4.2 $G = 4{,}9\,N$

$T = \frac{\pi}{4}\,s$

$\Delta l = 1\,cm = 0{,}01\,m$

$F - ?$

$T = \frac{2\pi}{\omega}$, $\omega = \sqrt{\frac{c}{m}}$, $G = mg \Rightarrow m = \frac{G}{g}$,

$c = \frac{F}{\Delta l}$, $\omega = \sqrt{\frac{\frac{F}{\Delta l}}{\frac{G}{g}}} = \sqrt{\frac{Fg}{G\Delta l}}$

$T = 2\pi \sqrt{\frac{G\Delta l}{Fg}}\,|^2$

$\frac{T^2}{4\pi^2} = \frac{G\Delta l}{Fg} \Rightarrow F = \frac{4\pi^2 G\Delta l}{T^2 g} = \frac{4 \cdot \pi^2 \cdot 4{,}9 \cdot 0{,}01}{\frac{\pi^2}{16} \cdot g}$

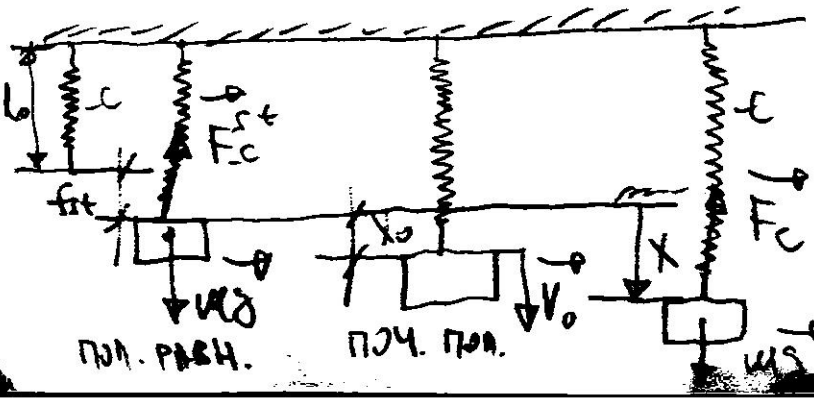
$\boxed{F = \frac{64 \cdot 4{,}9 \cdot 0{,}01}{g} = \frac{64 \cdot 0{,}5 \cdot g \cdot 0{,}01}{g} = 0{,}32\,N}$

4.5 $f_{st} = 10\,cm = 0{,}1\,m$

$t_0 = 0$ $\quad x_0 = 15\,cm = 0{,}15\,m$

$V_0 = 99\,cm/s = 0{,}99\,\frac{m}{s}$

$x(t) - ?$



$m\vec{a} = m\vec{g} + \vec{F_c}$ – ЗА ПРОИЗВОЛНИ ПОЛОЖБА

x: $m\ddot{x} = mg - c\Delta l$

$\Delta l = f_{st} + x$

$m\ddot{x} = mg - c(x + f_{st})$

У ПОЛОЖБА НА РАВНОТЕЖИЕ НА ВАНИ:

$\sum X_i = 0$, $mg - c f_{st} = 0 \Rightarrow mg = c f_{st}$

$m\ddot{x} = mg - cx - c f_{st}$ $/:m$

$\ddot{x} + \frac{c}{m} x = 0$, $\omega^2 = \frac{c}{m}$

$\ddot{x} + \omega^2 x = 0$

$f_{st} = 10\,cm = \frac{mg}{c} = \frac{g}{\frac{c}{m}} = \frac{g}{\omega^2} \Rightarrow$

$\Rightarrow \omega^2 = \frac{g}{f_{st}} = \frac{9{,}81}{0{,}1} = 98{,}1\,s^{-2} \Rightarrow$ $\boxed{\omega = 9{,}90\,s^{-1}}$

$x = C_1 \cos\omega t + C_2 \sin\omega t$

$\dot{x} = -C_1 \omega \sin\omega t + C_2 \omega \cos\omega t$

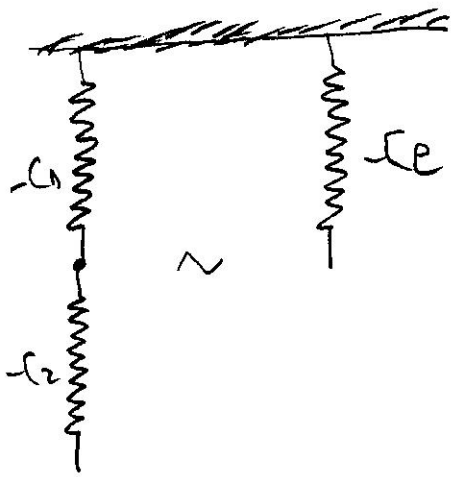
$t_0 = 0$ $x_0 = x(t_0) = 15\,cm$, $\dot{x}_0 = v_0 = \dot{x}(t_0) = 99\,\frac{cm}{s}$

$\boxed{15 = C_1}$

$99 = C_2 \omega \Rightarrow C_2 = \frac{99}{\omega} = \frac{99\,\frac{cm}{s}}{9{,}90\,\frac{1}{s}} = 10\,cm$

$x(t) = 15\cos(9{,}90t) + 10\sin(9{,}90t)$ [cm]

