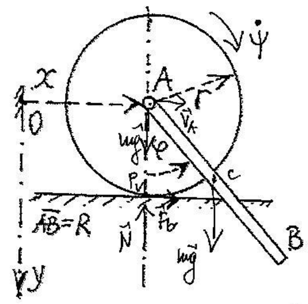


Za središte diska A, diska mase m i poluprečnika r, koji se kotrlja bez klizanja po horizontalnoj ravni, zglobno je vezan štap AB dužine R i mase m. Za date generalisane koordinate x, φ odrediti: 1) kinetičku energiju i rad sila sistema, 2) u linearnom slučaju (sinφ≈φ, cosφ≈1) diferencijalne jednačine kretanja, konačne jednačine kratanja (u početnom trenutku $t_0=0$ sistem mirovao φ(0)= φ_0, x(0)=0).

DVA STEPENA SLOBODE KRETANJA $q_1=x$, $q_2=\varphi$

(VEĆ SMO TEOREMAMA $\dot{\vec{K}}=\vec{F}^s_R$; $\dot{\vec{L}}_C=\vec{M}^s_R$ REŠILI OVAJ ZADATAK, POGLEDATI U SVESCI)

$$T=T_{DIS}+T_{ŠTAP}$$

RAVNO KRET.
$$\begin{cases} T_{DIS}=\frac{1}{2}m_A v_A^2+\frac{1}{2}J_{Az}\dot{\psi}^2 \\ T_{ŠTAP}=\frac{1}{2}m_S v_C^2+\frac{1}{2}J_{Cz}\dot{\varphi}^2 \end{cases}$$

$$\underbrace{\vec{v}_A, \vec{v}_C, \dot{\varphi}, \dot{\psi}}_{\text{APSOLUTNE BRZINE}}$$

$$v_A=\dot{x} \qquad \dot{\psi}=\frac{\dot{x}}{r}$$

$$C\begin{cases} x_C=x+\frac{R}{2}\sin\varphi \\ y_C=\frac{R}{2}\cos\varphi \end{cases} \qquad \begin{cases} \dot{x}_C=\dot{x}+\frac{R}{2}\dot{\varphi}\cos\varphi \\ \dot{y}_C=-\frac{R}{2}\dot{\varphi}\sin\varphi \end{cases}$$

$$v_C^2=\dot{x}_C^2+\dot{y}_C^2$$

$$T=\underbrace{\frac{1}{2}m\dot{x}^2+\frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{\dot{x}}{r}\right)^2}_{T_{DIS}}+\underbrace{\frac{1}{2}m\left(\dot{x}^2+\frac{R^2\dot{\varphi}^2}{4}+mR\dot{x}\dot{\varphi}\cos\varphi\right)+\frac{1}{2}\left(\frac{1}{12}mR^2\right)\dot{\varphi}^2}_{T_{ŠTAP}}$$

$$T=\frac{5}{4}m\dot{x}^2+\frac{1}{6}mR^2\dot{\varphi}^2+\frac{1}{2}mR\dot{x}\dot{\varphi}\cos\varphi$$

$$\sum_i A_i=A^s+\cancel{A^u}^{\,0}=A(m_S\vec{g})+\cancel{A(m_{dis}\vec{g})}^{\,0}+\cancel{A(\vec{N})}^{\,0}+\cancel{A(\vec{F}_{tr})}^{\,0}=-mg\frac{R}{2}(1-\cos\varphi)$$

$$T-T_0=A^s+\cancel{A^u}^{\,0}$$

$$\frac{5}{4}m\dot{x}^2+\frac{1}{6}mR^2\dot{\varphi}^2+\frac{1}{2}mR\dot{x}\dot{\varphi}\cos\varphi=-mg\frac{R}{2}(1-\cos\varphi) \quad \Big/ \frac{d}{dt}$$

$$\frac{5}{2}m\dot{x}\ddot{x}+\frac{1}{3}mR^2\dot{\varphi}\ddot{\varphi}+\frac{1}{2}mR\ddot{x}\dot{\varphi}\cos\varphi+\frac{1}{2}mR\dot{x}\ddot{\varphi}\cos\varphi-\frac{1}{2}mR\dot{x}\dot{\varphi}^2\sin\varphi=-\left(mg\frac{R}{2}\sin\varphi\right)\dot{\varphi}$$

GRUPIŠIMO DIF. FORMU PO NEZAVISNIM GEN. BRZINAMA $\dot{x}, \dot{\varphi}$ =>

$$\Rightarrow\begin{cases} \frac{5}{2}m\ddot{x}+\frac{1}{2}mR\ddot{\varphi}\cancel{\cos\varphi}^{\,1}-\frac{1}{2}mR\cancel{\dot{\varphi}^2\sin\varphi}^{\,\approx 0}=0 \\ \frac{1}{3}mR^2\ddot{\varphi}+\frac{1}{2}mR\ddot{x}\cancel{\cos\varphi}_{\,1}-\frac{1}{2}mR\cancel{\dot{x}\dot{\varphi}\sin\varphi}^{\,\approx 0}=-mg\frac{R}{2}\underbrace{\sin\varphi}_{\varphi} \end{cases}$$

{ LINEARIZACIJA DAJE:

