$m\vec a = \sum_i \vec F_i$

$\vec a = \vec a_p + \vec a_r + \vec a_{cor}$

$m(\vec a_p + \vec a_r + \vec a_{cor}) = \sum_i \vec F_i$

$m\vec a_r = \sum_i \vec F_i + (-m\vec a_p) + (-m\vec a_{cor})$

$\vec F_p^{in} = -m\vec a_p$ – ПРЕНОСНА ИНЕРЦИЈАЛНА СИЛА

$\vec F_{cor}^{in} = -m\vec a_{cor}$ – КОРИОЛИСОВА ИНЕРЦИЈАЛНА СИЛА

$m\vec a_r = \sum \vec F_i + \vec F_p^{in} + \vec F_{cor}^{in}$ – ДИФ. Ј-НА РЕЛАТИВНОГ КРЕТАЊА

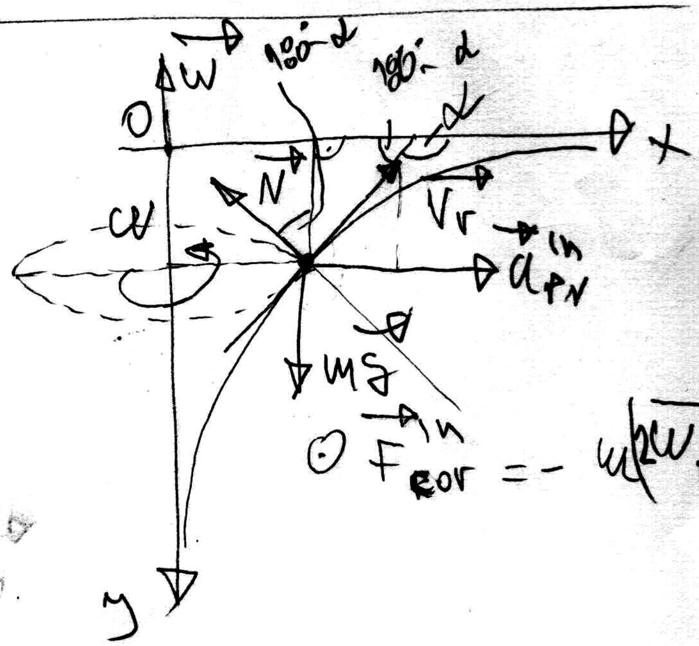
$\sum \vec F_i + \vec F_p^{in} = \vec 0$ $(\vec V_r = \vec 0)$ – РЕЛАТИВНО РАВНОТЕЖА

6.3 m

$x y = a^2$

$\omega = const.$

$x - y - N - ?$



$\odot\ \vec F_{cor}^{in} = -m(2\vec\omega_p \times \vec V_r)$

$m\vec g + \vec N + \vec F_p^{in} = \vec 0$

$\vec F_p^{in} = -m\vec a_p = -m(\vec a_{pN} + \underbrace{\vec a_{pT}}_{0})$ $\quad \omega = const.$

$\vec F_p^{in} = -m\vec a_{pN} = +m x \omega^2 \vec i$, $V_r = 0$

$$m\vec{g} + \vec{N} + \vec{F}_{ph}^{\,in} = \vec{0} \quad |\cdot\vec{i} \quad |\cdot\vec{j}$$

$$x:\ -N\sin(180^\circ - \alpha) + xm\omega^2 = 0 \quad (1)$$

$$-N\sin\alpha + xm\omega^2 = 0 \quad (1)$$

$$y:\ mg - N\cos(180^\circ - \alpha) = 0 \quad (2)$$

$$mg + N\cos\alpha = 0 \quad (2)$$

$$(1) \Rightarrow N\sin\alpha = xm\omega^2$$

$$(2) \Rightarrow N\cos\alpha = -mg$$

$$\frac{N\sin\alpha}{N\cos\alpha} = -\frac{x\omega^2}{g} \Rightarrow \boxed{\operatorname{tg}\alpha = -\frac{x\omega^2}{g}}$$