РЕДНО ВЕЗАНЕ ОПРУГИ



c_e - ЕКВИВАЛЕНТНА КРУТОСТ

$$F_1 = c_1 \Delta l_1 \Rightarrow \frac{1}{c_1} = \frac{\Delta l_1}{F_1}$$

$$F_2 = c_2 \Delta l_2 \Rightarrow \frac{1}{c_2} = \frac{\Delta l_2}{F_2}$$

$$F_e = c_e \Delta l_e \Rightarrow \frac{1}{c_e} = \frac{\Delta l_e}{F_e}$$

$F_1 = F_2 = F_e = F$ - УСЛОВ

$\Delta l_e = \Delta l_1 + \Delta l_2$

$$\frac{1}{c_1} + \frac{1}{c_2} = \frac{\Delta l_1}{F_1} + \frac{\Delta l_2}{F_2} = \frac{1}{F}(\Delta l_1 + \Delta l_2) = \frac{\Delta l_e}{F} = \frac{1}{c_e}$$

$$\boxed{\frac{1}{c_e} = \frac{1}{c_1} + \frac{1}{c_2}}$$

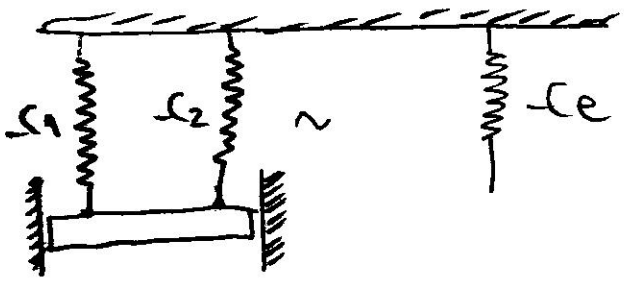
$$\frac{1}{c_e} = \frac{c_1 + c_2}{c_1 \cdot c_2} \Rightarrow$$

$$c_e = \frac{c_1 \cdot c_2}{c_1 + c_2} \Rightarrow c_e < \min\{c_1, c_2\}$$

ЗА СЛУЧАЙ ДА ИМАМЕ n ОПРУГИ ЧИЯТО СТ КРУТОСТИ $c_1 > c_2 > \dots > c_n$ РЕСПЕКТИВНО, ОНДА ЩЕ

$$\frac{1}{c_e} = \sum_{i=1}^{n} \frac{1}{c_i} \qquad c_e < \min\{c_1, c_2, \dots, c_n\}$$

Паралелно вezане опруге



$F_1 = -c_1 \Delta l_1$

$F_2 = -c_2 \Delta l_2$

$F_e = -c_e \Delta l_e$

$\Delta l_1 = \Delta l_2 = \Delta l_e = \Delta l$ – у односу

$F_1 + F_2 = F_e$

$F_1 + F_2 = F_e$

$-c_1 \Delta l_1 + -c_2 \Delta l_2 = -c_e \Delta l_e \quad / : \Delta l$

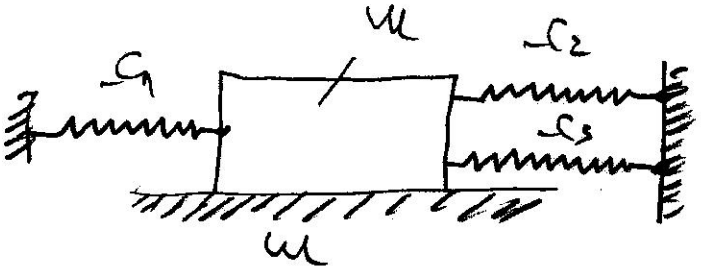
$\boxed{-c_1 + -c_2 = -c_e} \Rightarrow -c_e > \max\{-c_1, -c_2\}$

$-c_e = \sum_{i=1}^{n} -c_i \qquad -c_e > \max\{-c_1, -c_2, \ldots, -c_n\}$

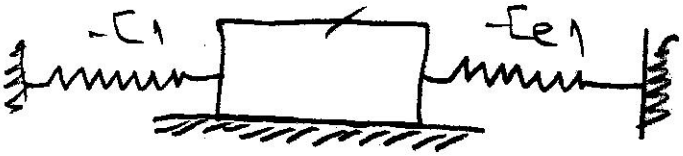
4.7 m

$-c_1, -c_2, -c_3$

$T = ?$

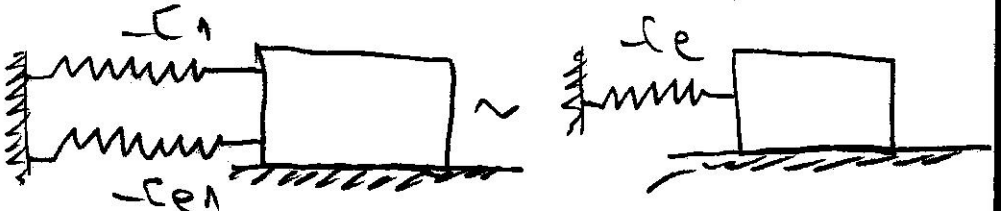


$-c_{e1} = -c_2 + -c_3$



$-c_e = -c_{e1} + -c_1 =$

$= -c_1 + -c_2 + -c_3$



$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{-c_e}{m}}} = 2\pi \sqrt{\frac{m}{-c_e}} = 2\pi \sqrt{\frac{m}{-c_1 + -c_2 + -c_3}}$