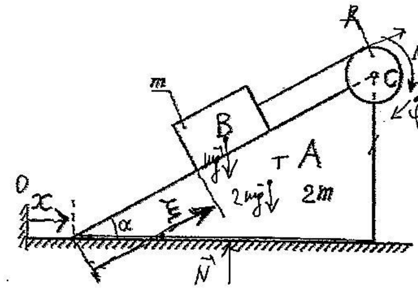
$$\begin{cases} \frac{5}{2}m\ddot{x}+\frac{1}{2}mR\ddot{\varphi}=0 \\ \frac{1}{3}mR^2\ddot{\varphi}+\frac{1}{2}mR\ddot{x}=-mg\frac{R}{2}\varphi \end{cases} \quad (*)$$

$$(*) \Rightarrow \quad \ddot{\varphi}+\left(\frac{15}{7}\frac{g}{R}\right)\varphi=0$$

REŠITI DIF. JEDNAČINU ZA $\dot{\varphi}(0)=0$ $\varphi(0)=\varphi_0$

POKAZATI DA JE I "x" FUNKCIJA (U LINEARNOM SLUČAJU) HARMONIJSKOG OSCILOVANJA; T.J. POTVRDITE DA JE OVO ZAISTA "KOTRLJAJUĆE KLATNO" (U LINEARNOM SLUČAJU)!

$$\left[x=\frac{R}{5}\varphi_0(1-\cos kt) \qquad k^2=\frac{15}{7}\frac{g}{R}\right]$$



Na glatkoj horizontalnoj ravni nalazi se prizma A, β=60°, mase 2m. Po hipotenuzi može bez klizanja da se kreće teret B mase m. Teret je vezan lakim neistegljivim užetom za kalem; kalem je zanemarljive mase, poluprečnika R, u C je zglobna veza, na kalem dejstvuje spreg sila momenta $M=(\sqrt{3}/4)mgR$. U početnom trenutku $t_o=0$ sistem je mirovao, $x(0)=0$, teret B je bio u podnožju tj. $\xi(0)=0$,. Za date generalisane koordinate x, ξ odrediti: 1) kinetičku energiju i rad sila sistema, 2) diferencijalne jednačine kretanja i ubrzanje prizme A.

DVA STEPENA SLOBODE KRETANJA: $q_1 = x$, $q_2 = \xi$ $\qquad \dot{\xi} = R\dot{\varphi}$

$$T = T_{prizme} + T_B \qquad T_{prizm} = \frac{1}{2}(2m)v_T^2 = m\dot{x}^2 \qquad T_B = \frac{1}{2}mV_B^2$$

APSOLUTNA BRZINA B MOŽE SE ODREDITI (VEKTORSKI) ILI KOORDINATNO/ANALITIČKI:

$$B\begin{cases} x_B = x + \xi\cos\alpha \\ y_B = \xi\sin\alpha \end{cases} \quad . \quad \begin{cases} \dot{x}_B = \dot{x} + \dot{\xi}\cos\alpha \\ \dot{y}_B = \dot{\xi}\sin\alpha \end{cases} \qquad V_B^2 = \dot{x}^2 + \dot{\xi}^2 + 2\dot{x}\dot{\xi}\cos\alpha \qquad (\alpha = 60^\circ)$$

$$T = \frac{1}{2}m\left(\dot{x}^2 + \dot{\xi}^2 + 2\dot{x}\dot{\xi}\frac{1}{2}\right) + m\dot{x}^2 \Rightarrow$$

$$T = \frac{3}{2}m\dot{x}^2 + \frac{1}{2}m\dot{\xi}^2 + \frac{1}{2}m\dot{x}\dot{\xi}$$

$$A = A^s + \cancel{A^u_0} = A(m_B\vec{g}) + \cancel{A(2m\vec{g})}^{0} + \cancel{A(\vec{N})}^{0} + \underbrace{A(M)}_{SMd\varphi} = -mg\cancel{\sin\alpha}^{\frac{\sqrt{3}}{2}}\xi + \frac{\sqrt{3}}{4}mgR\left(\frac{\xi}{R}\right)$$

$$A = -\frac{\sqrt{3}}{4}mg\xi$$

$$T - \cancel{T_0}^{0} = A^s + \cancel{A^u_0} \qquad \frac{3}{2}m\dot{x}^2 + \frac{1}{2}m\dot{\xi}^2 + \frac{1}{2}m\dot{x}\dot{\xi} = -\frac{\sqrt{3}}{4}mg\xi \ \Big/ \frac{d}{dt} \Rightarrow$$

$$3m\dot{x}\ddot{x} + m\dot{\xi}\ddot{\xi} + \frac{1}{2}m\ddot{x}\dot{\xi} + \frac{1}{2}m\dot{x}\ddot{\xi} = -\frac{\sqrt{3}}{4}mg\dot{\xi}$$

GRUPIŠIMO ČLANOVE DIFERENCIJALNE FORME PO NEZAVISNIM BRZINAMA "$\dot{x}$", "$\dot{\xi}$" $\Rightarrow$

$$\left(3m\ddot{x} + \frac{1}{2}m\ddot{\xi}\right)\dot{x} + \left(m\ddot{\xi} + \frac{1}{2}m\ddot{x} + \frac{\sqrt{3}}{4}mg\right)\dot{\xi} = 0 \qquad \Rightarrow$$

$$\begin{cases} 3m\ddot{x} + \frac{1}{2}m\ddot{\xi} = 0 \\ m\ddot{\xi} + \frac{1}{2}m\ddot{x} = -\frac{\sqrt{3}}{4}mg \end{cases} \Bigg| \Rightarrow \qquad \ddot{x} = \frac{\sqrt{3}}{22}g$$

$\boxed{\text{NAPOMENA}}$: ODREDITI BRZINU (APSOLUTNU) TAČKE B VEKTORSKI:

$$\vec{V}_{B\,aps} = \vec{V}_{B\,prenosno} + \vec{V}_{B\,relativno}$$