$$\operatorname{tg}\alpha = y' = -\frac{a^2}{x^2}$$

$$-\frac{a^2}{x^2} = -\frac{x\omega^2}{g} \Rightarrow x^3 = \frac{ga^2}{\omega^2}$$

$$x = \sqrt[3]{\frac{a^2 g}{\omega^2}}$$

$$\boxed{y} = \frac{a^2}{x} = \frac{a^2}{\sqrt[3]{\frac{a^2 g}{\omega^2}}} = a^2\sqrt[3]{\frac{\omega^2}{a^2 g}} = \boxed{a\sqrt[3]{\frac{a\omega^2}{g}}}$$

$$\cos\alpha = \frac{-1}{\sqrt{1+y'^2}} = -\frac{1}{\sqrt{1+\frac{a^4}{x^4}}} = \frac{-x^2}{\sqrt{x^4+a^4}} = -\frac{\left(\frac{a^2 g}{\omega^2}\right)^{2/3}}{\sqrt{\left(\frac{a^2 g}{\omega^2}\right)^{4/3}+a^4}}$$

$$N = -\frac{mg}{\cos\alpha} = +mg\,\frac{\sqrt{\left(\frac{a^2 g}{w^2}\right)^{4/3} + a^4}}{\left(\frac{a^2 g}{w^2}\right)^{2/3}} =$$

$$= mg\sqrt{\frac{\left(\frac{a^2 g}{w^2}\right)^{4/3} + a^4}{\left(\frac{a^2 g}{w^2}\right)^{4/3}}} = mg\sqrt{1 + a^4\left(\frac{w^2}{a^2 g}\right)^{4/3}} =$$

$$= mg\sqrt{1 + \left(\frac{a w^2}{g}\right)^{4/3}} = m\sqrt{g^2 + w^2 a\sqrt[3]{a w^2 g^2}}$$

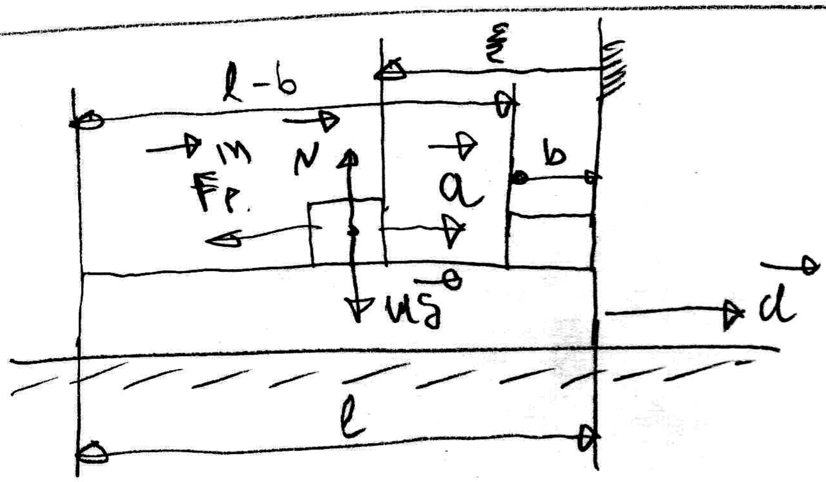
6.5 l

a

b

$V_{r0} = 0$

$t_1 - ?$



$$\vec{F}_p^{in} = -m\vec{a}$$

$$m\vec{a}_r = \sum_i \vec{F}_i^s + \vec{F}_p^{in} + \vec{F}_{cor}^{in}, \quad w_p = 0, \quad a_{cor} = 0$$

$$\cancel{m}\ddot{\xi} = \cancel{m}a$$

$$\ddot{\xi} = a \Rightarrow \dot{\xi} = \dot{\xi}_0 + at = \underbrace{V_{r0}}_{=0} + at = at$$

$$\xi = \underbrace{\xi_0}_{=0} + \frac{at^2}{2} = \frac{at^2}{2}, \quad \text{y } t = t_1 \quad \xi_1 = l - b$$

