Promena pritiska u dvofaznom toku

Bilans količine kretanja prema modelu dva fluida

- bilans količine kretanja za tečnu fazu

$$\frac{\partial(\alpha_1\rho_1 u_1)}{\partial t}+\frac{\partial\left(\alpha_1\rho_1 u_1^2\right)}{\partial z}=-\alpha_1\frac{\partial p}{\partial z}+a_{21}\tau_{21}-a_{1z}\tau_{1z}-\alpha_1\rho_1 g\sin\theta+\Gamma_{21}u_2-\Gamma_{12}u_1 \qquad (1)$$

- bilans količine kretanja za gasnu fazu

$$\frac{\partial(\alpha_2\rho_2 u_2)}{\partial t}+\frac{\partial\left(\alpha_2\rho_2 u_2^2\right)}{\partial z}=-\alpha_2\frac{\partial p}{\partial z}-a_{21}\tau_{21}-a_{2z}\tau_{2z}-\alpha_2\rho_2 g\sin\theta-\Gamma_{21}u_2+\Gamma_{12}u_1 \qquad (2)$$

Sabiranjem jednačina (1) i (2) za stacionarne uslove $\left(\frac{\partial}{\partial t}=0\right)$ i korišćenjem relacije

$\alpha_1+\alpha_2=1$

$$\frac{dp}{dz}=-\left(a_{1z}\tau_{1z}+a_{2z}\tau_{2z}\right)-\left(\alpha_1\rho_1+\alpha_2\rho_2\right)g\sin\theta-\frac{d}{dz}\left(\alpha_1\rho_1 u_1^2+\alpha_2\rho_2 u_2^2\right)$$

$$\rho=\alpha_1\rho_1+\alpha_2\rho_2$$

$$u_1=\frac{(1-\chi)G}{\alpha_1\rho_1}\Rightarrow \alpha_1\rho_1 u_1^2=\frac{(1-\chi)^2 G^2}{\alpha_1\rho_1}$$

$$u_2=\frac{\chi G}{\alpha_2\rho_2}\Rightarrow \alpha_2\rho_2 u_2^2=\frac{\chi^2 G^2}{\alpha_2\rho_2}$$

$$\frac{dp}{dz}=-\left(a_{1z}\tau_{1z}+a_{2z}\tau_{2z}\right)-\rho g\sin\theta-G^2\frac{d}{dz}\left(\frac{(1-\chi)^2}{\alpha_1\rho_1}+\frac{\chi^2}{\alpha_2\rho_2}\right)$$

$$\frac{dp}{dz}=\left(\frac{dp}{dz}\right)_{tr}+\left(\frac{dp}{dz}\right)_g+\left(\frac{dp}{dz}\right)_{ak}$$

Ukupna promena pritiska usled:

- gravitacije,
- trenja na zidovima,
- ubrzanja (akceleracije).

Međufazno trenje ne utiče direktno na promenu pritiska, ali utiče posredno jer međufazno trenje utiče na strukturu dvofaznog toka i α_k, a samim tim in a ostale članove u gornjoj jednačini.

Za homogeno strujanje $u_1=u_2\ i\ \chi=x$, i može se napisati

$$a_{1z}\tau_{1z}+a_{2z}\tau_{2z}=(a\tau)_{tr}$$

Pošto je po definiciji $x=\frac{\rho_2 V_2}{\rho_1 V_1+\rho_2 V_2}=\frac{\alpha_2\rho_2}{\alpha_1\rho_1+\alpha_2\rho_2}$

$$1-x=\frac{\rho_1 V_1}{\rho_1 V_1+\rho_2 V_2}=\frac{\alpha_1\rho_1}{\alpha_1\rho_1+\alpha_2\rho_2}$$

jednačina za jediničnu promenu pritiska se može napisati u sledećem obliku

$$\frac{dp}{dz} = -\left(a\tau\right)_{tr} - \rho g \sin\theta - G^2 \frac{d}{dz}\left(\frac{1}{\rho}\right)$$

što predstavlja promenu pritiska kako za homogeno dvofazno strujanje, tako i za jednofazno strujanje.

Množitelj pada pritiska usled trenja u dvofaznom toku, Φ^2

$$\left(-\frac{dp}{dz}\right)_{tr,DT} = \left(a\tau\right)_{tr} = \frac{4}{D} f \frac{\rho u^2}{2} = \frac{4}{D} f \frac{G^2}{2\rho} = \left(\frac{4}{D} f_{10} \frac{G^2}{2\rho_1}\right)\left(\frac{f}{f_{10}} \frac{\rho_1}{\rho}\right) = \left(-\frac{dp}{dz}\right)_{tr,10} \Phi_{10}{}^2$$

$$\Phi_{20}{}^2, \Phi_1{}^2, \Phi_2{}^2 \qquad f \approx f_{10}, \qquad \Phi_{10}{}^2 = \frac{\rho_1}{\rho} = \frac{v}{v_1} = \left[1 + x\left(\frac{v_2}{v_1} - 1\right)\right]$$

npr. $\left(-\frac{dp}{dz}\right)_{tr,DT} = \left(-\frac{dp}{dz}\right)_{tr,1} \Phi_1{}^2 = \frac{4}{D} f_1 \frac{(1-x)^2 G^2}{2\rho_1} \Phi_1{}^2$

