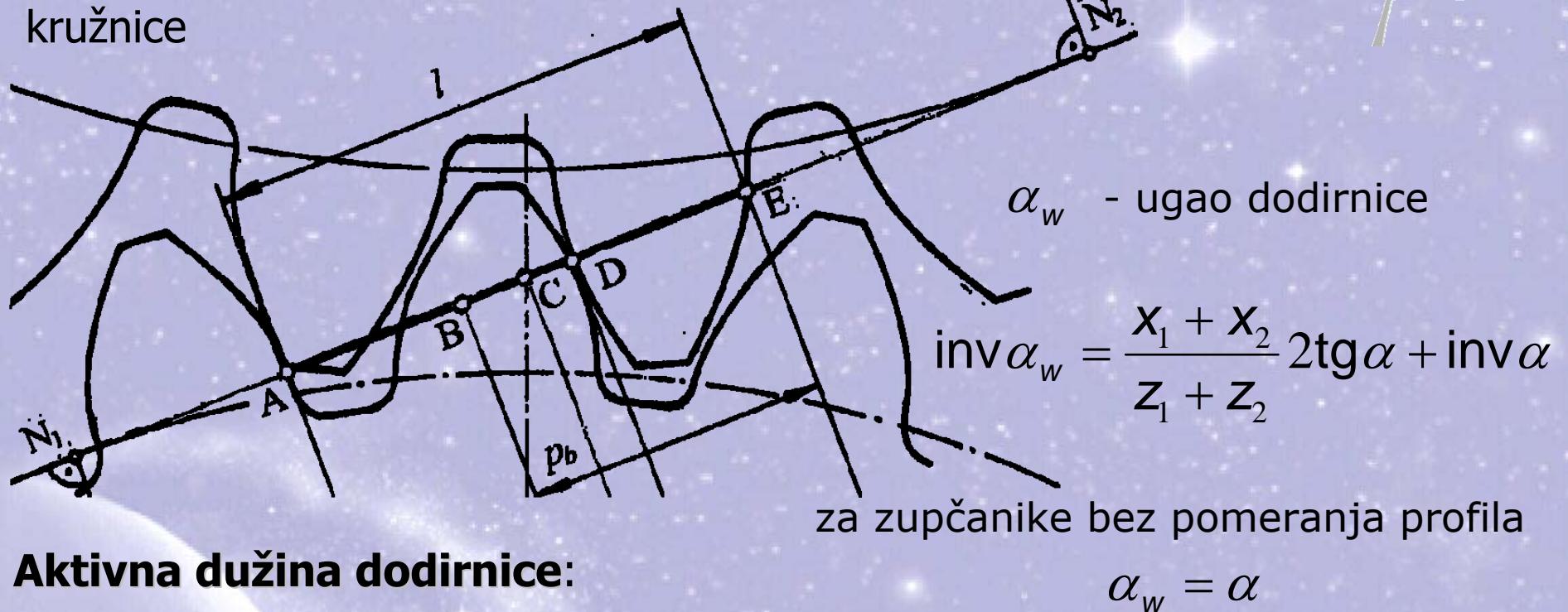
$r_{w1}$  i  $r_{w2}$  - poluprečnici kinematskih kružnica

$\alpha_w$  - ugao dodirnice

# Dodirnica

**Dodirnica - geometrijsko mesto tačaka dodira u nepokretnoj ravni**

Kod cilindričnih zupčanika - prava koja je zajednička tangenta na osnovne kružnice