$$t_1 = \sqrt{\frac{2(l-b)}{a}}$$

6.7 l

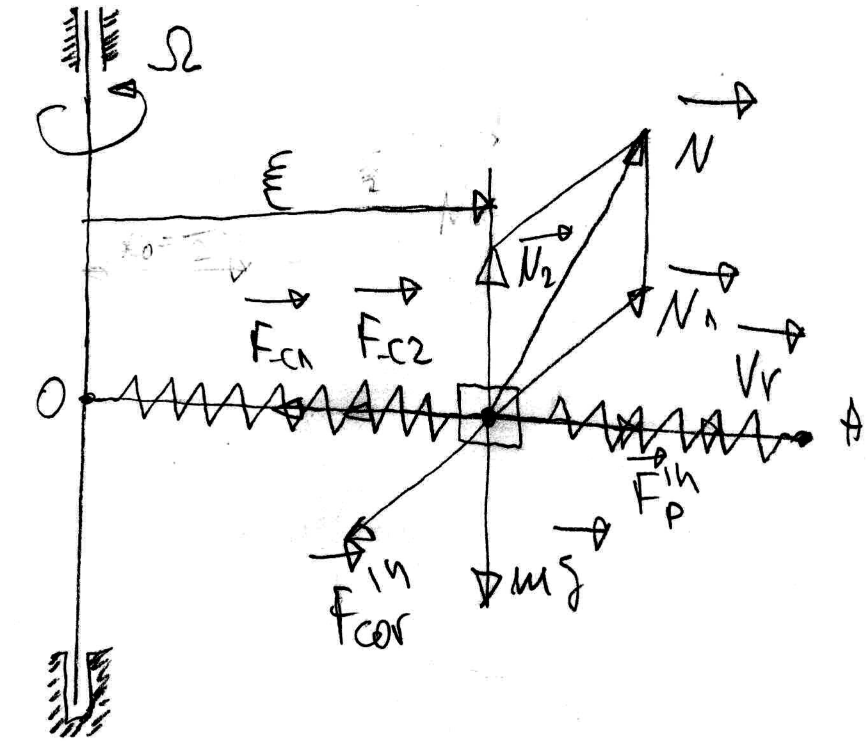
$\Omega = const.$

m

$l_0 = \frac{l}{2}$

c

$x(t) - ?$



$$m\vec{a}_r = \vec{F}_{c1} + \vec{F}_{c2} + m\vec{g} + \vec{N}_1 + \vec{N}_2 + \vec{F}_p^{in} + \vec{F}_{cor}^{in}$$

$$\xi: \quad m\ddot{\xi} = -c(\xi - l_0) - c(\xi - l_0) + m\Omega^2\xi \quad |:m$$

$$\ddot{\xi} + \left(2\frac{c}{m} - \Omega^2\right)\xi = 2cl_0\frac{1}{m}, \quad l_0 = \frac{l}{2}$$

$$\ddot{\xi} + \left(\frac{2c - m\Omega^2}{m}\right)\xi = \frac{cl}{m}, \quad w^2 = \frac{2c - m\Omega^2}{m}$$

$$\ddot{\xi} + w^2\xi = \frac{cl}{m}, \quad 2c > m\Omega^2, \quad c > \frac{m\Omega^2}{2}$$