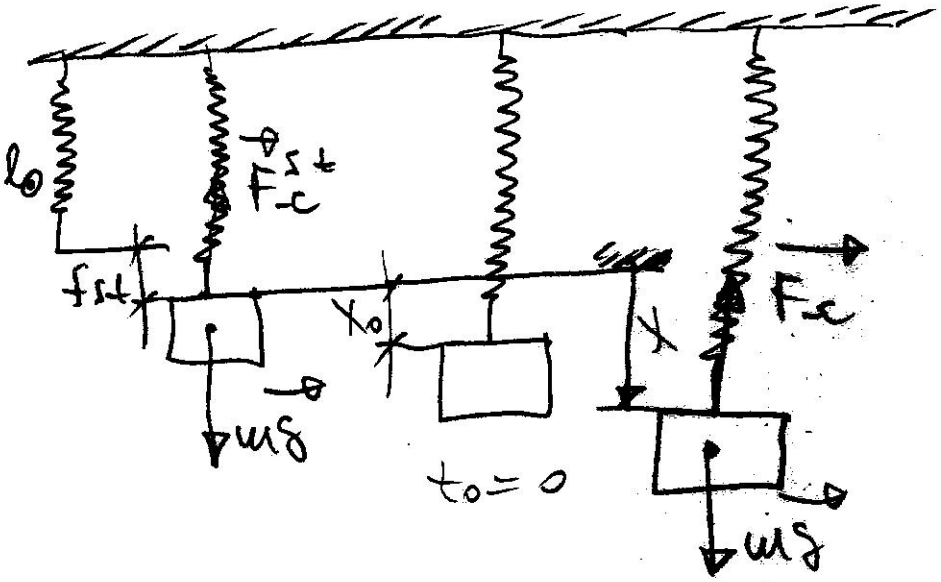
4.11 $m = 16\,kg$

$c = 4\,N/cm$

$x_0 = 8\,cm$

$a = 10\,cm$

$V_0 - ?$



$m\ddot{x} = +mg - c(f_{st} + x)$

$\ddot{x} + \frac{c}{m}x = 0$, $\omega^2 = \frac{c}{m} = \frac{4\,\frac{N}{cm}}{16\,kg} = \frac{4\,\frac{kg\cdot m}{s^2}\,\frac{1}{cm}}{16\,kg} = \frac{4\,\frac{1}{s^2}\,\frac{1}{0{,}01\,m}}{16} =$

$\ddot{x} + 25x = 0$ $= \frac{100}{4}\,\frac{1}{s^2} = 25\,\frac{1}{s^2} \Rightarrow \omega = 5\,s^{-1}$

$x = C_1\cos 5t + C_2 \sin 5t$

$\dot{x} = -5C_1\sin 5t + 5C_2\cos 5t$

$t_0 = 0$: $x(t_0) = x_0 = 8\,cm$

$\dot{x}(t_0) = \dot{x}_0 = V_0$

$8 = C_1$

$V_0 = 5C_2 \Rightarrow C_2 = \frac{V_0}{5}$

$x = 8\cos 5t + \frac{v_0}{5}\sin 5t$

$x = A\sin(5t+\alpha) = A\sin 5t\cos\alpha + A\cos 5t\sin\alpha$

$A\sin 5t\cos\alpha + A\cos 5t\sin\alpha = 8\cos 5t + \frac{v_0}{5}\sin 5t$

$$\left.\begin{aligned} A\cos\alpha &= \frac{v_0}{5} \;|^2 \\ A\sin\alpha &= 8 \;|^2 \end{aligned}\right\} +$$

$A^2 = \left(\frac{v_0}{5}\right)^2 + 8^2 \Rightarrow \left(\frac{v_0}{5}\right)^2 = A^2 - 64$, $A = a = 10\,cm$

$\boxed{v_0 = 5\sqrt{a^2-64} = 5\sqrt{10^2-64} = 5\cdot 6 = 30\,\frac{cm}{s}}$

$tg\alpha = \frac{8}{\frac{v_0}{5}} = \frac{8\cdot 5}{30} = \frac{8}{6} = \frac{4}{3} = 1{,}333$

$\boxed{\alpha = 53{,}13^\circ}$

4.118 l, a

а) φ_a T — ?

б) $\downarrow a$ T — ?

$m\vec{a} = m\vec{g} + \vec{S}$

$t: \; m a_t = -mg \sin\varphi \;\; | :m$

$a_t + g \sin\varphi = 0$

$\sin\varphi \approx \varphi$

$a_t + g\varphi = 0$

$a_t = \ddot{x}_M \cos\varphi - \ddot{y}_M \sin\varphi =$

$= l\ddot{\varphi}\cos^2\varphi - l\dot{\varphi}^2 \sin\varphi\cos\varphi$

$+ a\sin\varphi + l\ddot{\varphi}\sin^2\varphi +$

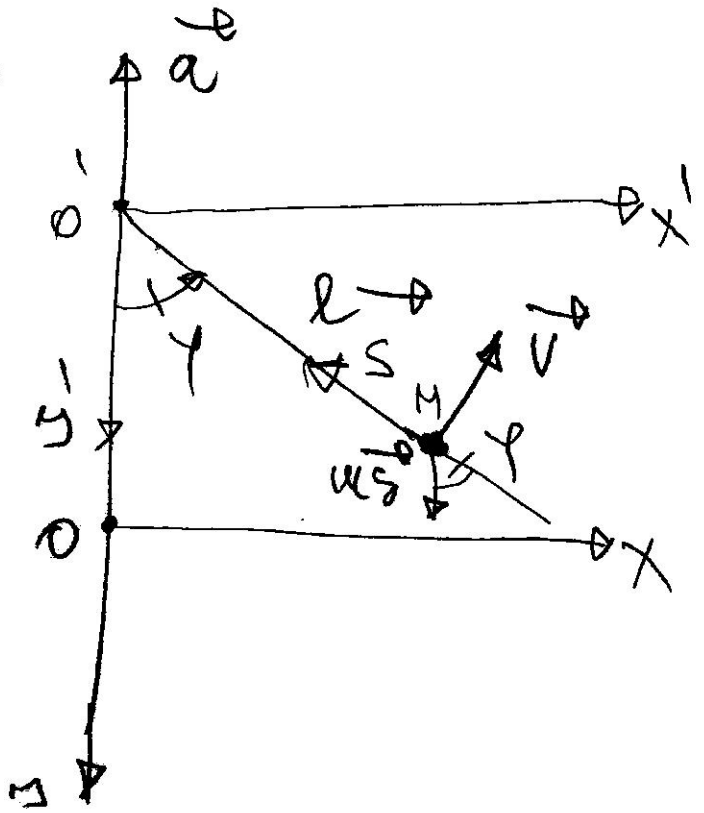
$+ l\dot{\varphi}^2 \sin\varphi\cos\varphi =$