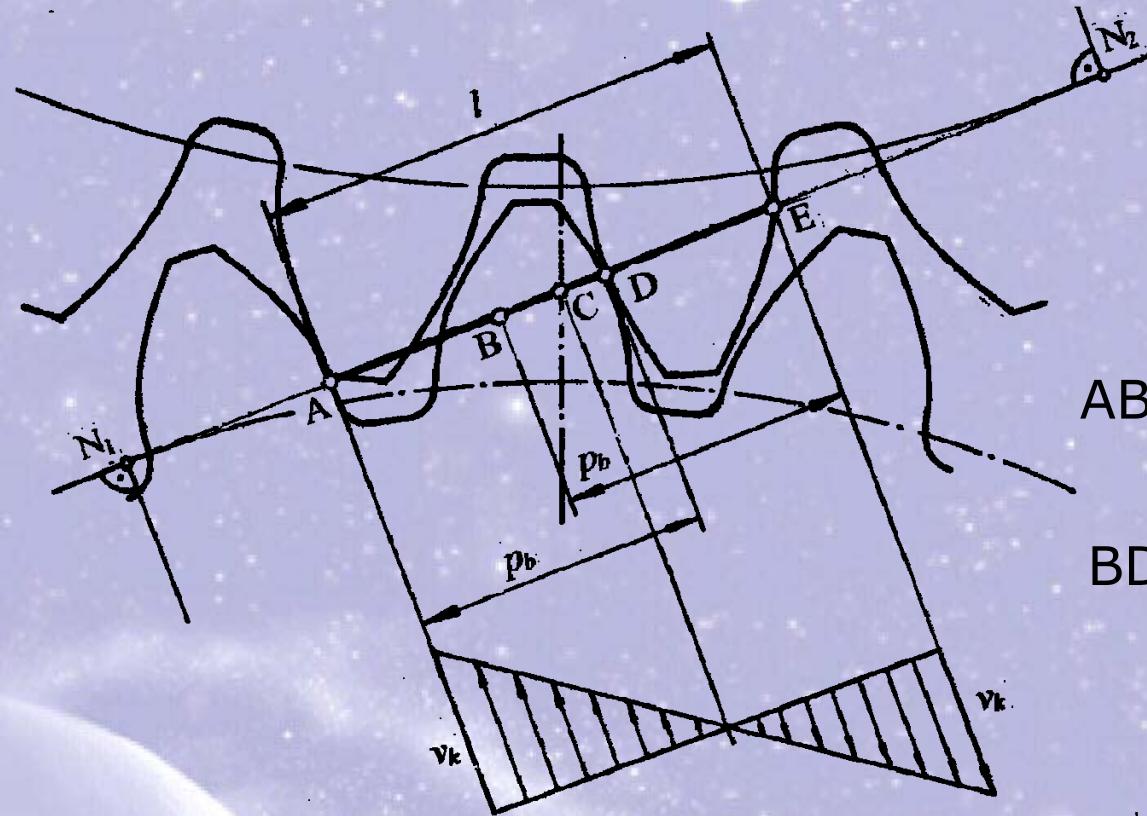
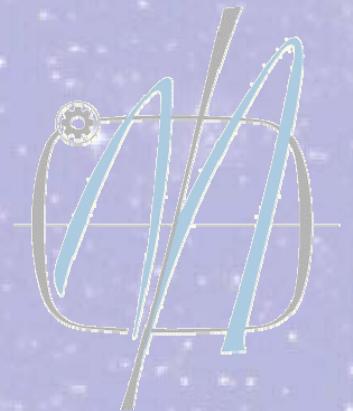
**Aktivna dužina dodirnice:**

$$\alpha_w = \alpha$$

$$AE = \ell = \sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a \sin \alpha_w$$

jer je:  $r_{w1} + r_{w2} = a$

# Stepen sprezanja profila



AB i DE – dvoparna sprega

BD – jednoparna sprega

$$\varepsilon_\alpha = \frac{\ell}{p_b} = \frac{\ell}{m\pi \cos \alpha} = \frac{g_\alpha}{p} = 1 \dots 2$$

# Dimenzije zupčanog para



## Međuosno rastojanje

$$a = r_{w1} + r_{w2} = \frac{r_{b1}}{\cos \alpha_w} + \frac{r_{b2}}{\cos \alpha_w} = (r_1 + r_2) \frac{\cos \alpha}{\cos \alpha_w}$$

$$a = m \frac{z_1 + z_2}{2} \frac{\cos \alpha}{\cos \alpha_w}$$

## Prečnici kinematskih kružnica

$$u = \frac{r_{w2}}{r_{w1}} = \frac{r_{b2}/\cos \alpha_w}{r_{b1}/\cos \alpha_w} = \frac{r_2 \cos \alpha}{r_1 \cos \alpha} = \frac{r_2}{r_1} = \frac{z_2}{z_1}$$

$$r_{w1} = \frac{a}{u+1}$$

za zupčanike bez pomeranja profila       $\alpha_w = \alpha$

$$d_{w1} = d_1 = m z_1$$

$$r_{w2} = u r_{w1}$$

$$d_{w2} = d_2 = m z_2$$

# Dimenzijs zupčanog para



## Prečnici temenih kružnica

$$r_{a1} + cm + r_{f2} = a$$

$$r_{a1} = a - r_{f2} - cm$$

cm - temeni zazor

$$r_{a2} = a - r_{f1} - cm$$

## Prečnici podnožnih kružnica

$$r_f = r - m(1 + C_{a0} - x)$$

gde je  $C_{a0}=0,25$

$$r_{f1} = r_1 - m(1,25 - x_1)$$

$$r_{f2} = r_2 - m(1,25 - x_2)$$