$$\xi = \xi_h + \xi_p$$

$$\xi = C_1\cos wt + C_2\sin wt + \frac{cl}{mw^2}$$

$$-C = \frac{m\Omega^2}{2} \Rightarrow \xi = \frac{-Cl}{2m}t^2 + A_1 t + A_2$$

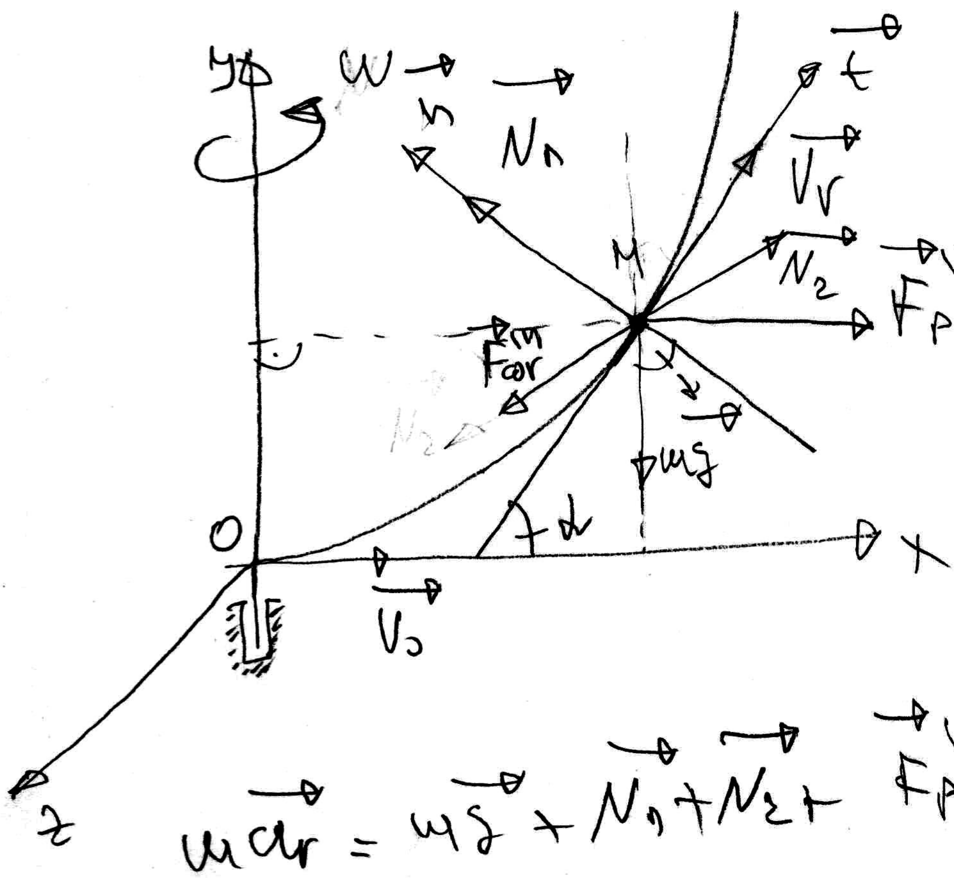
$$-C < \frac{m\Omega^2}{2}, \quad \frac{2-C-m\Omega^2}{m} = -k^2, \quad x = B_1 e^{wt} + B_2 e^{-wt} - \frac{Cl}{mk^2}$$

6.10 $y = \frac{x^2}{2a}$

$$\omega = \sqrt{\frac{g}{a}} = const.$$

m, $t_0 = 0$

$M_0(0,0)$, $V_0 = \sqrt{ag}$



$$m\vec{a}_r = m\vec{g} + \vec{N}_1 + \vec{N}_2 + \vec{F}_p^{in} + \vec{F}_{cor}^{in}$$

$$T - T_0 = \sum_i A(\vec{F}_i^e) + A(\vec{F}_p^{in})$$

$$\frac{1}{2}mV_r^2 - \frac{1}{2}mV_0^2 = -mgy + m\omega^2\int_0^x x\,dx \quad \Big|\cdot\frac{2}{m}$$

$$V_r^2 = V_0^2 - 2gy + \cancel{2}\omega^2 \frac{1}{\cancel{2}} x^2$$

$$V_r^2 = ag - \cancel{2}g\,\cancel{\frac{x^2}{2a}} + \cancel{\frac{g}{a} x^2} = ag$$

$$V_r^2 = V_0^2 = ag$$

$$\boxed{V_r = \sqrt{ag}}$$

$$n: \quad \frac{m}{R_K} V_r^2 = -mg\cos\alpha + N_n + F_{in}^{u} \sin\alpha$$

$$y' = \frac{x}{a}, \quad y'' = \frac{1}{a}$$

$$\frac{1}{R_K} = \frac{|y''|}{(1+y'^2)^{3/2}} = \frac{\frac{1}{a}}{\left(1+\frac{x^2}{a^2}\right)^{3/2}} = \frac{a^2}{(a^2+x^2)^{3/2}}$$