$= l\ddot{\varphi} + a\sin\varphi$

$l\ddot{\varphi} + a\varphi + g\varphi = 0$

$\ddot{\varphi} + \frac{a+g}{l}\varphi = 0$

$\ddot{\varphi} + \omega^2\varphi = 0$

б) 

$x_M = l\sin\varphi$

$y_M = +y_0' + l\cos\varphi$

$\dot{x}_M = l\dot{\varphi}\cos\varphi$

$\dot{y}_M = +\dot{y}_0' - l\dot{\varphi}\sin\varphi$

$\ddot{x}_M = l\ddot{\varphi}\cos\varphi - l\dot{\varphi}^2\sin\varphi$

$\ddot{y}_M = +\ddot{y}_0' - l\ddot{\varphi}\sin\varphi - l\dot{\varphi}^2\cos\varphi$

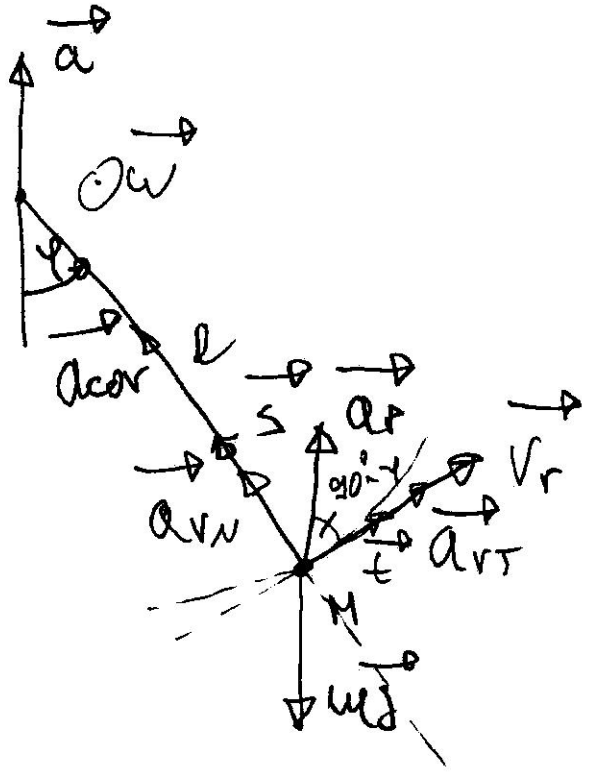
$\ddot{y}_0' = -a$

$| : l \quad \omega^2 = \frac{a+g}{l}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{a+g}}$

б) .... $T = 2\pi\sqrt{\frac{l}{g-a}}$

Други начин



$$m\vec{a} = m\vec{g} + \vec{S}$$

$$\vec{a} = \vec{a}_p + \vec{a}_r + \vec{a}_{cor}$$

$$\vec{a}_r = \vec{a}_{rN} + \vec{a}_{rT}$$

$$\vec{a} = \vec{a}_p + \vec{a}_{rN} + \vec{a}_{rT} + \vec{a}_{cor}$$

$$m(\vec{a}_p + \vec{a}_{rN} + \vec{a}_{rT} + \vec{a}_{cor}) = m\vec{g} + \vec{S}$$

$a_p = a$

$t: \quad m\left(\frac{dV_r}{dt} + a\sin\varphi\right) = -mg\sin\varphi \quad | : m$

$V_r = l\dot{\varphi}$

$l\ddot{\varphi} + a\sin\varphi + g\sin\varphi = 0 \quad | : l$

$\ddot{\varphi} + \frac{a+g}{l}\sin\varphi = 0, \quad \sin\varphi \approx \varphi$

