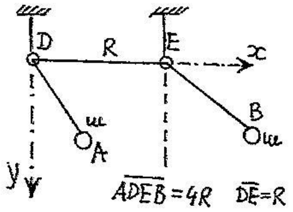
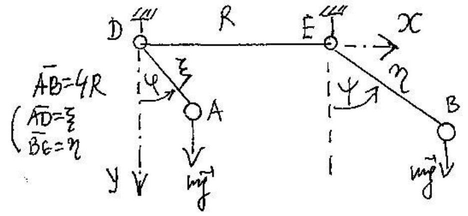


Ispitni zadatak ① Sistem je u vertikalnoj ravni; osa oy inercijalnog sistema 0xy je vertikalna. Dve materijalne tačke A i B, svaka je mase m, povezane su neistegljivim užetom (zanemarljive mase) dužine 4R koje prolazi kroz nepomične glatke prstenove D i E; DE=R. Odrediti diferencijalne jednačine kretanja primenom vektorskih teorema dinamike.
(Ovde ćemo ga rešavati analitičkim metodama.)



$$T=\frac{1}{2}mV_A^2+\frac{1}{2}mV_B^2 \qquad q_1=\xi;\ q_2=\varphi;\ q_3=\psi$$

(GENERALISANE KOORDINATE)

$$T(\dot{x}_A,\dot{y}_A,\dot{x}_B,\dot{y}_B) \longrightarrow T(\dot{\xi},\dot{\varphi},\dot{\psi},\xi,\varphi,\psi)$$

KINEMATSKA VEZA: $\xi+R+\eta=4R$

T.j. $\eta=3R-\xi$

$\dot{\eta}=-\dot{\xi} \qquad \ddot{\eta}=-\ddot{\xi}$

$$\begin{cases} x_A=\xi\sin\varphi & \dot{x}_A=\dot{\xi}\sin\varphi+\xi\dot{\varphi}\cos\varphi \\ y_A=\xi\cos\varphi & \dot{y}_A=\dot{\xi}\cos\varphi-\xi\dot{\varphi}\sin\varphi \end{cases} \qquad V_A^2=\dot{x}_A^2+\dot{y}_A^2$$

$$V_A^2=\dot{\xi}^2+\xi^2\dot{\varphi}^2$$

$$\begin{cases} x_B=\eta\sin\psi=(3R-\xi)\sin\psi \\ y_B=\eta\cos\psi=\underbrace{(3R-\xi)}_{\text{OSLOBAĐAMO SE ZAVISNE KOORDINATE }\eta}\cos\psi \end{cases}$$

$$\begin{cases} \dot{x}_B=-\dot{\xi}\sin\psi+(3R-\xi)\dot{\psi}\cos\psi \\ \dot{y}_B=-\dot{\xi}\cos\psi-(3R-\xi)\dot{\psi}\sin\psi \end{cases} \qquad V_B^2=\dot{x}_B^2+\dot{y}_B^2=\dot{\xi}^2+(3R-\xi)^2\dot{\psi}^2$$

$$\boxed{T=\frac{1}{2}m\left(\dot{\xi}^2+\xi^2\dot{\varphi}^2+(3R-\xi)^2\dot{\psi}^2\right)}$$

$$\delta A=m\vec{g}\cdot\delta\vec{r}_A+m\vec{g}\cdot\delta\vec{r}_B \qquad \delta A=mg\delta(\xi\cos\varphi)+mg\delta[(3R-\xi)\cos\psi]$$

$$\delta A=\underbrace{mg\cos\varphi\delta\xi-mg\xi\sin\varphi\delta\varphi}_{A}\ \underbrace{-mg\cos\psi\delta\xi+(3R-\xi)(mg)(-\sin\psi\delta\psi)}_{B}$$

$$\delta A=mg(\cos\varphi-\cos\psi)\delta\xi-(mg\xi\sin\varphi)\delta\varphi-mg(3R-\xi)\sin\psi\delta\psi$$

$$\begin{cases} Q_\xi=mg(\cos\varphi-\cos\psi) \\ Q_\varphi=-mg\xi\sin\varphi \\ Q_\psi=-mg(3R-\xi)\sin\psi \end{cases}$$

* * *

(1) $\frac{d}{dt}\frac{\partial T}{\partial\dot{\xi}}-\frac{\partial T}{\partial\xi}=Q_\xi \quad\Rightarrow\quad 2\ddot{\xi}-\xi\dot{\varphi}^2+(3R-\xi)\dot{\psi}^2=g(\cos\varphi-\cos\psi)$

(2) $\frac{d}{dt}\frac{\partial T}{\partial\dot{\varphi}}-\frac{\partial T}{\partial\varphi}=Q_\varphi \quad\Rightarrow\quad \xi\ddot{\varphi}+2\dot{\xi}\dot{\varphi}=-g\sin\varphi$

(3) $\frac{d}{dt}\frac{\partial T}{\partial\dot{\psi}}-\frac{\partial T}{\partial\psi}=Q_\psi \quad\Rightarrow\quad (3R-\xi)\ddot{\psi}-2\dot{\xi}\dot{\psi}=-g\sin\psi$

(IZRAČUNATI)
T.J.
(PROVERITI)