$$\cos\alpha = \frac{1}{\sqrt{1+y'^2}} = \frac{a}{(a^2+x^2)^{1/2}}$$

$$\sin\alpha = \frac{y'}{\sqrt{1+y'^2}} = \frac{x}{(a^2+x^2)^{1/2}}$$

$$N_n = \frac{m a^2 \cdot ag}{(a^2+x^2)^{3/2}} + mg\frac{a}{(a^2+x^2)^{1/2}} - mx\omega^2 \frac{x}{(a^2+x^2)^{1/2}}$$

$$N_1 = \frac{mg}{a} \frac{a^4 + (x^2+a^2)^2}{(x^2+a^2)^{3/2}}$$

b: $0 = F_{cor}^{in} - N_2$

$$N_2 = F_{cor}^{in} = 2m\,\omega V_r \sin\angle(\vec{\omega}, \vec{V_r}) =$$

$$= 2m \sqrt{\frac{g}{a}} \sqrt{ag} \sin(90° - \alpha) =$$

$$= 2mg\cos\alpha = 2mg \frac{a}{(a^2+x^2)^{1/2}}$$

$$N = \sqrt{N_1^2 + N_2^2}$$

6.12 $\alpha = 60°$

ω

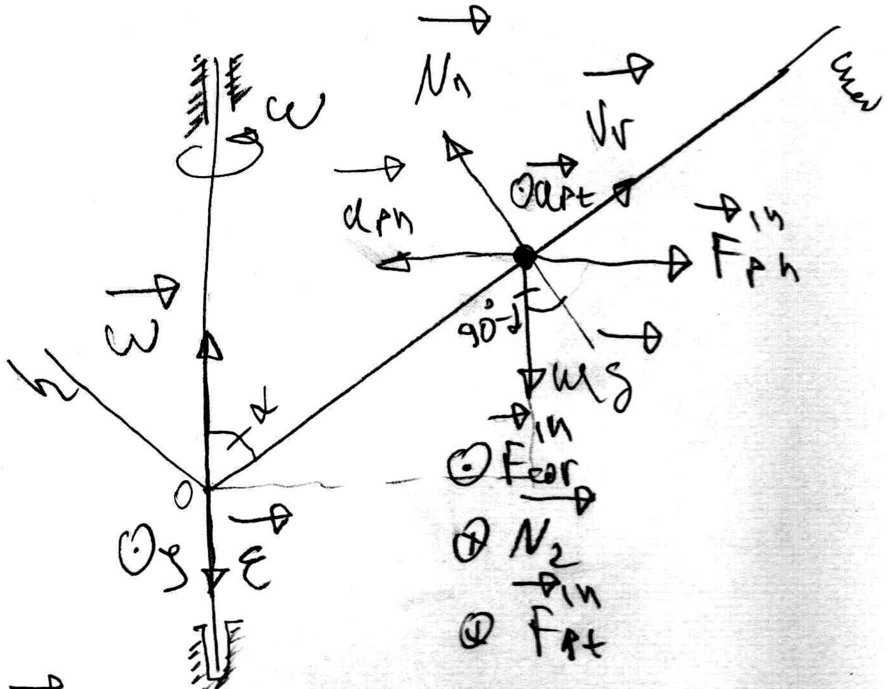
m

$t_0 = 0$

$\overline{OM_1} = a$

$V_r = V_0 = const.$

ω - ?