$\ddot{\varphi} + \omega^2\varphi = 0, \quad \omega^2 = \frac{a+g}{l}$

4.19

r

h

$c_1 = c_2 = c$

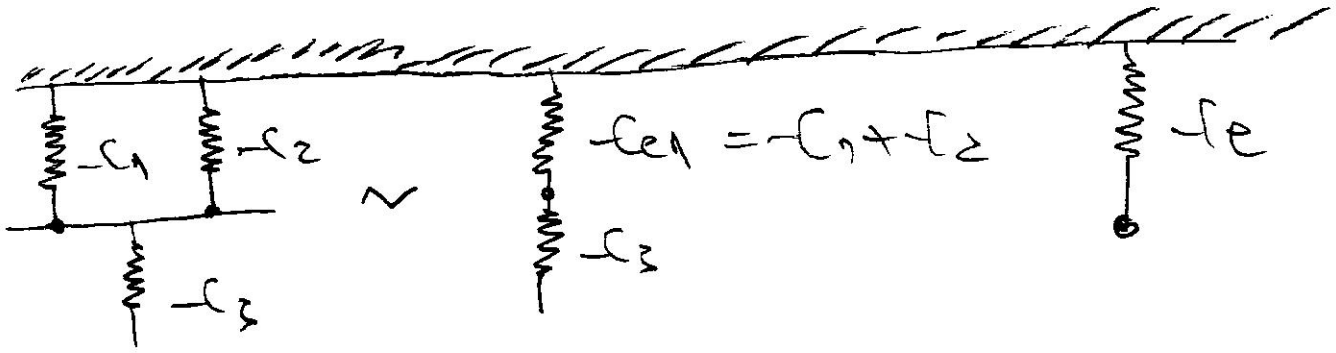
$c_3 = 2c$

r

$h_{st} = \frac{h}{2}$

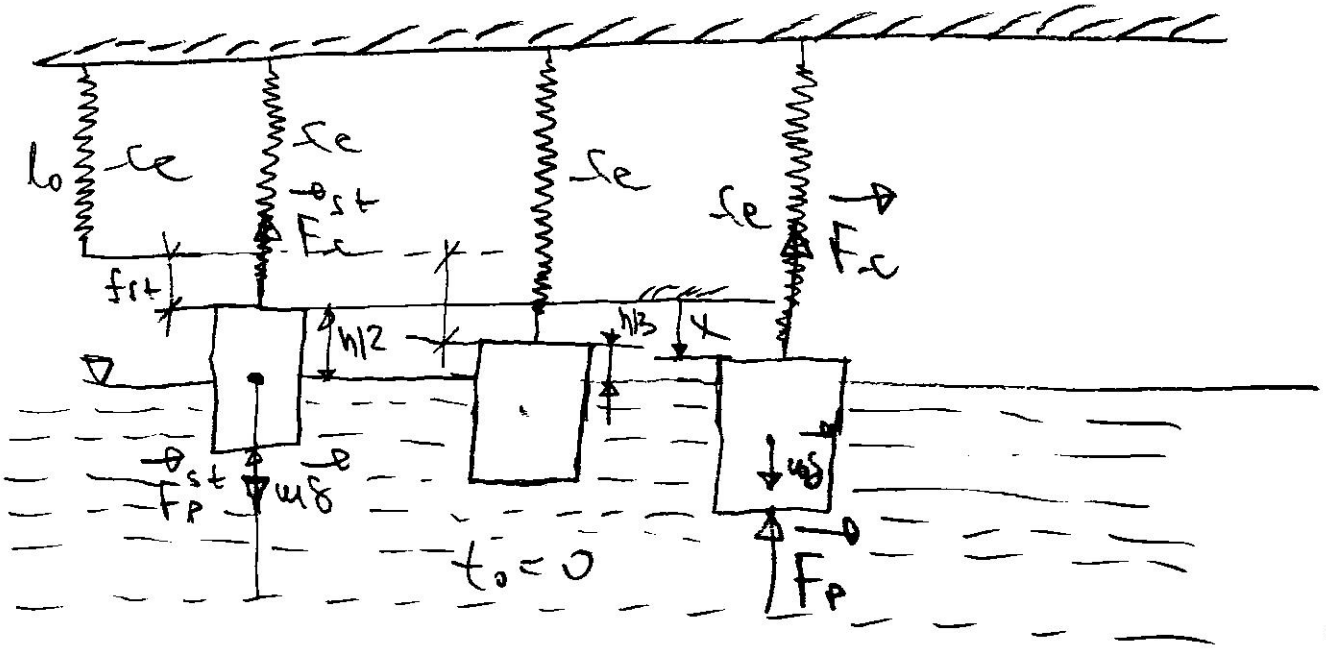
$t_0 = 0 \quad h(t_0) < \frac{2}{3}h \;,\; V_0 = 0$

$y(t) = ?$



$$c_{e1} = c_1 + c_2 = 2c$$

$$\frac{1}{c_e} = \frac{1}{c_{e1}} + \frac{1}{c_3} = \frac{1}{2c} + \frac{1}{2c} = \frac{1}{c} \Rightarrow \boxed{c_e = c}$$



$$m\vec{a} = m\vec{g} + \vec{F_p} + \vec{F_c}$$

$$x: \quad m\ddot{x} = mg - \rho r^2\pi\left(\frac{h}{2} + x\right) - c(f_{st} + x)$$

Положај стат. равнотеже:

$$0 = mg - \rho r^2\pi\frac{h}{2} - c f_{st} = 0$$

$$m\ddot{x} = -\rho r^2\pi x - cx \quad / : m$$

$$\ddot{x} + \frac{\rho r^2\pi + c}{m}x = 0, \quad \omega = \frac{g}{f}$$

$$\ddot{x} + \frac{(\rho r^2\pi + c)f}{G}x = 0, \quad \omega^2 = \frac{(\rho r^2\pi + c)g}{G}$$

$$\ddot{x} + \omega^2 x = 0$$

$$x = C_1\cos\omega t + C_2\sin\omega t$$

$$\dot{x} = -C_1\omega\sin\omega t + C_2\omega\cos\omega t$$

$$t_0 = 0 \quad x_0 = \frac{h}{2} - \frac{h}{3} = \frac{h}{6}, \quad \dot{x}_0 = 0$$