Martineli parametar

$$X^2 = \frac{\left(\frac{dp}{dz}\right)_{tr,1}}{\left(\frac{dp}{dz}\right)_{tr,2}}$$

$$\Phi_1{}^2 = 1 + \frac{C}{X} + \frac{1}{X^2}$$

$$\Phi_2{}^2 = 1 + CX + X^2$$

$$\left(-\frac{dp}{dz}\right)_{tr,DT} = \left(-\frac{dp}{dz}\right)_{tr,1} + \overbrace{C\left[\left(-\frac{dp}{dz}\right)_{tr,1}\left(-\frac{dp}{dz}\right)_{tr,2}\right]^{1/2}}^{\text{novi član koji je karakteristika dvofaznog toka}} + \left(-\frac{dp}{dz}\right)_{tr,2} \qquad (3)$$

$$\left(-\frac{dp}{dz}\right)_{tr,DT} = f \frac{G^2}{2\rho} \frac{4}{D}$$

$$\left(-\frac{dp}{dz}\right)_{tr,1} = f_1 \frac{(1-x)^2 G^2}{2\rho_1} \frac{4}{D}$$

$$\left(-\frac{dp}{dz}\right)_{tr,2} = f_2 \frac{x^2 G^2}{2\rho_2} \frac{4}{D}$$

Ako je $f \approx f_1 \approx f_2$ tada se iz jednačine (3) dobija

$$\frac{1}{\rho} = \frac{(1-x)^2}{\rho_1} + C\left[\frac{(1-x)^2}{\rho_1} \frac{x^2}{\rho_2}\right]^{1/2} + \frac{x^2}{\rho_2} \qquad (4)$$

$$\frac{1}{\rho} = \frac{1-x}{\rho_1} + \frac{x}{\rho_2}$$

u gornju jednačinu (4) dobija se

$$\frac{1-x}{\rho_1} + \frac{x}{\rho_2} = \frac{(1-x)^2}{\rho_1} + \frac{x^2}{\rho_2} + C \frac{x(1-x)}{(\rho_1 \rho_2)^{1/2}}$$

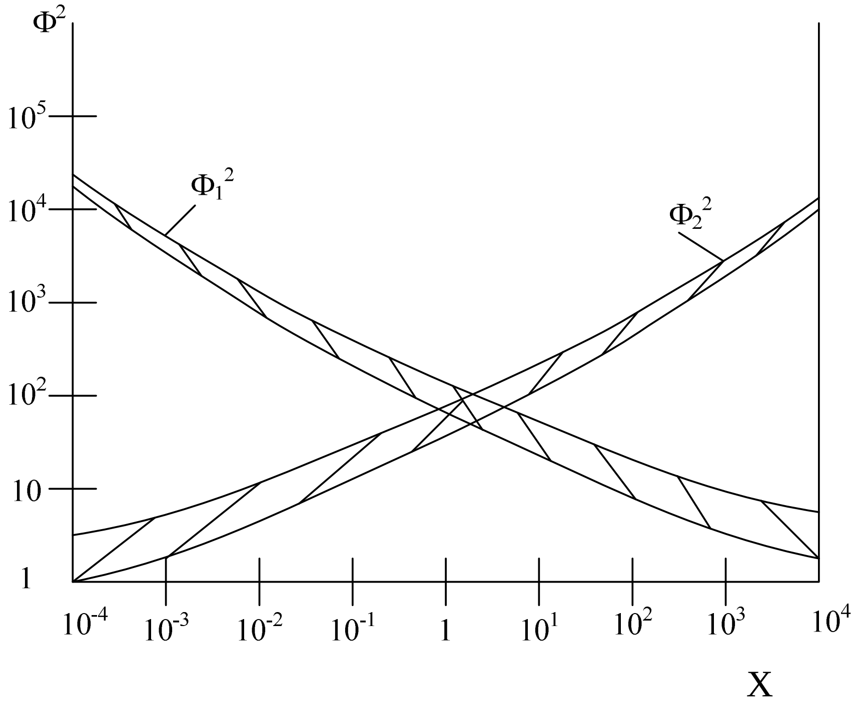
$$\frac{1-x}{\rho_1}(1-1+x)+\frac{x}{\rho_2}(1-x)=C\frac{x(1-x)}{(\rho_1\rho_2)^{1/2}}$$

$$x(1-x)\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)=C\frac{x(1-x)}{(\rho_1\rho_2)^{1/2}}$$

$$C=\left(\frac{\rho_2}{\rho_1}\right)^{1/2}+\left(\frac{\rho_1}{\rho_2}\right)^{1/2}$$

Za strujanje vode i vazduha na atmosferskom pritisku C=28,9.

Eksperimentalna zavisnost Lockhart – Martinelli-a



Dobro se može opisati sa sledećim vrednostima koef. C

tečnost	gas	C
turbulentno	turbulentno	20
laminarno	turbulentno	12
turbulentno	laminarno	10
laminarno	laminarno	5

Za laminarno strujanje ili turbulentno u glatkoj cevi

$$X^2=\frac{f_1\dfrac{(1-x)^2G^2}{2\rho_1}\dfrac{4}{D}}{f_2\dfrac{x^2G^2}{2\rho_2}\dfrac{4}{D}}=\frac{\dfrac{k}{\mathrm{Re}_1^n}\dfrac{(1-x)^2}{\rho_1}}{\dfrac{k}{\mathrm{Re}_2^n}\dfrac{x^2}{\rho_2}}=\frac{\rho_2}{\rho_1}\frac{x^n\mu_1^n}{(1-x^n)\mu_2^n}\frac{(1-x)^2}{x^2}=\frac{\rho_2}{\rho_1}\left(\frac{\mu_1}{\mu_2}\right)^n\left(\frac{1-x}{x}\right)^{2-n}$$

$$\mathrm{Re}_1=\frac{\alpha_1\rho_1u_1D}{\mu_1}=\frac{(1-x)GD}{\mu_1},\ \mathrm{Re}_2=\frac{xGD}{\mu_2}$$

za laminarno n=1, za turbulentno n=0,25.