$$m\vec{a_r} = m\vec{g} + \vec{N_1} + \vec{N_2} + \vec{F_p^{in}} + \vec{F_{cor}^{in}}$$

$$\xi: m\ddot{\xi} = -mg\cos\alpha + m\omega^2 \sin\alpha \sin\alpha \, \xi$$

$$\dot{\xi} = V_0 \quad , \quad \ddot{\xi} = 0$$

$$\xi = \xi_0 + V_0 t = a + V_0 t$$

$$0 = -\cancel{m}g\cos\alpha + \cancel{m}\omega^2 \sin\alpha \sin\alpha \, \xi$$

$$\cos\alpha \, g = \omega^2 \xi \sin^2\alpha$$

$$\omega^2 = \frac{\cos\alpha \, g}{\xi \sin^2\alpha} = \frac{g \frac{1}{2}}{(a+V_0t)\frac{3}{4}} = \frac{2g}{3(a+V_0t)}$$

$$\omega = \sqrt{\frac{2g}{3(a+V_0t)}}$$

$$\varepsilon = \frac{d\omega}{dt} = \sqrt{\frac{2g}{3}} \frac{d}{dt}\left(\frac{1}{\sqrt{a+V_0t}}\right) =$$

$$= \sqrt{\frac{2g}{3}} \frac{-\frac{V_0}{2\sqrt{a+V_0t}}}{a+V_0t} = -\sqrt{\frac{2g}{3}} \frac{V_0}{2(a+V_0t)^{3/2}}$$

$$\eta: 0 = -mg\sin\alpha + N_a - m\omega^2 \sin\alpha\cos\alpha \, \xi$$

$$\boxed{N_a} = mg\sin\alpha + m\omega^2 \xi \sin\alpha\cos\alpha =$$

$$= mg\frac{\sqrt{3}}{2} + m\frac{\cancel{2}g}{3\cancel{\xi}}\cancel{\xi}\frac{\sqrt{3}}{\cancel{2}}\frac{1}{2} = mg\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6}\right) = \boxed{\frac{2\sqrt{3}}{3}mg}$$

$\xi: \quad 0 = -N_2 + 2\omega \dot{\xi} m \sin\alpha + m \ddot{\xi} \sin\alpha \quad \mathcal{E} \quad 0$

$$N_2 = \cancel{2} m \sqrt{\frac{2g}{\cancel{3}(a+V_0t)}}\, V_0 \frac{\cancel{\sqrt{3}}}{\cancel{2}} - m(a+V_0t)\frac{\cancel{\sqrt{3}}}{2}\sqrt{\frac{2g}{\cancel{3}}}\frac{V_0}{2(a+V_0t)^{3/2}}$$

$$\boxed{N_2} = mV_0\sqrt{\frac{2g}{a+V_0t}} - \frac{V_0 m}{4(a+V_0t)^{1/2}}\sqrt{2g} =$$

$$= \frac{mV_0\sqrt{2g}}{\sqrt{a+V_0t}}\left(1 - \frac{1}{4}\right) = \boxed{\frac{3\sqrt{2g}}{4}\frac{mV_0}{\sqrt{a+V_0t}}}$$

$$N = \sqrt{N_1^2 + N_2^2} = m\sqrt{\frac{4}{3}g^2 + \frac{9gV_0^2}{8(a+V_0t)}}$$