$$\boxed{\frac{h}{6} = C_1} \qquad 0 = C_2\omega \Rightarrow \boxed{C_2 = 0}$$

$$x = \frac{h}{6}\cos\left(\sqrt{\frac{(\rho r^2\pi + c)g}{G}}\,t\right)$$

4.30

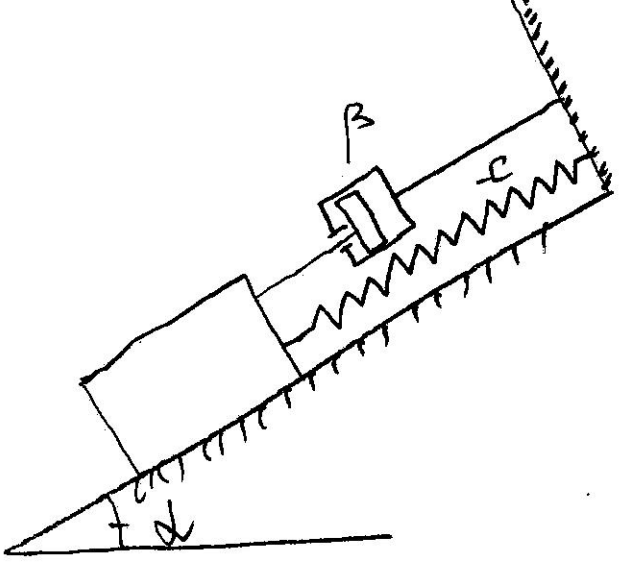
m

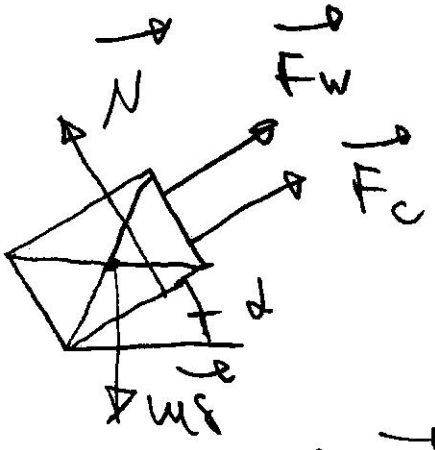
c

$\alpha = 30^\circ$

$T_\beta = 2T_\omega$

$\beta - ?$





$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_c + \vec{F}_w$$

$$x: \; m\ddot{x} = mg\sin\alpha - c(x + f_{st}) - \dot{x}\beta$$

$$\sum X_i = 0, \quad mg\sin\alpha - c f_{st} = 0$$

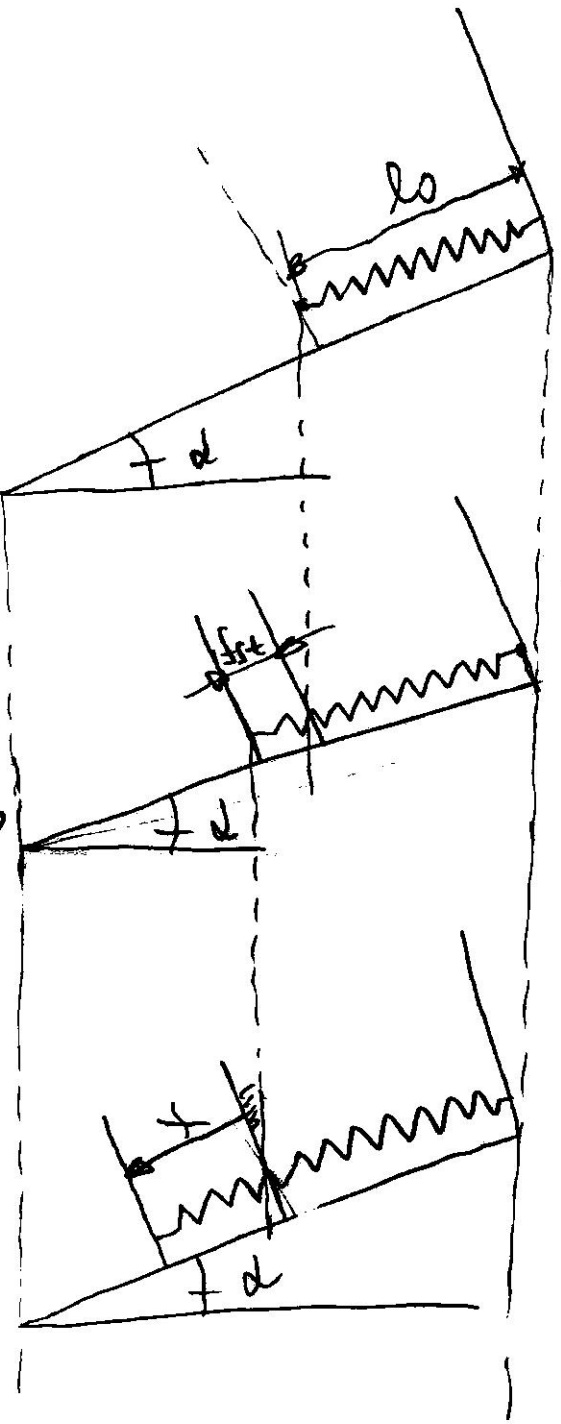
$$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{c}{m}x = 0$$

$$\ddot{x} + 2\nu\dot{x} + \omega^2 x = 0$$

$$\omega^2 = \frac{c}{m}, \quad 2\nu = \frac{\beta}{m}$$

$$T_\omega = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{c}}$$

$$T_\beta = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{\omega^2 - \delta^2}}$$



$T_p = 2T_w$

$$\frac{2\pi}{\sqrt{\omega^2 - \delta^2}} = 2 \cdot 2\pi \sqrt{\frac{4}{c}}$$

$$\frac{1}{\omega^2 - \delta^2} = 4\left(\frac{4}{c}\right)$$

$$4 = 4\,\frac{4}{c}\left(\omega^2 - \delta^2\right)$$

$$1 = 4\,\frac{4}{c}\left(\frac{c}{4} - \frac{\beta^2}{4\omega^2}\right)$$

$$1 = 1 - \frac{\beta^2}{4c} \Rightarrow \beta^2 = 3\omega c$$

$$\boxed{\beta = \sqrt{3\omega c}}$$

$$\lambda^2 + 2\delta\lambda + \omega^2 = 0$$

$$\lambda_{1,2} = \frac{-2\delta \pm \sqrt{4\delta^2 - 4\omega^2}}{2} = -\delta \pm \sqrt{\delta^2 - \omega^2}$$

