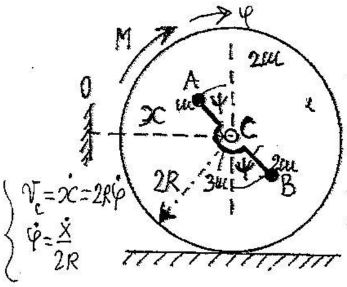
Sistem je u vertikalnoj ravni, čine ga disk, štap AB i dve materijalne tačke zavarene za krajeve štapa. Veza između centra diska i centra štapa (u tački C) je zglobna. Disk je poluprečnika 2R, mase 2m i kotrlja se bez klizanja po horizontalnoj ravni; štap AB je mase 3m, AB=2R; tačka A je mase m, tačka B je mase 2m. Na disk dejstvuje spreg sila momenta M. Za date generalisane koordinate x, ψ odrediti: 1) diferencijalne jednačine kretanja, 2) kako treba da se menja moment sprega sila M, u funkciji ugla ψ, da bi disk imao konstantnu ugaonu brzinu?



$\left\{ v_C = \dot{x} = 2R\dot{\varphi},\quad \dot{\varphi} = \frac{\dot{x}}{2R} \right.$

$\overline{AB} = 2R$, $\overline{AC} = \overline{CB}$

$\left\{ q_1 = x,\quad q_2 = \psi \right.$

$T = T_{dis} + T_{AB} + T_A + T_B$

$T_{dis} = \frac{1}{2} m_{dis} v_C^2 + \frac{1}{2} J_{Cz} \omega_{dis}^2$ (RAVNO KRET.) $\quad J_{Cz} = \frac{1}{2}(2m\,4R^2)$

$T_{dis} = \frac{1}{2}(2m)\dot{x}^2 + \frac{1}{2}(4mR^2)\left(\frac{\dot{x}}{2R}\right)^2 = \frac{3}{2} m \dot{x}^2$

$T_{AB} = \frac{1}{2} m_{AB} v_C^2 + \frac{1}{2} J_{Cz} \omega_{AB}^2$ (RAVNO KRET.) $\quad J_{Cz} = \frac{1}{12}(3m\,4R^2)$

$T_{AB} = \frac{1}{2}(3m)\dot{x}^2 + \frac{1}{2}(mR^2)\dot{\psi}^2$

$T_A = \frac{1}{2} m_A v_A^2$ (TAČKA) $\quad T_B = \frac{1}{2} m_B v_B^2$ (TAČKA) $\quad \vec{v}_A, \vec{v}_B$ NAĆI APSOLUTNE BRZINE.

$A \begin{cases} x_A = x - R\sin\psi \\ y_A = 2R + R\cos\psi \end{cases} \quad \begin{cases} \dot{x}_A = \dot{x} - R\dot{\psi}\cos\psi \\ \dot{y}_A = -R\dot{\psi}\sin\psi \end{cases} \quad v_A^2 = \dot{x}^2 + R^2\dot{\psi}^2 - 2R\dot{x}\dot{\psi}\cos\psi$

$B \begin{cases} x_B = x + R\sin\psi \\ y_B = 2R - R\cos\psi \end{cases} \quad \begin{cases} \dot{x}_B = \dot{x} + R\dot{\psi}\cos\psi \\ \dot{y}_B = R\dot{\psi}\sin\psi \end{cases} \quad v_B^2 = \dot{x}^2 + R^2\dot{\psi}^2 + 2R\dot{x}\dot{\psi}\cos\psi$

$T_A = \frac{1}{2} m\left(\dot{x}^2 + R^2\dot{\psi}^2 - 2R\dot{x}\dot{\psi}\cos\psi\right) \quad T_B = \frac{1}{2}(2m)\left[\dot{x}^2 + R^2\dot{\psi}^2 + 2R\dot{x}\dot{\psi}\cos\psi\right]$

$$T = \frac{1}{2} m\left(9\dot{x}^2 + 4R^2\dot{\psi}^2 + 2R\dot{x}\dot{\psi}\cos\psi\right)$$

$\frac{\partial T}{\partial \dot{x}} = 9m\dot{x} + mR\dot{\psi}\cos\psi$

$\frac{\partial T}{\partial \dot{\psi}} = 4mR^2\dot{\psi} + mR\dot{x}\cos\psi$

$\frac{\partial T}{\partial \psi} = -mR\dot{x}\dot{\psi}\sin\psi$

$\delta A = m_A\vec{g}\cdot\delta\vec{r}_A + m_B\vec{g}\cdot\delta\vec{r}_B + M\delta\varphi \quad \delta A = -mg\,\delta(2R + R\cos\psi) - 2mg\,\delta(2R - R\cos\psi) + M\left(\frac{\delta x}{2R}\right)$

$\delta A = -mgR\sin\psi\,\delta\psi + \frac{M}{2R}\delta x \Rightarrow Q_x = \left[\frac{\delta}{\delta x}(A)\right]_{\delta\psi=0} = \frac{M}{2R}; \quad Q_\psi = \left[\frac{\delta}{\delta\psi}(A)\right]_{\delta x=0} = -mgR\sin\psi$

$$\begin{aligned} &(1)\ \frac{d}{dt}\frac{\partial T}{\partial \dot{x}} - \cancel{\frac{\partial T}{\partial x}}^{\,0} = Q_x \Rightarrow \quad 9m\ddot{x} + mR\left(\ddot{\psi}\cos\psi - \dot{\psi}^2\sin\psi\right) = \frac{M}{2R} \\ &(2)\ \frac{d}{dt}\frac{\partial T}{\partial \dot{\psi}} - \frac{\partial T}{\partial \psi} = Q_\psi \Rightarrow \quad 4mR^2\ddot{\psi} + mR\ddot{x}\cos\psi = -mgR\sin\psi \end{aligned}$$

$\omega_{dis} = \omega_0 = \text{const} \Rightarrow \dot{x} = 2R\omega_0 \,/\frac{d}{dt} \Rightarrow \ddot{x} = 0 \quad (1) \Rightarrow \quad M = 2mR^2\left(\ddot{\psi}\cos\psi - \dot{\psi}^2\sin\psi\right)$

$(2)\ \ddot{\psi} = -\frac{g}{4R}\sin\psi \Rightarrow \int\dot{\psi}\,d\dot{\psi} = -\int\frac{g}{4R}\sin\psi\,d\psi \quad \dot{\psi}^2 = \dot{\psi}^2_{(0)} - \frac{g}{2R}(1 - \cos\psi)$

$$M = 2mR^2\left[\ddot{\psi}(\psi)\cos\psi - \dot{\psi}^2(\psi)\sin\psi\right]$$