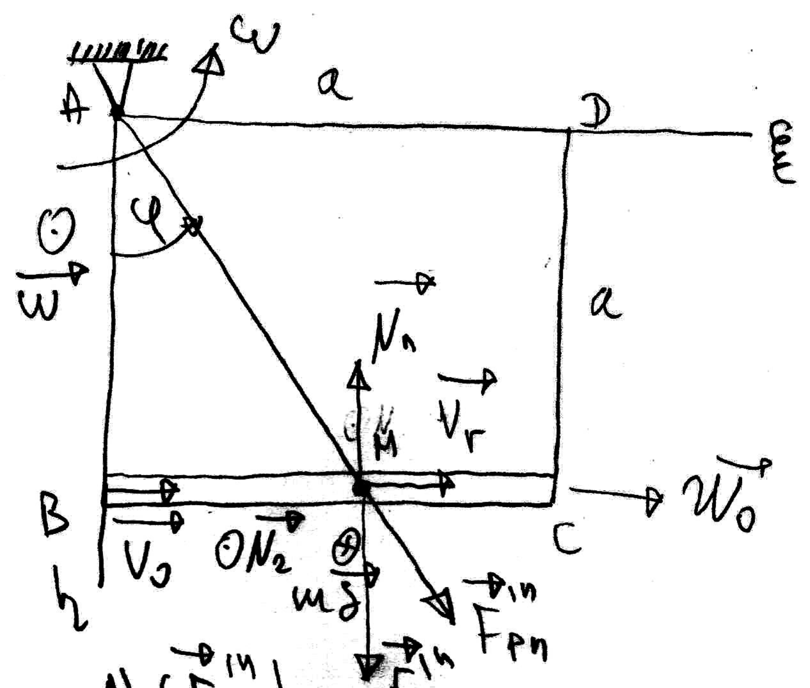
6.15 $\omega = const.$

a

V_0

$V_1 = 2V_0$

$\omega - ?$



$$T_C - T_B = \sum_{i=1}^{n} A(\vec{F}_i^{\,s}) + A(\vec{F}_{pn}^{in})$$

$$\frac{m}{2}(V_C^2 - V_0^2) = A(\vec{F}_{pn}^{in}) \quad (*)$$

$$d'A(\vec{F}_{pn}^{in}) = \vec{F}_{pn}^{in} \cdot \tilde{d}\vec{s}_M = \vec{F}_{pn}^{in} \cdot \vec{V}_r\, dt =$$

$$= \vec{F}_{pn}^{in} \cdot \vec{V}_r\, dt + \vec{F}_{pn}^{in} \cdot \vec{V}_p\, dt = \left| \vec{F}_{pn}^{in} \cdot \vec{V}_p = 0,\ \angle(\vec{F}_{pn}^{in}, \vec{V}_p) = \pi/2 \right|$$

$$= \vec{F}_{pn}^{in} \cdot (\vec{V}_p + \vec{V}_r)\, dt = \vec{F}_{pn}^{in} \cdot \vec{V}\, dt = \vec{F}_{pn}^{in} \frac{d\vec{r}}{dt}\, dt =$$

$$= \vec{F}_{pn}^{in} \cdot d\vec{r} = m\omega^2 \vec{r} \cdot d\vec{r} = \left| \begin{array}{l} \vec{r} \cdot \vec{r} = r^2 \ | \cdot d \\ 2\vec{r} \cdot d\vec{r} = 2r\,dr \\ \vec{r} \cdot d\vec{r} = r\,dr \end{array} \right| =$$

$$= m\omega^2 r\,dr = d'A(\vec{F}_{pn}^{in}) \ | \int$$

$$A(\vec{F}_{pn}^{in}) = m\int_{r_0}^{r_1} \omega^2 r\,dr = |\omega = const| = m\omega^2 \frac{1}{2} r^2 \Big|_{r_B}^{r_C}$$

$$= \frac{1}{2} m\omega^2 (r_C^2 - r_B^2) = \frac{1}{2} m\omega^2 (2a^2 - a^2) = \frac{m\omega^2 a^2}{2}$$

(*)

$$\frac{\cancel{m}}{\cancel{2}}\left(4V_0^2 - V_0^2\right) = \frac{\cancel{m}\omega^2 a^2}{\cancel{2}}$$

$$3V_0^2 = a^2\omega^2 \Rightarrow \boxed{\omega = \sqrt{\frac{3V_0^2}{a^2}} = \sqrt{3}\,\frac{V_0}{a}}$$

$$d'A(\vec{F}^{in}_{pn}) = \vec{F}^{in}_{pn} \cdot d\tilde{\vec{r}}_M = m\omega^2 \vec{r} \cdot d\xi \vec{\lambda} =$$

$$= m\omega^2 \left(\xi\vec{\lambda} + h\vec{\mu}\right) \cdot d\xi \vec{\lambda} = m\omega^2 \xi\, d\xi$$

$$A(\vec{F}^{in}_{pn}) = m\omega^2 \int_0^a \xi\, d\xi = \frac{1}{2} m\omega^2 \xi^2 \Big|_0^a = \frac{m\omega^2}{2} a^2$$