ЗА $\omega > \delta$ – МАЛО ПРИГУШЕНИЕ $\delta^2 - \omega^2 = -\mu^2$

ЗА $\delta > \omega$ – ВЕЛИКО ПРИГУШЕНИЕ $\delta^2 - \omega^2 = \nu^2$

ЗА $\delta = \omega$ – ГРАНИЧЕН СЛУЧАЙ

$\lambda_{1,2} = -\delta \pm i\mu$ – МАЛО ПРИГУШЕНИЕ

$\lambda_{1,2} = -\delta \pm \nu$ – ВЕЛИКО ПРИГУШЕНИЕ

$\lambda_1 = \lambda_2 = -\delta$ – ГРАНИЧЕН СЛУЧАЙ

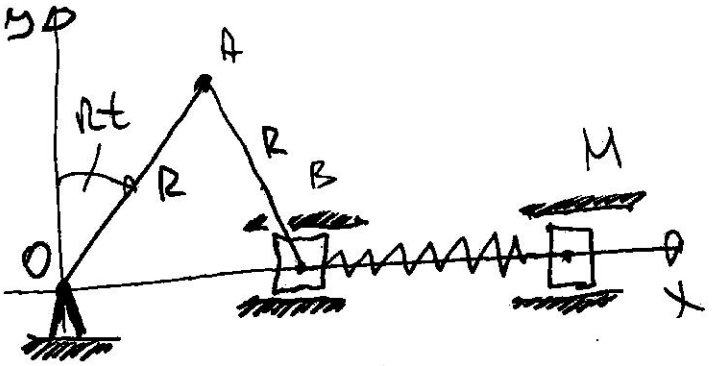
$$\gamma = \frac{\beta}{2m} = \frac{\sqrt{3mc}}{2m} = \sqrt{\frac{3mc}{4m^2}} = \sqrt{\frac{3c}{4m}}$$

$$\gamma^2 - \omega^2 = \frac{3c}{4m} - \frac{c}{m} = -\frac{c}{4m}, \quad \gamma = \frac{1}{2}\sqrt{\frac{c}{m}}$$

$$\lambda_{1,2} = -\frac{1}{2}\sqrt{\frac{3c}{m}} \pm i\,\frac{1}{2}\sqrt{\frac{c}{m}}$$

$$x = e^{-\frac{1}{2}\sqrt{\frac{3c}{m}}t}\left(C_1 \cos\left(\frac{1}{2}\sqrt{\frac{c}{m}}t\right) + C_2 \sin\left(\frac{1}{2}\sqrt{\frac{c}{m}}t\right)\right)$$

4.38 $\Omega = \sqrt{\frac{c}{m}}$



c

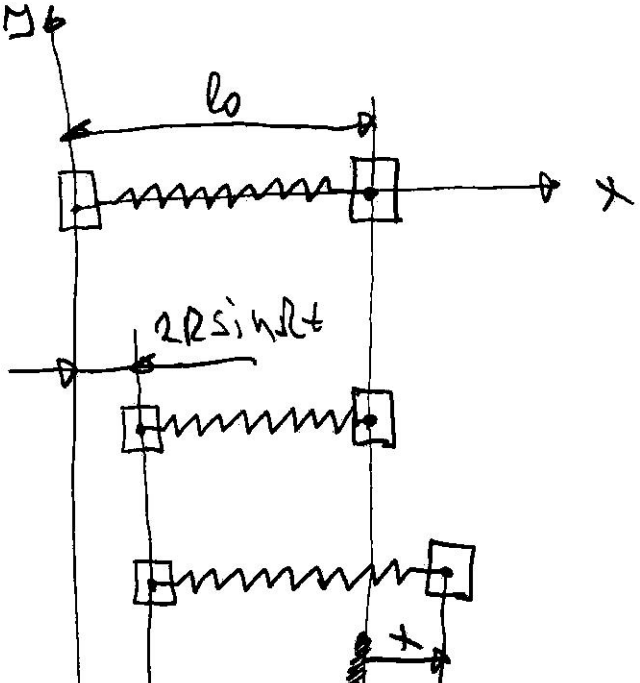
m

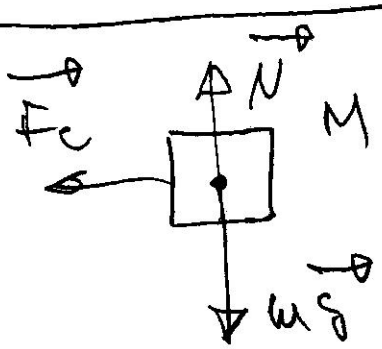
$t_0 = 0 \quad x_0 = 0, \; v_0 = 0$

$\overline{OA} = \overline{AB} = R$

$x(t) - ?$

$$\Delta R = x - 2R\sin\Omega t$$





$$m\ddot{x} = -c(x - 2R\sin\Omega t) : m$$

$$\ddot{x} + \frac{c}{m}x = \frac{2cR}{m}\sin\Omega t, \quad \omega^2 = \frac{c}{m}, \quad h = \frac{2cR}{m}$$

$x = x_h + x_p \quad \left(\ddot{x} + w^2 x = h \sin \Omega t \right)$

$x_h = C_1 \cos wt + C_2 \sin wt$

$w = \Omega$ - РЕЗОНАНСНЫЙ СЛУЧАЙ !!!

$x_p = At \sin \Omega t + Bt \cos \Omega t$

$\dot{x}_p = A \sin \Omega t + A\Omega t \cos \Omega t + B \cos \Omega t - B\Omega t \sin \Omega t$

$\ddot{x}_p = A\Omega \cos \Omega t + A\Omega \cos \Omega t - A\Omega^2 t \sin \Omega t - B\Omega \sin \Omega t$

$- B\Omega \sin \Omega t - B\Omega^2 t \cos \Omega t =$

$= 2\Omega A \cos \Omega t - 2\Omega B \sin \Omega t - \Omega^2 t (A \sin \Omega t + B \cos \Omega t)$

$2\Omega A \cos \Omega t - 2\Omega B \sin \Omega t - \Omega^2 t (A \sin \Omega t + B \cos \Omega t) +$

$+ w^2 (At \sin \Omega t + Bt \cos \Omega t) \equiv h \sin \Omega t$

$2\Omega A = 0 \Rightarrow A = 0$

$-2\Omega B = h \Rightarrow B = -\frac{h}{2\Omega} = -\frac{2 - \Omega}{4\Omega} = -\frac{w^2 R}{\Omega}$