$$\xi^2 + a^2 = r^2 \,/\, \cdot d$$

$$\cancel{2}\xi\, d\xi = \cancel{2} r\, dr$$

$$\boxed{\xi\, d\xi = r\, dr}$$

6.18 $\omega = \text{const.}$

m

$t_0 = 0, \quad V_{r0} = l\omega$

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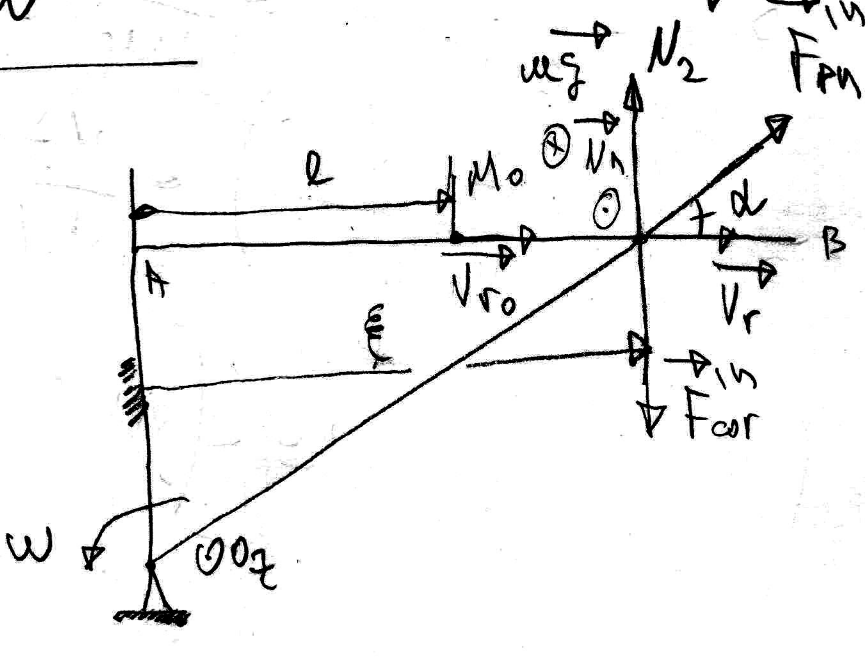
a) $t_1 - ?$

b) $N_1 - ?$



$$m\ddot{\xi} = F_{pn}^{in}\cos\alpha = m\omega^2 \cancel{OM}\,\frac{AM}{\cancel{OM}} = m\omega^2\xi \quad /:m$$

$$\ddot{\xi} - \omega^2\xi = 0$$

$$\xi = A_1 e^{\omega t} + A_2 e^{-\omega t}$$

$$\dot{\xi} = A_1\omega e^{\omega t} - A_2\omega e^{-\omega t}$$

$$t_0 = 0,\ \xi_0 = l,\ \dot{\xi}_0 = l\omega$$

$$l = A_1 + A_2$$

$$l\omega = A_1\omega - A_2\omega$$

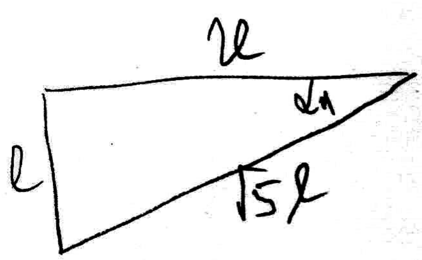
$$A_1 = l,\quad A_2 = 0$$

$$\xi = l e^{\omega t},\quad \dot{\xi} = l\omega e^{\omega t}$$

$$\xi_1 = 2l = l e^{\omega t_1} \Rightarrow \ln 2 = \omega t_1 \Rightarrow \boxed{t_1 = \frac{1}{\omega}\ln 2}$$

b) $F_{cor1}^{in} = 2m V_{r1}\omega\cdot 1$

$$\dot{\xi}_1 = l\omega e^{\omega t_1} = 2l\omega = V_{r1}$$



$$F_{cor1}^{in} = 4ml\omega^2$$

$$F_{pn1}^{in} = ml^2 l\sqrt{5}$$

$$N_2 = F_{cor1}^{in} - F_{pn1}^{in}\sin\alpha_1 = ml\omega^2(4-1) = 3ml\omega^2$$