$\boxed{B = -wR}$

$\boxed{x_p = -wRt \cos \Omega t}$

$x = C_1 \cos wt + C_2 \sin wt - wRt \cos \Omega t$

$\dot{x} = -wC_1 \sin wt + wC_2 \cos wt - wR \cos \Omega t + w^2 Rt \sin \Omega t$

$t_0 = 0: x_0 = 0, \dot{x}_0 = 0$

$0 = C_1$

$0 = \omega C_2 - \omega R \Rightarrow \boxed{C_2 = R}$

$x = R\left[\sin\sqrt{\frac{c}{m}}t - \sqrt{\frac{c}{m}}t + \cos\sqrt{\frac{c}{m}}t\right]$

4.39 $V = 10\ м/с$

$y = b\sin(0{,}2x)$

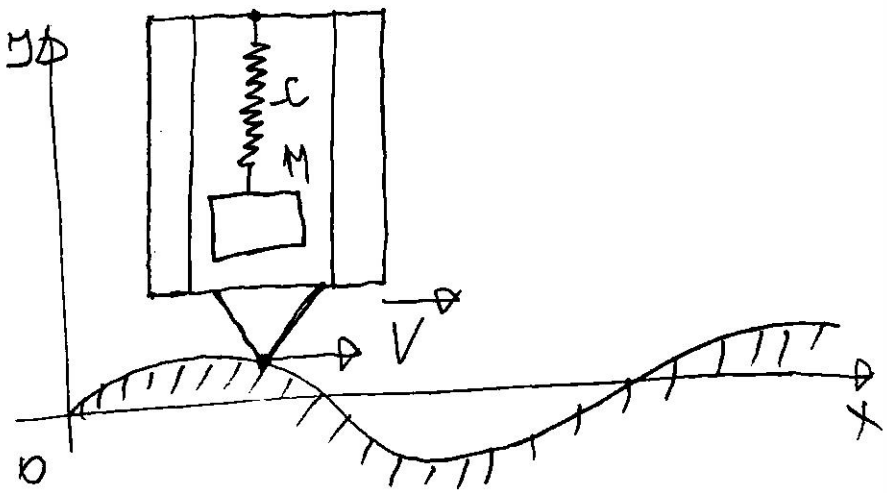
$c = 0{,}4\ кН/м$

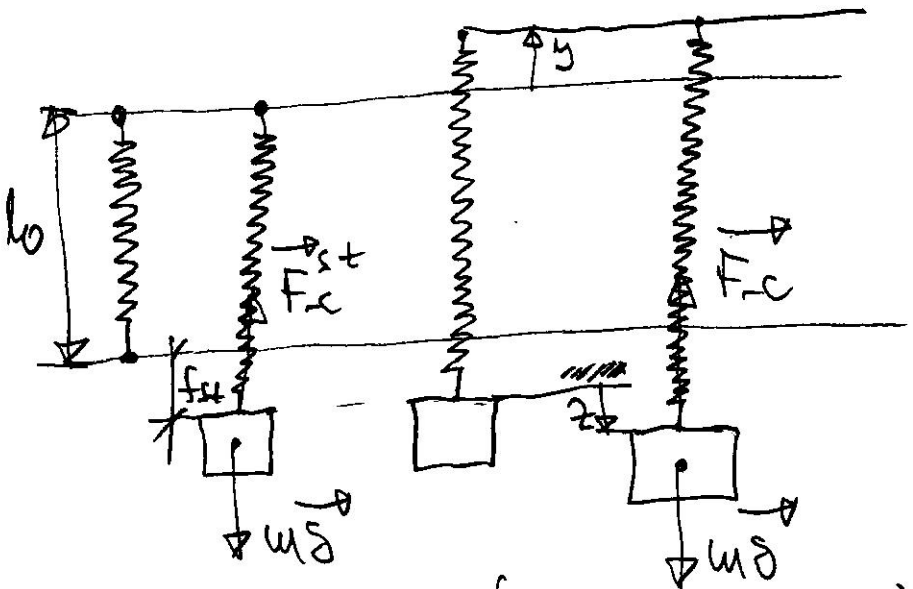
$m = 10\ кг$

$x_0 = 0$

$A = 2\ см$

$b - ?$





$l = b + f_{ст} + y + z$

$\Delta l = l - b = z + y + f_{ст}$

$m\ddot{z} = mg - c(z + y + f_{ст})$

$\sum Z_i = 0,\ mg - c f_{ст} = 0$

$\ddot{z} + \frac{c}{m}z = -\frac{c}{m}y = -\frac{c}{m}b\sin(0{,}2x)$

$V = \frac{dx}{dt} \Rightarrow dx = V\,dt \int$

$x = x_0 + Vt\Big|_0^t$ $\quad \boxed{x = Vt = 10t}$

$x_0 = 0$

$\ddot{z} + \omega^2 z = 4\sin(2t)$, $\omega^2 = \frac{-c}{m} = \frac{100}{10} = 10$

$4 = -\frac{-c}{\omega} b = -10b$

$z_p = A\sin(2t) + B\cos(2t)$

$\dot{z}_p = 2A\cos(2t) - 2B\sin(2t)$

$\ddot{z}_p = -4A\sin(2t) - 4B\cos(2t)$

$-4A\sin(2t) - 4B\cos(2t) + 10A\sin(2t) +$

$+ 10B\cos(2t) = -10b\sin(2t)$

$6A = -10b \Rightarrow \boxed{A = -\frac{5}{3}b}$

$6B = 0 \Rightarrow \boxed{B = 0}$

$z_p = -\frac{5}{3}b\sin(2t)$

$\frac{5}{3}b = 2 \Rightarrow \boxed{b = \frac{6}{5} = 1{,}2\ cm}$